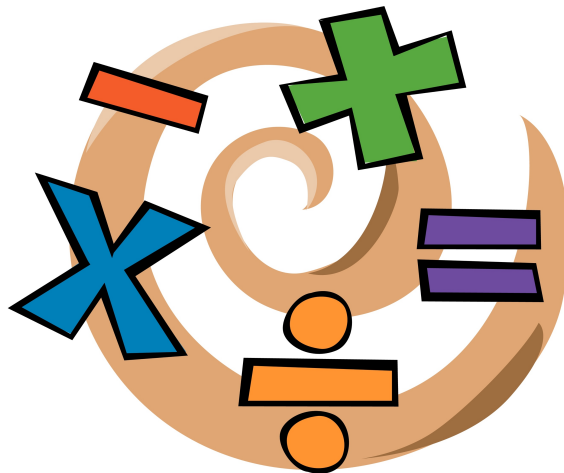




UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational B • 2021



DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $[(1 + 1)^2]! \div 3 - 5 \times 8 + 13$

- (A) -20 (B) -19 (C) 29 (D) 37 (E) 63

2. Willie Prawfette bought four sets of used golf clubs at a garage sale for \$75.00 each. He sold one set at his garage sale for \$120.00, a second set for \$85.00, a third set for \$145.00, and the fourth set for \$55.00. What was his percent profit?

- (A) 65% (B) 60% (C) 50% (D) 45% (E) 35%

3. Three-fourths is to two-thirds as one-half is to what?

- (A) $\frac{1}{3}$ (B) $\frac{4}{9}$ (C) $\frac{5}{6}$ (D) 1 (E) $1\frac{1}{2}$

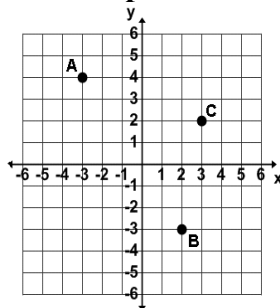
4. Forty percent of the Mustangs were added to twenty percent of the Plainsmen and the total was 33. Twice the number of Mustangs was fifteen less than three times the number of Plainsmen. How many were Plainsmen?

- (A) 40 (B) 45 (C) 50 (D) 55 (E) 60

5. 150 students registered for Mr. White's problem writing camp. 85 students signed up for the math sessions. 70 signed up for the number sense sessions. 50 signed up for both the math and number sense sessions. How many students signed up for neither math nor number sense?

- (A) 20 (B) 35 (C) 45 (D) 50 (E) 55

6. Points A, B, and C are plotted below. Find point D such that $\overline{AB} \perp \overline{CD}$ and D is in quadrant III.



- (A) $(-4, -3)$ (B) $(-3, -2)$ (C) $(-2, -4)$ (D) $(-3, -4)$ (E) $(-2, -3)$

7. Simplify: $\frac{4x^2 - 25}{x + 1} \div \frac{6x^2 + 13x - 5}{3x^2 + 2x - 1}$

- (A) $2x - 5$ (B) $3x - 5$ (C) $\frac{2x - 5}{3x - 1}$ (D) $\frac{1}{2x + 5}$ (E) $2x + 5$

8. Let $(4x - 1)(ax + b)(cx - 1) = 8x^3 + dx^2 + ex + 3$, where a, b, c, d, and e are integers and $a > c$. Find $a + b + c + d + e$.

- (A) -16 (B) -13 (C) -5 (D) 0 (E) 1

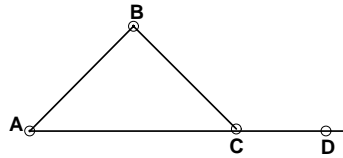
9. Two trains, the *Clickety* and the *Clack*, left the home depot at the same time going in opposite directions. The *Clickety* reached the next depot in 1.5 hours traveling at an average speed of 50 mph. The *Clack* reached the next depot in 2 hours traveling at an average speed of 60 mph. How far apart were the two depots?

(A) 385 miles (B) 190 miles (C) 295 miles (D) 195 miles (E) 115 miles

10. Line m and line n are skew. Point A is on line m . How many lines containing point A can be drawn perpendicular to line n ?

