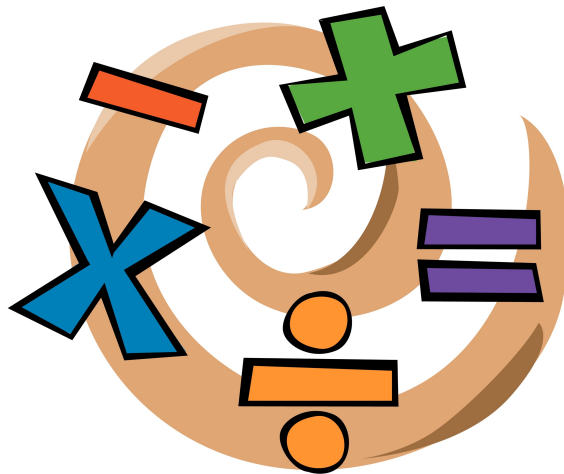




UNIVERSITY INTERSCHOLASTIC LEAGUE

# Mathematics

Invitational A • 2021



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1. Solve for k:  $k - 20 \div (3 + 1) \times 2 + 0! = 22$

- (A) 25                      (B) 26                      (C) 28                      (D) 30                      (E) 31

2. Phil Detank travels 25 miles each way driving to and from work every day, Monday through Friday. His vehicle has a 15 gallon tank and averages 20 mpg. The tank is full when he heads to work on Monday. How much will it cost him to fill his tank when he gets home from work on Friday if gas costs \$2.35 a gallon? (nearest cent)

- (A) \$11.75                      (B) \$14.69                      (C) \$28.13                      (D) \$29.38                      (E) \$35.25

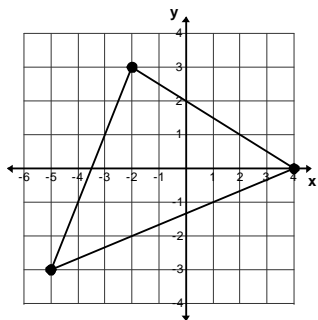
3. If  $P = \{2,3,5,7,11\}$ ,  $T = \{1,3,6,10,15\}$ ,  $F = \{1,2,3,5,8\}$ , and  $L = \{1,2,3,4,7\}$ , then  $(P \cup T) \cap (F \cup L)$  contains how many elements?

- (A) 1                      (B) 4                      (C) 5                      (D) 6                      (E) 10

4. A line with a slope of  $-\frac{5}{6}$  intersects the x-axis at  $x = 4$  and intersects the y-axis at  $y = ?$

- (A)  $2\frac{1}{2}$                       (B)  $3\frac{1}{6}$                       (C)  $3\frac{1}{3}$                       (D)  $3\frac{2}{3}$                       (E)  $4\frac{5}{6}$

5. The triangle shown is considered to be which of the following types of triangles?

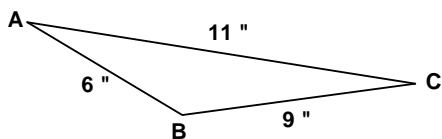


- (A)cute                      (O)btuse                      (R)ight                      (E)quilateral                      (S)calene                      (I)sosceles  
 (A) O & I                      (B) O & S                      (C) A & S                      (D) R & S                      (E) R & I

6. Papa Jawn is three times as old as Dom Knowles and Lil Seizer is 3 years younger than Dom. The sum of their ages 3 years ago was 63 years. How long from now will Papa's age be equal to the sum of the ages of Lil and Dom?

