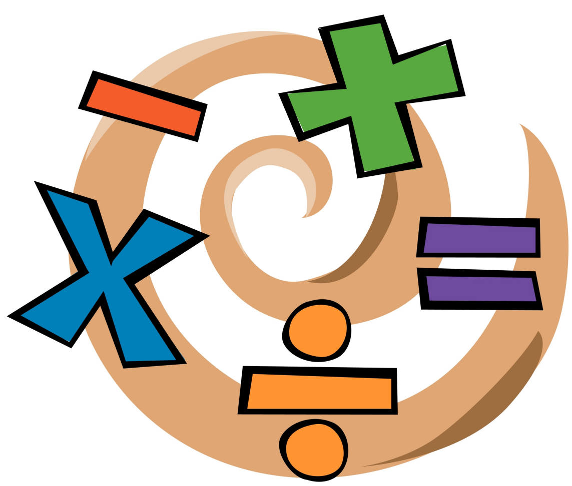
UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

District • 2021



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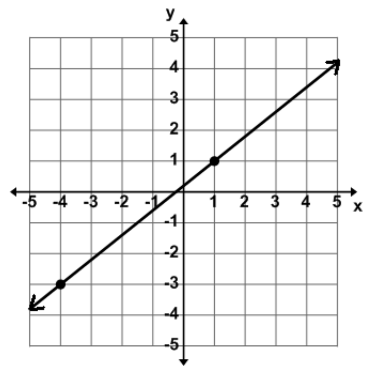
1. **Solve for k: $(1 + 3)^2 \div 4! - 5 \times k + 7 = 8$**

   (A) $-\frac{3}{5}$ **(B)** $-\frac{3}{15}$ **(C)** $-\frac{2}{15}$ **(D)** $-\frac{1}{15}$ **(E)** $-\frac{1}{30}$

2. **Lotta Dough baked a batch of cookies. She put $\frac{1}{3}$ of them in a box for snacks at work, shared 75% of the remaining ones with her family, and took the remaining 5 cookies to her grandmother's house. How many cookies did she bake?**

   (A) 18 **(B)** 24 **(C)** 30 **(D)** 36 **(E)** 42

3. **Find the slope of the line perpendicular to the line shown and through the point $(-6, 7)$?**



   (A) $-\frac{5}{4}$ **(B)** $-\frac{6}{7}$ **(C)** $-\frac{4}{5}$ **(D)** $-\frac{7}{6}$ **(E)** $-\frac{5}{6}$

4. **Simplify:** $\left(\frac{3x - 3y}{x^2 + 2xy + y^2}\right) \times \left(\frac{x + y}{x - y}\right) \div \left(\frac{6}{x^2 - y^2}\right)$

   (A) $\frac{x - y}{2}$ **(B)** $2x + 2y$ **(C)** $2x - 2y$ **(D)** $\frac{x + y}{2}$ **(E)** 2

5. **Two complementary angles have measures of $2x - 1$ degrees and $5x + 3$ degrees. What would the measure of an angle be if it is supplementary to the smaller of the two complementary angles? (nearest whole degree)**

   (A) 90° **(B)** 114° **(C)** 124° **(D)** 138° **(E)** 156°

6. **If $\frac{5 + 4x}{3x + 2} - \frac{x - 2}{2x - 1} = \frac{Ax^2 + Bx + C}{Px^2 + Qx + R}$, where A, B, C, P, Q, and R are integers. Find $A + B + C + P + Q + R$.**

   (A) 3 **(B)** 9 **(C)** 15 **(D)** 19 **(E)** 25

7. **Let $(a^5 \div b^2)^3 \times a \times b^{-4} \div (a^{-6}) \times b^0 = a^m \times b^n$. Find $m + n$.**

   (A) -6 **(B)** 9 **(C)** 12 **(D)** 4.5 **(E)** 64

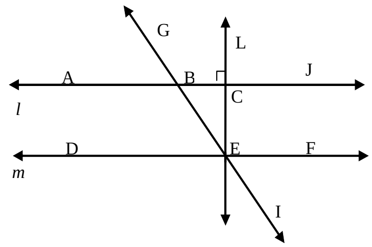
8. **Determine the phase shift of $f(\theta) = 2 + 3\cos(\frac{4\pi}{5}\theta - 6)$. (nearest tenth)**

   (A) 2.4 **(B)** 2.5 **(C)** 3 **(D)** 5.2 **(E)** 6

9. Which of the following is considered to be the first known female mathematician. Based on the works that have survived, it is thought that she worked on the Golden Mean and the Golden Rectangle.