(A) none (B) 1 (C) 2 (D) 3 (E) infinite

11. Given: $AB = CD = 12$, $AD = 36$ and $\angle ABC = 90^\circ$. Find $\angle BCD$.



(A) 120° (B) 130° (C) 140° (D) 150° (E) 160°

12. Paulie Gawn drew an isosceles trapezoid with base lengths of 10 feet and 5 feet and a height of 3 feet. If she increased the smaller base 1 foot and decreased the longer base 2 feet, how much would the height have to be for the area of the second trapezoid to be 80% of the area of the original trapezoid? (nearest half inch)

(A) 2' 9" (B) 2' 8" (C) 2' 7.5" (D) 2' 7" (E) 2' 6.5"

13. Let $(a^3 \div b)^2 \times a^{-1} \times b^4 \div (a^{-1}) \times b^0 = a^m \times b^n$. Find $(m)^n$.

(A) 8 (B) 9 (C) 12 (D) 36 (E) 64

14. Find the domain of $f(x) = \frac{\sqrt{12-2x}}{2x^2 + 13x + 21}$.

(A) $x \in \mathbb{R} \mid x \leq 6$ (B) $x \in \mathbb{R} \mid x \leq 6, x \neq -3.5$ (C) $x \in \mathbb{R} \mid x \leq 6, x \neq -3$
 (D) $x \in \mathbb{R} \mid x < 6, x \neq -3.5, -3$ (E) $x \in \mathbb{R} \mid x \leq 6, x \neq -3.5, -3$

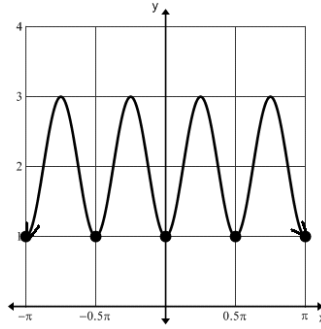
15. Mr. White likes to take a daily walk at 3 miles per hour. Mrs. White picks him up in their golf cart at the end of his walk and returns home over the same route at 10 miles per hour. The time allotted for the entire trip is 2.5 hours. How far does Mr. White walk? (nearest tenth)

(A) 2.5 miles (B) 1.2 miles (C) 5.8 miles (D) 4.5 miles (E) 3.6 miles

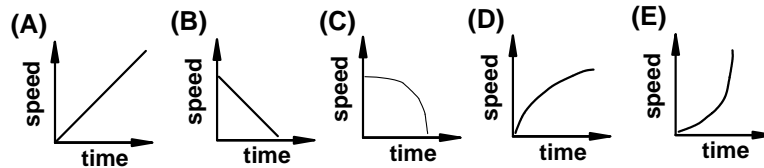
16. Chip Chawt has three white golf balls and two yellow ones. He wants to arrange them in random order and find the probability of having only two of the same colored balls next to each other given that a yellow ball is first or last. How many elements are in the total sample space?

(A) 5 (B) 7 (C) 8 (D) 12 (E) 24

17. Given the graph shown, what would the equation of this graph become if the amplitude is increased by 2, the phase shift is not changed, the period is doubled, and the displacement is cut in half. $y = ?$

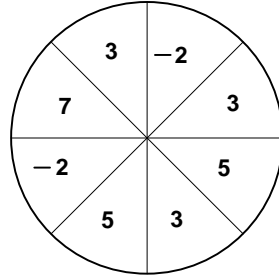


- (A) $1 + 3\sin(2x - \frac{\pi}{4})$ (B) $4 - \frac{1}{2}\sin(8x - \frac{\pi}{2})$ (C) $1 + 2\sin(\pi x - \frac{\pi}{4})$
 (D) $2 + \sin(2x - \frac{\pi}{2})$ (E) $1 + 3\sin(x - \frac{\pi}{4})$
18. The remainder when the polynomial $2x^3 - 5x^2 + kx + 6$ is divided by $x - 4$ is 98. Find k .
- (A) 4 (B) 11 (C) 38 (D) 100 (E) 300
19. $[(22A_{14}) - (BC_{14})] \times 7_{14} = \underline{\hspace{2cm}} 14 \cdot$
- (A) 1,064 (B) 757 (C) 1,848 (D) 132 (E) 960
20. Ms. Daisy is driving to the market. She is waiting for the red light to turn green. When the light turns green she accelerates. Which graph best shows her speed versus time as she reaches the speed limit?