- (A) 12 yrs                      (B) 15 yrs                      (C) 18 yrs                      (D) 27 yrs                      (E) 45 yrs

7. Find the area of  $\triangle ABC$ . (nearest tenth)



- (A)  $26.0 \text{ in}^2$                       (B)  $27.0 \text{ in}^2$                       (C)  $24.8 \text{ in}^2$                       (D)  $22.4 \text{ in}^2$                       (E)  $21.2 \text{ in}^2$

8. If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is a(n) \_\_\_\_\_ triangle.

- (A) obtuse      (B) scalene      (C) acute      (D) isosceles      (E) right

9. If  $-2, -1,$  and  $3$  are the roots of  $2x^3 + bx^2 + cx + d = 0$ , then  $b + c + d$  equals?

- (A)  $-26$       (B)  $-13$       (C)  $12$       (D)  $13$       (E)  $14$

10. Sir Cal Puhl is pouring a rectangular concrete patio to put his circular hot tub on. The diameter of the hot tub is 10 feet. The dimensions of the patio is 14 feet by 18 feet. What percent of the area of the patio is covered by the tub? (nearest whole percent)

- (A) 8%      (B) 12%      (C) 19%      (D) 21%      (E) 31%

11. If  $\frac{2x-1}{x-2} + \frac{3x+2}{x+1} = \frac{ax^2+bx+c}{dx^2+ex+f}$ , then  $(a + b + c) \div (d + e + f)$  equals:

- (A)  $-2.5$       (B)  $-0.5$       (C)  $1.5$       (D)  $0.75$       (E)  $2$

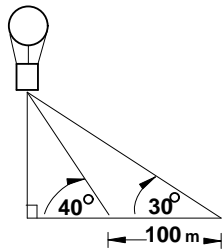
12. Let  $a^4 \div b^3 \times a^{-5} \times b^6 \div (a^1)^7 \times b^2 = a^m \times b^n$ . Find  $m \times n$ .

- (A)  $-1$       (B)  $-9$       (C)  $-40$       (D)  $-60$       (E)  $-88$

13.  $\triangle ABC$  is an isosceles triangle, where segment  $\overline{BX}$  is the median to the base  $\overline{AC}$ . Find  $AB$  if  $AC = 8$  in and  $BX = 4$  in. (nearest tenth)

- (A) 8.9"      (B) 3.5"      (C) 6"      (D) 5.7"      (E) 4"

14. Three cables from an observation balloon are attached to the level ground below as show below. How high is the observation balloon? (nearest meter)



- (A) 221 m      (B) 160 m      (C) 120 m      (D) 321 m      (E) 185 m

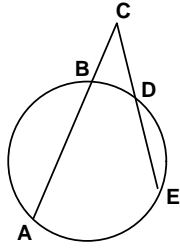
15. Find the focus of the graph of  $y^2 + 6y + 4x + 25 = 0$

- (A)  $(-1, -3)$       (B)  $(-2, -3)$       (C)  $(-3, -3)$       (D)  $(-4, -3)$       (E)  $(-5, -3)$

16. Which type of conic is the graph of the equation  $x + 7y^2 + 3y + 5 = 0$ ?

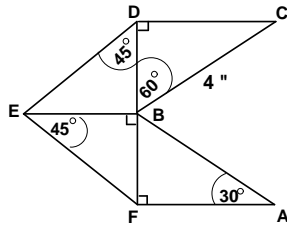
- (A) circle      (B) ellipse      (C) hyperbola      (D) parabola      (E) not a conic