(A) Agnesi (B) Hypatia (C) Lady Lovelace (D) Freda Porter (E) Theano

10. The four lines in the figure are coplanar with $m \| l$. Which of the following are true statements?



1. $\angle ABE$ & $\angle JBG$ are congruent
2. $m\angle DEI + m\angle ABG = 180°$
3. $\angle JBI$ & $\angle BEF$ are vertical angles
4. $m\angle CBE = 45°$

(A) 3 only (B) 1 & 2 (C) 1 & 4 (D) 3 & 4 (E) 4 only

11. Given: $\begin{bmatrix} \frac{1}{2} & -1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Find $(a + d) - (b + c)$.

(A) 1 (B) 0 (C) -1 (D) -2 (E) -3

12. If $f''(x) = 6x - 12$, $f'(1) = 0$, and $f(-2) = -49$, then $f(-1) = ?$

(A) 6 (B) 3 (C) -7 (D) -15 (E) -16

13. Wynn Zenn's science team consists of 5 seniors, 6 juniors, 3 sophomores, and 6 freshmen. In how many ways can he form a 6-member science team consisting of 2 seniors, 2 juniors, a sophomore, and a freshman?

(A) 2,700 (B) 59 (C) 38,760 (D) 10,800 (E) 34

14. Given that the set of natural numbers continues in the triangular pattern shown below, find the 3^{rd} number in row 10.

			1				(row 1)
		2	3	4			(row 2)
	5	6	7	8	9		(row 3)
10	11	12	13	14	15	16	(row 4)
			...				(...)

(A) 103 (B) 97 (C) 91 (D) 88 (E) 84

15. A string is 5 feet long. Three smaller strings with lengths of 1 foot 10 inches, 1 foot 8 inches, and 11 inches are cut from the original string. How long is the original string after the three cuts?

(A) 7 inches (B) 8 inches (C) 9 inches (D) 11 inches (E) 13 inches

16. Simplify: $\frac{(n-1)!}{(n)!} \times \frac{(n+1)!}{(n+2)!} \times n$

(A) $n-1$ (B) $n+1$ (C) $\frac{1}{n}$ (D) $\frac{n}{n+1}$ (E) $\frac{1}{n+2}$

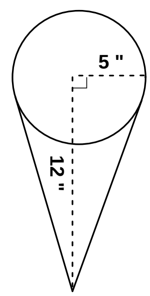
17. Let $(3x+A)(4x+B) = Cx^2 - 23x - 24$. Find $A - B + C$.

(A) 23 (B) 17 (C) 12 (D) 7 (E) 1

18. Jack is twice as old as Jill. Seven years ago, the sum of their ages was 13. What will be the sum of their ages in five years?

(A) 20 yrs (B) 27 yrs (C) 32 yrs (D) 37 yrs (E) 46 yrs

19. The least number of cups of water needed to fill the conic cup and spill over is:



(A) 20 cups (B) 21 cups (C) 22 cups (D) 23 cups (E) 24 cups

20. A regular heptagonal prism has how many edges?

(A) 12 (B) 14 (C) 18 (D) 21 (E) 24

21. If $\sqrt[4]{x^2\left(\sqrt[3]{x\left(\sqrt{x^6}\right)}\right)} = \sqrt[n]{x^k}$, where k and n are relatively prime, then k = ?

(A) 12 (B) 10 (C) 6 (D) 5 (E) 3

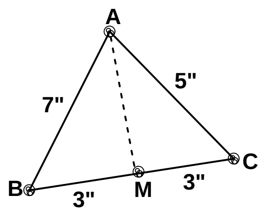
22. Willett Staupp has a horse trough that holds 400 gallons of water with a tiny hole in it. The trough loses a cup of water every hour. What percent of the total gallons of water will still be in the trough 120 days after Willett fills it?