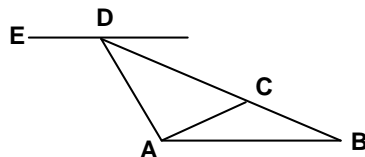


21. Let $f(x) = x^2 - 4x + 3$ be continuous on $[1, 3]$ and differentiable on $(1, 3)$. Find the equation of the line tangent to the graph of $f(x)$ when the slope of the tangent line is 0.
- (A) $f(x) = 2$ (B) $f(x) = 0$ (C) $f(x) = -1$ (D) $f(x) = 2x$ (E) $f(x) = 2x - 4$
22. If $f''(x) = 24x + 6$, $f'(-1) = 4$, and $f(1) = 6$, find $f(-2)$.
- (A) -15 (B) -16 (C) -23 (D) -37 (E) -42
23. Which of the following mathematicians is known as the "father of geometry"?
- (A) Ptolemy (B) Archimedes (C) Rene Descartes (D) Leonard Euler (E) Euclid of Alexandria
24. How many positive digits in base 10 are considered to be "unhappy odious" numbers?
- (A) 5 (B) 4 (C) 3 (D) 2 (E) 1

25. Willie When spins the wheel. The wheel consists of eight congruent sectors as shown. What is the mathematical expectation of any one spin?



- (A) 3 (B) 2.2 (C) 2.25 (D) 2 (E) 2.75
26. Tai Bowe needs ribbons to wrap her presents. Each ribbon needs to be 5 feet 4 inches in length. How many ribbons of that length can she get from a 50-yard spool of ribbon?
- (A) 28 (B) 29 (C) 30 (D) 31 (E) 32
27. How many 4-digit even numbers between 2,500 and 5,000 can be created using the digits 2, 1, 3, 4, and 7? A digit cannot be used more than once in any of the 4-digit numbers.
- (A) 54 (B) 50 (C) 36 (D) 20 (E) 12
28. Which of the following equations has no real roots?
- (A) $x^2 + 4x - 5 = 0$ (B) $4x^2 + 12x + 9 = 0$ (C) $-2x^2 + 6x + 8 = 0$
(D) $4x^2 + 6x + 1 = 0$ (E) $2x^2 + 8x + 15 = 0$
29. Given: $m\angle CBA = 30^\circ$, $m\angle ADE = 110^\circ$, $ED \parallel AB$, and $BC = AC$. Find $m\angle CAD$.



- (A) 30° (B) 40° (C) 60° (D) 80° (E) 110°
30. The roots of the equation $8x^2 - kx - 15 = 0$ are -1.25 and R . Find k .
- (A) -2 (B) -0.75 (C) 1.5 (D) 2 (E) 2.75
31. $\sin(\frac{\pi}{6}) + \sin(\frac{\pi}{3}) + \sin(\frac{\pi}{2}) + \sin(\frac{2\pi}{3}) + \dots + \sin(\frac{7\pi}{3}) + \sin(\frac{5\pi}{2}) = ?$ (nearest hundredth)
- (A) 2.37 (B) 2.5 (C) 2.73 (D) 2.87 (E) 3
32. Given the function $f(x) = 5\sin(x) - 1$, find the slope of the secant line between $x = \frac{\pi}{2}$ and $x = \pi$.
- (A) $-\frac{10}{\pi}$ (B) $-\frac{5}{\pi}$ (C) $-\frac{1}{\pi}$ (D) zero slope (E) no slope


33. Find the remainder when 31^{30} is divided by 29.
- (A) 1 (B) 2 (C) 4 (D) 9 (E) 16
34. Let $f_1 = 3, f_2 = 8, f_3 = 11, f_4 = 19, \dots, f_{12} = 877, \dots$ be the terms of a Fibonacci characteristic sequence. Find f_{20} .
- (A) 25,463 (B) 66,663 (C) 107,863 (D) 94,797 (E) 41,200
35. The *Bait Yor Hook* fishing pond has bass, crappie, catfish, perch, gar, and carp. The limit of fish you can keep is five and at least one of them has to be a perch. How many different stringers of fish could you keep?
- (A) 252 (B) 120 (C) 210 (D) 126 (E) 600
36. The figures below are made up of little squares. The side length of each little square is 1 cm. If the pattern of the shapes continues what will the perimeter of figure 18 be?
- 