17. Given:  $AB = 8$  cm,  $ED = x$  cm,  $BC = 3$  cm, and  $DC = x - 1$  cm. Find  $CE$ . (nearest cm)



- (A) 9 cm      (B) 8 cm      (C) 7 cm      (D) 6 cm      (E) 5 cm
18. The average rate of change of  $f(x) = x^2 + bx + 1$  from  $x = 2$  to  $x = 8$  is 15. Find the value of  $b$ .
- (A) 3      (B) 3.75      (C) 5      (D) 6      (E) 7.5
19. Which of the following statements is/are true for  $f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ -x^2 & \text{if } 0 < x \end{cases}$  ?
- I.  $f$  is continuous at  $x = 0$     II.  $f'_-(0)$  exists    III.  $f'_+(0)$  exists    IV.  $f$  is differentiable at  $x = 0$
- (A) I only      (B) I & IV      (C) I, II, & III    (D) All of them    (E) none of them
20. I. C. Yew wants to find the probability that his class officers consisting of a president, treasurer, and secretary are randomly selected such that one of them has glasses, one has contacts, and one has no eyewear. How many members will be in the successful event's sample space?
- (A) 3      (B) 6      (C) 8      (D) 9      (E) 27
21. Which of the following mathematicians developed formulas to calculate the sum of the first  $n$  integers, their squares and their cubes?
- (A) Diophantus    (B) Agnesi      (C) Aryabhata    (D) Eratosthenes    (E) Bigollo
22.  $(1^3 + 2^3 + 3^3 + 4^3 + 5^3 + \dots + 20^3) \div (1 + 2 + 3 + 4 + 5 + \dots + 20) = ?$
- (A) 55      (B) 210      (C) 400      (D) 1,100      (E) 2,870
23. The operation  $\odot$  is defined as  $x \odot y = x^y - y^x$ , where  $x, y$  are integers. Find the value of  $(1 \odot 2) \odot (2 \odot 1)$ .
- (A)  $-2$       (B)  $-1$       (C)  $0$       (D)  $\frac{1}{2}$       (E)  $2$
24. Sluggy Snail crawled 2.5 feet in 45 seconds. How many inches will Sluggy crawl in 3 minutes?
- (A) 3,240 in    (B) 337.5 in    (C) 200 in      (D) 120 in      (E) 37.5 in
25. Let  $(ax^2 - 5x + 1)(x + d) = 4x^3 + bx^2 + cx - 3$ . Find  $a + b - c + d$ .
- (A)  $-32$       (B)  $-26$       (C)  $-8$       (D)  $0$       (E)  $34$

26. Find the perimeter of the hexagon ABCDEF. (nearest tenth)



- (A) 17.7 in      (B) 24.7 in      (C) 18.9 in      (D) 26.6 in      (E) 20.6 in

27. If  $a_1 = 2$ ,  $a_2 = -1$ ,  $a_3 = -3$  and  $a_n = (a_{n-1})(a_{n-3}) - (a_{n-2})$ , where  $n \geq 4$ , then  $a_6$  equals:

- (A) -29      (B) -19      (C) -13      (D) -5      (E) 8

28. Linda has a collection of green and red marbles. The ratio of greens to reds is 7 to 5 and the number of greens exceeds the number of reds by 48. How many red marbles does Linda have?

- (A) 120      (B) 132      (C) 144      (D) 156      (E) 168

29. Given: Points P, Q, R, and S lie on a circle with center O. Point T lies outside of the circle such that point R lies on segment TQ and point S lies on segment TP. Which of the following is true about  $\angle RSP$ ?

- I.  $m\angle RSP = m\angle QPT + m\angle QTP$       II.  $\angle RSP$  and  $\angle PQT$  are supplementary  
 III.  $\angle RSP$  is a right angle      IV.  $m\angle RSP = m\angle RST + m\angle SRT$

- (A) I only      (B) II & IV      (C) I & III      (D) I & II      (E) III & IV

30. Find the domain of  $f(x) = \frac{\sqrt{3x}}{|3x| - 2}$ .

- (A)  $[0, \infty)$       (B)  $[0, \frac{2}{3}) \cup (\frac{2}{3}, \infty)$       (C)  $[0, 1) \cup (1, \infty)$   
 (D)  $(0, \frac{2}{3}) \cup (\frac{2}{3}, \infty)$       (E)  $(-\infty, \infty)$

31. Three ships left a tiny island at the same time. The *Friend* ship left on a bearing of  $320^\circ$  for 200 km then dropped anchor. The *Scholar* ship left on a bearing of  $210^\circ$  for 200 km and dropped anchor. The *Citizen* ship left on a bearing of  $70^\circ$  for 200 km and dropped anchor. The next day the *Citizen* sent out an SOS. How much closer to the *Citizen* was the closest ship than the other ship? (nearest km)