(A) 55% (B) 45% (C) $33\frac{1}{3}$% (D) 25% (E) 10%

23. Allie Gater is a zoologist studying crocodiles. She spots two crocodiles on the bank. She estimates the distance from her boat is 30 meters to one croc and 40 meters to the other. She estimates the measure of the angle between her two lines of sight to be 28°. How far apart are the two crocs? (nearest foot).

(A) 20 ft (B) 30 ft (C) 35 ft (D) 45 ft (E) 50 ft

24. Find AM. (nearest tenth)



(A) 5.0" (B) 5.1" (C) 5.3" (D) 5.5" (E) 5.6"

25. Let $f(x) = 2x - 5$ and $g(x) = 3x + 1$. Find $f(g(x)) - g(f(-x))$.

(A) $12x + 11$ (B) -17 (C) $12x - 17$ (D) 11 (E) $12x - 11$

26. Which of the following is neither an even nor an odd function?

I. $\sqrt{x^4 - x^2} + 4$ II. $\sqrt[3]{x}$ III. $x\sqrt{x^2 - 1}$

(A) I & III (B) II only (C) I, II, & III (D) II & III (E) none of them

27. Find the distance between the absolute maximum and the absolute minimum of $h(t) = 2t^3 + 3t^2 - 12t + 4$ on the interval [0, 2]. (nearest whole number)

(A) 7 (B) 9 (C) 11 (D) 12 (E) 15

28. Which of the points lie on the line that is tangent to the curve $x^2 + 2y^2 = 9$ at point (1, 2).

(A) $(-40, 13)$ (B) $(-12, 5)$ (C) $(2, 2)$ (D) $(25, -4)$ (E) $(32, -6)$

29. Penni Lesse has two nickels and two dimes. She wants to arrange them in random order and find the probability of having two of the same coins next to each other given that a nickel is first or a dime is last. How many elements are in the successful sample space?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

30. The units digit of $13^{(2021)}$ is _______.

(A) 1 (B) 3 (C) 6 (D) 7 (E) 9

31. What value of k will make $x^2 + \frac{4}{5}x + k$ a trinomial square?

(A) .75 (B) 1 (C) .06 (D) .16 (E) .8

32. The vertex of the graph of the function $y = -x^2 - 3x + 5$ is (x, y). Find x + y.

(A) 5.25 (B) 5.75 (C) 7.25 (D) 8.75 (E) 11.75

33. $[(2A8_{12}) + (9B_{12})] \times 5_{12} =$ _____________$_{12}$.

(A) 1839 (B) 166B (C) 2675 (D) 843 (E) 226B

34. Given:

x	-2	-1	1	a	3	5	0
g(x)	7	1	1	31	17	49	b

. Find $a + b$.

(A) 11 (B) 7 (C) 5 (D) 3 (E) 2

35. How many positive digits k exists such that $k < 24$ and $24^k \div 7$ has a remainder of 1?

(A) 3 (B) 4 (C) 6 (D) 7 (E) 9

36. How many positive digits in base 10 are considered to be "happy" and/or "evil" numbers?

(A) 2 (B) 3 (C) 5 (D) 6 (E) 7

37. Given the Fibonacci characteristic sequence $f_0 = 2, f_1 = 5, f_2 = 7, f_3 = 12, \ldots$, find the sum of the f_5, f_7, and f_{11}.

(A) 667 (B) 636 (C) 586 (D) 555 (E) 524

38. The digits 1, 2, 3, 4, and 5 are used once each to form the smallest possible five-digit odd number less than 40,000. What is the digit in the tens place?

(A) 5 (B) 4 (C) 3 (D) 2 (E)

39. Thirty students at Venn U. took a survey about their favorite class. Twelve marked science, fourteen marked math, and seventeen marked English. Four students marked all three classes. Two marked math and science, but not English. Six marked math and English, but not science. Six marked only science. How many students did not mark any of these three classes?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

40. Kay Ack paddled up a river for 3 hours 30 minutes. The return trip took 2 hours 45 minutes. If the speed of the current was 2 miles per hour, what was the speed of Kay's boat in still water?