fig. 1




fig. 2

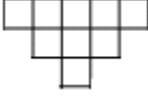


fig. 3

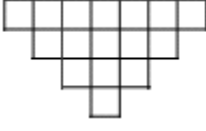


fig. 4
- (A) 108 (B) 106 (C) 104 (D) 102 (E) 100
37. The top surface of a square board is painted red. A circle is inscribed on the board and its area is painted blue. What per cent of the square board remained red? (nearest tenth)
- (A) 31.4% (B) 78.5% (C) 12.6% (D) 21.5% (E) 68.6%
38. A regular polygon of n sides exists such that its central angle is 24° and each side length is 4". Find the perimeter of the polygon. (nearest inch)
- (A) 4' 0" (B) 4' 4" (C) 4' 6" (D) 4' 8" (E) 5' 0"
39. The *Wildflower* seed company's research data shows that the probability that a seed will germinate and grow into a plant is 85%. What are the odds that it won't germinate?
- (A) $\frac{17}{20}$ (B) $\frac{5}{8}$ (C) $\frac{3}{17}$ (D) $\frac{3}{20}$ (E) $\frac{5}{13}$
40. Mark Ex and Drew Oh are playing Tic-tac-toe. Mark has won 5 games and Drew has won 3 games. If they continue to play, what is the least number of games they will have to play in order for Mark to have a winning probability of 75%?
- (A) 7 (B) 6 (C) 5 (D) 4 (E) 3
41. If $a_1 = 2, a_2 = -3, a_3 = -1$ and $a_n = (a_{n-2})^{(a_{n-3})} - (a_{n-1})$, where $n \geq 4$, then a_6 equals:
- (A) 12.1 (B) 11.1 (C) 1 (D) -1 (E) -11

42. Which type of conic is the graph of the equation $Ax^2 + By^2 + Cy + D = 0$, where A, B, C, and D are integers and $A, C > 0$ and $B, D < 0$?
- (A) circle (B) ellipse (C) hyperbola (D) parabola (E) not a conic
43. Find the units digit of $37^{(35)} - 33$.
- (A) 0 (B) 2 (C) 4 (D) 6 (E) 8
44. If $f(x) = 4x - 1$, $g(x) = 5x + 2$, $h(x) = 3 - 6x$, and $g(h(f(3x + 4))) = ax + b$, then $a + b = ?$
- (A) - 840 (B) - 793 (C) - 773 (D) - 465 (E) - 384
45. Let R_1 and R_2 be the roots of $2x^2 - 3x + c = 0$, where the ratio of R_1 to R_2 is 5. Find c.
- (A) 0.15625 (B) 0.3125 (C) 0.625 (D) 1.25 (E) 2.5
46. Two circles, $(x - 4)^2 + (y + 5)^2 = 16$ and $x^2 + y^2 = 16$, intersect at two points. Find the slope of the line passing through the two points of intersection.
- (A) $\frac{4}{5}$ (B) $\frac{9}{16}$ (C) $\frac{1}{16}$ (D) $\frac{16}{25}$ (E) $1\frac{1}{4}$
47. If r, s, and t are real numbers such that $r + s + t = 20$, $t^2 = r^2 + s^2$, and $rs = 10$, find the value of t.
- (A) 8.5 (B) 9 (C) 9.5 (D) 10 (E) 10.5
48. Bill Durr is a surveyor working on a bridge that will span a small lake from point A to point B. The line of sight distances from him to point B and from him to point A are 456.2 ft. and 429.8 ft., respectively. He measures the angle between the two lines of sight to be 48.7° . How long did he compute the length of the bridge to be? (nearest foot).
- (A) 496 ft (B) 443 ft (C) 391 ft (D) 366 ft (E) 313 ft
49. Given: $8\cos^2(4A) + 10\sin(4A) - 11 = 0$, where $0 \leq A \leq 4$ (rad). Find the largest value for A, in degrees. (nearest degree)
- (A) 188° (B) 213° (C) 218° (D) 223° (E) 230°
50. The vertical asymptote and the oblique asymptote of $f(x) = \frac{2x^2 + 3x + 5}{x - 1}$ intersect at point (x, y). Find the value of y.
- (A) 10 (B) 7 (C) 4 (D) 3 (E) 1
51. Find the sum of the first ten terms of an arithmetic sequence if the fourth term is 4 and the thirteenth term is 34.
- (A) $88\frac{2}{3}$ (B) $89\frac{1}{3}$ (C) 90 (D) $90\frac{2}{3}$ (E) $91\frac{1}{3}$