- (A) 45 km      (B) 48 km      (C) 58 km      (D) 65 km      (E) 93 km

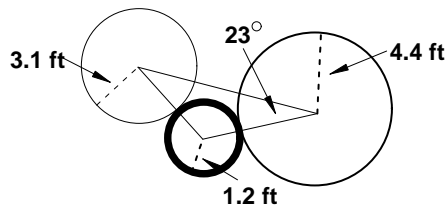
32. The polar equation  $r = 2 \cos \theta$  written in rectangular coordinate form is:

- (A)  $x^2 - y^2 = 2x$       (B)  $y - x = 2$       (C)  $x^2 + y^2 = 2y$       (D)  $x^2 + y^2 = 2x$       (E)  $x + y = 2$

33. How many four-letter arrangements of the letters CAPSULE having three consonants and one vowel can be formed if no letter is repeated?

- (A) 24      (B) 72      (C) 96      (D) 288      (E) 840

- 34 Three gears needed to run a particular machine need to be arranged and fit the specifications shown below. Find the measure of the central angle of the smallest gear.



- (A)  $126^\circ$       (B)  $112^\circ$       (C)  $140^\circ$       (D)  $123^\circ$       (E)  $131^\circ$
35. Let  $X$  represent the number of people in a randomly selected household in Statville. Use the chart data to find the mean of the discrete random variable  $X$ .

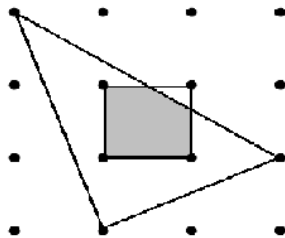
Size of Households in Statville, USA

Number of People	1	2	3	4	5	6
Probability	.18	.36	.21	.15	.07	.03

- (A) 2.62      (B) 2.64      (C) 2.66      (D) 2.68      (E) 2.70
36. Mei Telfone has a cell phone with a 7 digit number. The first and the last digits are 2's. The sum of any three consecutive digits is 9. What is the median digit?

- (A) 2      (B) 3      (C) 4      (D) 5      (E) 6

37. The dots below are one unit apart vertically and horizontally. Find the area of the shaded region. (square units)



- (A) 0.888...      (B) 0.900      (C) 0.91666...      (D) 0.9333...      (E) .9375
38. Jack went up a hill to fetch a 5-gallon pail of water. He drank a pint of water on his way back down. He used three gallons to water his bean stalk and two quarts to make some curds and whey. He gave Jill four cups of water to boil some eggs. How much water was left in the 5-gallon pail?

- (A) 2 gals 1 qt 1 pt      (B) 1 gal 1 pt      (C) 1 qt 1 pt      (D) 1 gal      (E) 2 qts 1 pt

39. Let  $f(x) = -3x^3 + 2x^2 - x + 4$ . Which of the following is true about the end behavior of the graph?
- I. As  $x$  gets larger,  $f(x)$  gets smaller      II. As  $x$  gets larger,  $f(x)$  gets larger  
 III. As  $x$  gets smaller,  $f(x)$  gets larger      IV. As  $x$  gets smaller,  $f(x)$  gets smaller

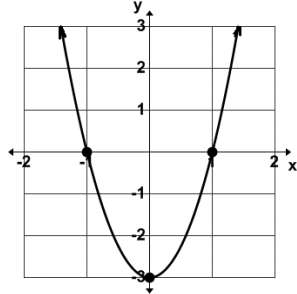
- (A) I only      (B) II & IV      (C) I & III      (D) III only      (E) I & IV