(A) $12\frac{1}{2}$ mph (B) $15\frac{1}{3}$ mph (C) $16\frac{2}{3}$ mph (D) 17 mph (E) $18\frac{1}{2}$ mph

41. Anne Teak took her history class on a bus trip to the Smithsonian museum. The expenses for the trip totaled $540.00 and were to be shared by the teacher and the students. When five students were unable to go due to bad grades, $1.50 was added to the cost per person going on the trip. How many people went on the trip?

(A) 45 (B) 25 (C) 36 (D) 50 (E) 40

42. The letters from the word NUMBER are put into a bag. Two letters are selected at random without replacement. What is the probability that the second letter selected is a vowel?

(A) 75% (B) $66\frac{2}{3}$% (C) 50% (D) $33\frac{1}{3}$% (E) 25%

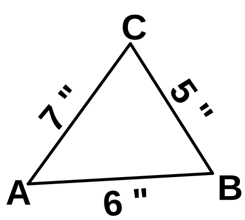
43. Chip Picker selects three chips without replacement from a bag containing four red chips and two white chips. What is the probability that he selects a red chip followed by a white chip followed by a red chip? (nearest whole percent)

(A) 24% (B) 18% (C) 15% (D) 12% (E) 20%

44. Reid Allot has 5 math books, 3 science books, and 4 literature books to be arranged on a shelf. In how many ways can the books be arranged if they are to be grouped by topic?

(A) 17,280 (B) 51,840 (C) 103,680 (D) 311,040 (E) 479,001,600

45. Find the area of $\triangle ABC$. (nearest tenth)



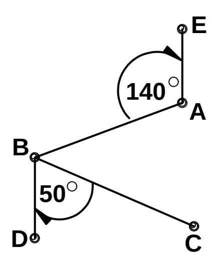
(A) 12.3 in^2 (B) 14.5 in^2 (C) 14.7 in^2 (D) 17.0 in^2 (E) 17.4 in^2

46. Let $f(x) = 4x^3 - 3x^2 - x - 2$. Which of the following is true about the end behavior of the graph?

I. As x gets larger, f(x) gets smaller
II. As x gets larger, f(x) gets larger
III. As x gets smaller, f(x) gets larger
IV. As x gets smaller, f(x) gets smaller

(A) II only (B) II & IV (C) II & III (D) IV only (E) I & IV

47. Given: $\overline{BD} \parallel \overline{AE}$, BC = 30 cm, and AB = 20 cm. Find AC. (nearest tenth)



(A) 33.0 cm (B) 38.8 cm (C) 34.2 cm (D) 37.5 cm (E) 36.1 cm

48. Two circles, $(x-3)^2 + (y-2)^2 = 25$ and $(x+1)^2 + (y-1)^2 = 9$, intersect at two points. Find the slope of the line passing through the two points of intersection.

(A) -4 (B) $-\frac{5}{8}$ (C) $-2\frac{1}{2}$ (D) $-\frac{1}{2}$ (E) -2

49. If you start at $(-1, 0)$ on a unit circle and travel clockwise 44 radians, where will you come to a stop on the unit circle?

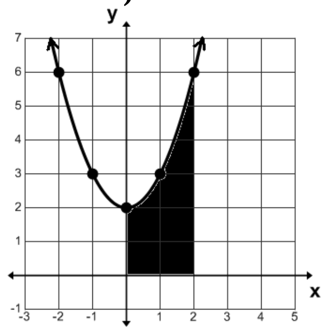
(A) QI (B) QII (C) QIII (D) QIV (E) x-axis

50. Professor Stats' probability class of 20 students took a pretest. The following chart shows the number of errors and the distribution. Find the standard deviation. (nearest hundredth)

Number of errors	0	1	2	3	4	5
Frequency	1	5	4	5	2	3

(A) 0.92 (B) 0.98 (C) 1.08 (D) 1.47 (E) 2.15

51. The area (in square units) of the shaded region below is:



(A) $5\frac{2}{3}$ (B) $6\frac{1}{3}$ (C) $6\frac{2}{3}$ (D) $7\frac{1}{3}$ (E) $7\frac{2}{3}$

52. Which of the equations in rectangular form describes the parametric equations $x = 5 - 3\cos(t)$ and $y = 4 + 2\sin(t)$, where $0 \leq t \leq 2\pi$?