52. Which of the following is closest to the length of the segment bounded by the parametric equations: $x = 3t$, $y = 5t - 1$, $0 \leq t \leq 1$.
- (A) 4.2 (B) 4.3 (C) 5.0 (D) 5.8 (E) 5.9
53. Find the area of the region bounded by the curve $8y = 5x^2 + 16$ and the lines $y = 0$, $x = 0$, $y = 12$, and $x = 6$.
- (A) $45\frac{1}{3}$ (B) 46 (C) $46\frac{2}{3}$ (D) $47\frac{1}{3}$ (E) 48
54. The *Brite Lite* company statistics shows that only 3% of the light bulbs shipped out are defective. If a shipment of 24 bulbs is randomly selected for testing what is the probability that exactly 2 of them are defective? (nearest percent)
- (A) 1% (B) 4% (C) 6% (D) 7% (E) 13%
55. The probability distribution of X , the number of hours Landon works on Thursdays is:

Hours (h)	$P(X=h)$
8	0.5
9	0.2
10	0.15
11	0.1
12	0.05

Find the variance of the probability distribution.

- (A) 1.5 (B) 1.65 (C) 1.8 (D) 1.95 (E) 2.1
56. Point $P(-1, 6)$ lies on the x - y plane. P is reflected across the line $x = 1$ to point Q . Point Q is rotated 90° counterclockwise about the point $(1, 0)$ to point R . Point R translated 4 units vertically and -2 units horizontally to point $S(x, y)$. Find the distance from point S to point P . (nearest tenth)
- (A) 4.5 (B) 6.0 (C) 7.0 (D) 7.5 (E) 9.2
57. Wendy Dae sailed her sloop from her pier traveling $S50^\circ W$ for 4 nautical miles. She turned and traveled $N40^\circ E$ for 4 nautical miles. Then the wind stopped and Wendy used the outboard motor to head straight back to the pier. What course did she need to take back to the pier?
- (A) $W40^\circ N$ (B) $S45^\circ E$ (C) $E50^\circ S$ (D) $N45^\circ W$ (E) $S35^\circ E$
58. $\triangle ABC$ is a right triangle with $\angle ABC$ being the right angle. Point D lies outside of $\triangle ABC$ such that $m\angle BCD = 90^\circ$, $m\angle BAC = 59^\circ$, and $\angle ACD$ is an obtuse angle. If $AB = 6''$ and $CD = 9''$, then $BD = ?$ (nearest tenth)
- (A) 9.8'' (B) 10.0'' (C) 10.8'' (D) 12.7'' (E) 13.4''

59. Points A and B lie on a circle with center C and a radius of 2.5". Point T lies outside of the circle such that segments AT and BT are tangent to the circle and $m\angle ACB = 130^\circ$. Find the area outside the circle and bounded by segments AT and BT. (nearest tenth)

- (A) 3.6 sq. in (B) 4.0 sq. in (C) 6.3 sq. in (D) 7.5 sq. in (E) 8.0 sq. in

60. Given that the set of natural numbers continue in the triangular pattern shown below, find the sum of the 3rd term and the 9th term in row 11.

			2				(row 1)					
			3		3		(row 2)					
		4		5		4	(row 3)					
		5		7		7		5	(row 4)			
	6		9		12		9		6	(row 5)		
	7		11		16		16		11		7	(row 6)
					...							(...)

- (A) 72 (B) 68 (C) 64 (D) 70 (E) 80

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**University Interscholastic League
MATHEMATICS CONTEST
HS • Invitation B • 2021
Answer Key**

- | | | |
|-------|-------|-------|
| 1. B | 21. C | 41. B |
| 2. E | 22. A | 42. C |
| 3. B | 23. E | 43. A |
| 4. B | 24. C | 44. B |
| 5. C | 25. E | 45. C |
| 6. A | 26. A | 46. A |
| 7. A | 27. D | 47. C |
| 8. C | 28. E | 48. D |
| 9. D | 29. D | 49. C |
| 10. B | 30. D | 50. B |
| 11. D | 31. A | 51. C |
| 12. D | 32. A | 52. D |
| 13. D | 33. C | 53. A |
| 14. E | 34. E | 54. E |
| 15. C | 35. D | 55. A |
| 16. B | 36. B | 56. B |
| 17. A | 37. D | 57. B |
| 18. B | 38. E | 58. E |
| 19. E | 39. C | 59. C |
| 20. D | 40. D | 60. A |