40. Moe Tell and Fran Tick are dishwashers at the *Hot Plate* diner. Moe washes 10 dishes per minute and Fran washes 14 dishes per minute. Moe starts washing the 800 dirty dishes twenty minutes before Fran starts. How many of the 800 dishes did Fran wash?
- (A) 330            (B) 350            (C) 380            (D) 430            (E) 450
41. Point A(2, 4) lies on the x-y plane. A is reflected across the y-axis to point B. Then B is reflected across the line  $y = -x$  to point C. Then C is translated horizontally 5 units to point D. Then D is translated vertically  $-4$  units to point E. Then E is rotated  $90^\circ$  clockwise about the origin to point F(x, y). Find  $x + y$ .
- (A)  $-6$             (B)  $-3$             (C)  $-1$             (D) 0            (E) 2
42. If Lotta Cache invested  $x$  dollars at a 4% simple interest rate and  $y$  dollars at a 3.5% simple interest rate, she would get \$185.00 in interest in a year. However, if she invested  $x$  dollars at a 3.5% simple interest rate and  $y$  dollars at a 4% simple interest rate, she would get \$199.00 in interest in a year. What is the total amount of her investment?
- (A) \$5,120.00    (B) \$5,000.00    (C) \$3,960.00    (D) \$3,840.00    (E) \$3,480.00
43. Max Kutter has a piece of cardboard that is 2 feet long and 1 foot wide. He cuts a square out of each corner in order to create a cardboard box that has maximum volume. What is the area of the base of the cardboard box Max created? (nearest tenth)
- (A) 131.1 sq. in    (B) 132.9 sq. in    (C) 134.5 sq. in    (D) 138.2 sq. in    (E) 150.2 sq. in
44. Let  $(a, b)$  be a point on the graph of  $y = 1 + 2\cos 3x$ . Which of the following is a point on the graph of  $y = -1 + 2\cos 3x$ ?
- (A)  $(a, b - 2)$     (B)  $(a, b + 2)$     (C)  $(a - 2, b)$     (D)  $(a + 2, b)$     (E)  $(a + 2, b - 2)$
45. When Doe gets to select one bill from a box containing bills as follows:
- | <u>Denomination of Bill</u> | <u>Number of Bills in the Box</u> |
|-----------------------------|-----------------------------------|
| 1                           | 16                                |
| 5                           | 10                                |
| 10                          | 8                                 |
| 20                          | 6                                 |
- Based on the information, find the mathematically expected value.
- (A) \$10.00            (B) \$2.66            (C) \$16.07            (D) \$9.00            (E) \$6.65
46. Which of the following are considered to be *impolite* numbers?
- I. 16    II. 48    III. 64    IV. 8,192
- (A) IV only            (B) II only            (C) I, III, & IV            (D) I, II, & III            (E) II & IV

47. A couple of years ago 80 young men and 40 young ladies representing Class AA took the state number sense test. Of the 120 students, 20% of the boys used pencils and 40% of the girls used pencils. If one of the students was chosen at random, what is the probability that the student used a pencil or is a young lady? (nearest whole percent)

- (A) 9%            (B) 30%            (C) 45%            (D) 47%            (E) 60%

48. The graph of  $h'(x)$  is shown below. Find  $h(10)$ , when  $h(1) = 2$ .



- (A) 791            (B) 803            (C) 970            (D) 974            (E) 997

49. Let  $(x, y)$  be a solution to the equation  $3(x - y) = xy + 1$ , where  $x$  and  $y$  are positive digits. Find the sum of all such  $x$  and  $y$  digits.

- (A) 12            (B) 9            (C) 8            (D) 7            (E) 5

50.  $[(31F_{16}) + (A2B_{16})] \div 7_{16} = \underline{\hspace{2cm}}_{16} \cdot$

- (A) 19E            (B) 1E6            (C) 196            (D) D16            (E) D4A

51. The sequence  $1 - \frac{5^2}{2!} + \frac{5^4}{4!} - \frac{5^6}{6!} + \frac{5^8}{8!} + \dots$  simplifies to the decimal number 0.ABCDEFG..., where the letters represent digits. What digit does letter G represent?