(A) $\frac{(x+5)^2}{9} + \frac{(y+4)^2}{4} = 1$ (B) $\frac{(y+4)^2}{2} - \frac{(x+5)^2}{3} = 1$ (C) $\frac{(y-4)^2}{2} - \frac{(x-5)^2}{3} = 1$
(D) $\frac{(x-5)^2}{3} + \frac{(y+4)^2}{2} = 1$ (E) $\frac{(x-5)^2}{9} + \frac{(y-4)^2}{4} = 1$

53. Penni Lesse has two nickels and two dimes. She arranges them in random order. What are the odds of having two of the same coins next to each other, given that a nickel is first?

(A) 2:5 (B) 2:1 (C) 2:3 (D) 3:5 (E) 3:2

54. The coordinates of the vertices of a triangle are (0, 0), (0, 12), and (5, 0). The coordinates of the centroid of this triangle is (x, y). Find $x + y$.

(A) $6\frac{2}{3}$ (B) $1\frac{4}{13}$ (C) $1\frac{1}{2}$ (D) $4\frac{8}{13}$ (E) $5\frac{2}{3}$

55. Given the harmonic sequence $\frac{1}{12}, \frac{1}{19}, \frac{1}{26}, \frac{1}{33}, \ldots$, which of the following would be an element of this sequence?

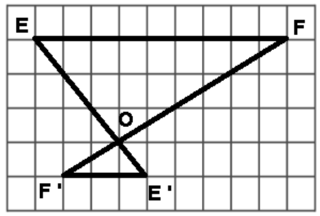
(A) $\frac{1}{336}$ (B) $\frac{1}{338}$ (C) $\frac{1}{340}$ (D) $\frac{1}{348}$ (E) $\frac{1}{352}$

56. Use following table to calculate a midpoint Riemann sum on [0, 6], n = 3.

x	0	1	2	3	4	5	6
f(x)	4	8	5	3	7	4	8

(A) 15 (B) 30 (C) 39 (D) 45 (E) 60

57. The dilation shown is?



(A) $D_O, -\frac{1}{2}$ **(B)** $D_O, -\frac{1}{3}$ **(C)** $D_O, \frac{2}{3}$ **(D)** $D_O, 2$ **(E)** $D_O, 3$

58. Find the sum of the values of a and b so that f(x) is continuous for all real values of x.

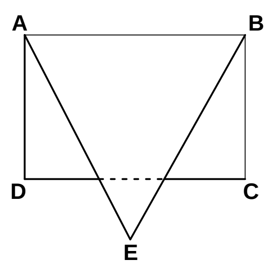
$$f(x) = \begin{cases} -x^2 + 2x + 4, & \text{if } x \geq 1 \\ ax + b, & \text{if } -1 \leq x \leq 1 \\ -x, & \text{if } x \leq -1 \end{cases}$$

(A) -1.5 **(B)** -1 **(C)** 1 **(D)** 1.5 **(E)** 5

59. Find a positive number c whose existence is guaranteed by the Mean Value Theorem for the function $f(x) = x^3 + x$ on the interval $[-1, 1]$.

(A) $\frac{1}{3}$ **(B)** $\frac{\sqrt{3}}{3}$ **(C)** $\frac{\sqrt{2}}{2}$ **(D)** 1 **(E)** 2

60. In rectangle ABCD, AB = 2 × BC. Equilateral triangle ABE overlaps rectangle ABCD. What percent of rectangle ABCD is covered by triangle ABE? (nearest whole percent)



(A) 60% **(B)** 65% **(C)** 67% **(D)** 71% **(E)** 73%

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University Interscholastic League
MATHEMATICS CONTEST
HS ● District ● 2021
Answer Key

1. D	21. D	41. E
2. C	22. A	42. D
3. A	23. A	43. E
4. A	24. C	44. C
5. E	25. A	45. C
6. D	26. E	46. B
7. C	27. C	47. E
8. A	28. D	48. A
9. E	29. C	49. B
10. B	30. B	50. D
11. C	31. D	51. C
12. D	32. B	52. E
13. A	33. B	53. B
14. E	34. D	54. E
15. A	35. A	55. D
16. E	36. D	56. B
17. E	37. A	57. B
18. D	38. B	58. E
19. C	39. B	59. B
20. D	40. C	60. D