- (A) 0            (B) 1            (C) 2            (D) 5            (E) 6

52. The graph of the function  $y = 5x + \sin(x) - 2$ , where  $-\pi \leq x \leq \pi$ , lies in which quadrants?

- (A) II & IV            (B) I & III            (C) I & IV            (D) II, III, & IV            (E) I, III, & IV

53. The graph of  $f(x) = \frac{8x-3}{\sqrt{5x^2+1}}$  has horizontal asymptotes  $y = \frac{a}{b}$  and  $y = \frac{-a}{b}$ .

Find the value of  $\left| \frac{a}{b} \right|$  to four decimal places.

- (A) 3.5333            (B) 3.5555            (C) 3.5777            (D) 3.5999            (E) 3.6111

54. If  $53^{(45)} \cong x \pmod{12}$ , then  $x = ?$

- (A) 1            (B) 3            (C) 5            (D) 8            (E) 9

55. Which of the points lie on the line that is tangent to the curve  $x^3 + y^2 = 2$  at point  $(1, 1)$ ?

- (A)  $(-9, 11)$             (B)  $(-20, -10.5)$             (C)  $(-15, 20)$             (D)  $(4, -3)$             (E)  $(10, -12.5)$



56. Dee Grader has 5 different pens with black ink and 3 different pens with red ink. How many different sets of 5 pens can she make such that at least 3 of them are black ink pens?
- (A) 46            (B) 75            (C) 56            (D) 13            (E) 30
57. Betty Duzett is taking this math test. If the odds of scoring 200 or higher on this test is  $\frac{3}{5}$ , what is the probability that Betty will score less than 200 on this test?
- (A) 37.5%        (B) 40%        (C) 60%        (D) 62.5%        (E) 80%
58. Let  $f_1 = 2, f_2 = 7, f_3 = 9, f_4 = 16, \dots$  be the terms of a Fibonacci characteristic sequence. If the ratio of  $f_n$  to  $f_{n-1}$  is the first ratio to equal the Golden Ratio rounded to the thousandth place, then  $f_n - f_{n-1} = ?$
- (A) 66            (B) 107            (C) 162            (D) 173            (E) 280
59. The positive even numbers are arranged as shown. What is the sum of the numbers in row 8?
- |  |    |    |     |     |         |
|--|----|----|-----|-----|---------|
|  |    |    | 2   |     | (row 1) |
|  |    | 4  | 6   |     | (row 2) |
|  | 8  | 10 | 12  |     | (row 3) |
|  | 14 | 16 | ... | ... | (row 4) |
- (A) 350            (B) 536            (C) 738            (D) 720            (E) 520
60. The base of a pyramid is a regular hexagon with each side equal to 24 cm. If the altitude of the pyramid is 42 cm, find the total surface area of the pyramid. (nearest  $\text{cm}^2$ )
- (A)  $4820 \text{ cm}^2$     (B)  $4837 \text{ cm}^2$     (C)  $4854 \text{ cm}^2$     (D)  $4871 \text{ cm}^2$     (E)  $4888 \text{ cm}^2$

**DO NOT DISTRIBUTE TO STUDENTS BEFORE OR DURING THE CONTEST**

**University Interscholastic League  
MATHEMATICS CONTEST  
HS • Invitation A • 2021  
Answer Key**

- |       |       |       |
|-------|-------|-------|
| 1. E  | 21. C | 41. B |
| 2. D  | 22. B | 42. A |
| 3. C  | 23. A | 43. A |
| 4. C  | 24. D | 44. A |
| 5. E  | 25. A | 45. E |
| 6. C  | 26. E | 46. C |
| 7. B  | 27. B | 47. D |
| 8. C  | 28. A | 48. D |
| 9. A  | 29. D | 49. A |
| 10. E | 30. B | 50. B |
| 11. C | 31. B | 51. B |
| 12. C | 32. D | 52. E |
| 13. D | 33. D | 53. C |
| 14. E | 34. A | 54. C |
| 15. E | 35. C | 55. E |
| 16. D | 36. A | 56. A |
| 17. A | 37. C | 57. D |
| 18. C | 38. B | 58. B |
| 19. D | 39. C | 59. E |
| 20. B | 40. B | 60. D |