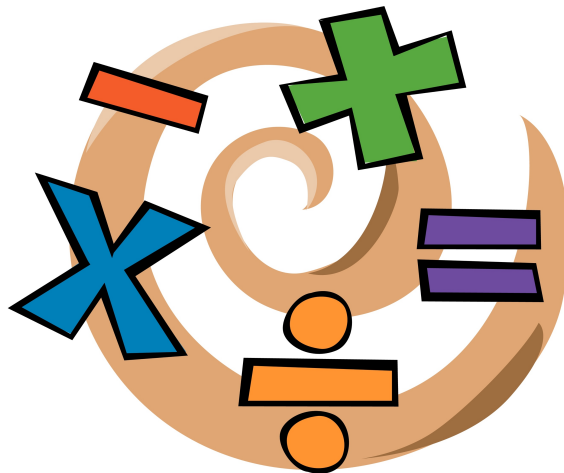UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Region • 2021

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1. **Solve for k:** $(k - 4) \div 16 + 20 - 2 \times 1! = 17$

   (A) -12 (B) -13 (C) -15 (D) -17 (E) -20

2. *Cheap Flicks* streams movies online. There is a special price for any new members renting 5 movies. They can rent 2 movies for the regular price of 1, another one at 20% off the regular price, and two others at $\frac{2}{3}$ of the regular price. If they give them their email address, they will get $5.00 off of their total order. What would it cost a new member to rent 5 movies if the regular rental price is $7.50 per movie and they give them their email? (nearest cent)

   (A) $13.50   (B) $ 16.00   (C) $18.50   (D) $22.50   (E) $26.00
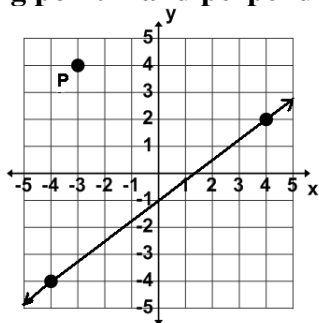
3. If $\frac{3}{4}$ of A is $\frac{4}{5}$ of B, then B is what percent of A?

   (A) $106\frac{2}{3}\%$ (B) 60%   (C) $166\frac{2}{3}\%$ (D) 93.75%   (E) 6.25%

4. Let O = {o, s, p, r, e, y}, P = {p, e, l, i, c, a, n}, A = {a, v, o, c, e, t} and B = {b, a, r, n, o, w, l}. The number of elements in $(A \cup O) \cap (P \cup B)$ is:

   (A) 6   (B) 7   (C) 10   (D) 13   (E) 15

5. The line containing point P and perpendicular to the line shown contains the point (6, y). Find y.

   (A) $-6\frac{2}{3}$ (B) $-7\frac{1}{3}$ (C) -8 (D) $-8\frac{1}{3}$ (E) $-9\frac{1}{3}$

6. Which of the following equations has real roots?

   I. $4x^2 + 6x + 1 = 0$ II. $-\frac{1}{2}x^2 - 5x + 6 = 0$ III. $-x^2 + x - 1 = 0$

   (A) I only   (B) I & II   (C) II & III   (D) I & III   (E) I, II, & III

7. Let $(Ax + 3)(Bx + C) = 16x^2 + Dx + 12$, where A, B, C, and D are natural numbers and one of the roots of the equation $16x^2 + Dx + 12 = 0$ is -2. Find D.

   (A) 19   (B) 28   (C) 32   (D) 38   (E) 52

8. Which of the following mathematicians are thought of when working with prime numbers?

   I. Erastosthenes   II. Marin Mersenne   III. Sophie Germain

   (A) I only   (B) I & II   (C) I, II, & III   (D) I & III   (E) III only

9. **Willit Gorown is trying to open the combination lock. He has to turn it clockwise to a prime number, then twice around counterclockwise to a Fibonacci number, then clockwise to a factor of 40. How many unique 3-number combinations of natural numbers fit the criteria?**
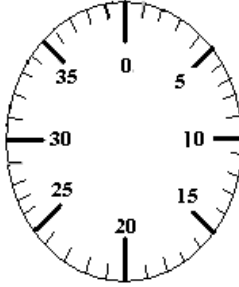
    (A) 28   (B) 144   (C) 672   (D) 768   (E) 864

10. **Two supplementary angles have measures of 3x — 7 degrees and 4x + 5 degrees. What would the measure of an angle be if it is complementary to the smaller of these two angles? (nearest whole degree)**

    (A) 19°   (B) 26°   (C) 38°   (D) 64°   (E) 71°

11. **Rose Pedler has a square garden with a perimeter of 80 feet. Rose wants to increase the area of her garden by at least 30%. What is the least number of feet does she need to add to the length if she adds 2 feet to the width? (whole feet)**

    (A) 1 foot   (B) 2 feet   (C) 3 feet   (D) 4 feet   (E) 6 feet

12. **Find the total surface area of the cylinder shown, where both ends are closed. (nearest sq. inch)**
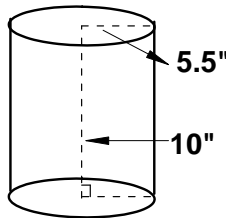
    (A) 346 sq. in   (B) 380 sq. in   (C) 536 sq. in   (D) 570 sq. in   (E) 691 sq. in

13. **How many real roots less than zero exist given the equation $2x^4 + 9x^3 - 7x^2 - 54x - 40 = 0$?**

    (A) 0   (B) 1   (C) 2   (D) 3   (E) 5

14. **Les Dough, Noah Dough, and Lotta Dough went to the local market to buy bread and milk. Les paid $3.47 for 2 loaves of bread and one quart of milk. Noah paid $3.07 for one loaf of bread and 2 quarts of milk. Based on these prices, how much should it cost Lotta for 4 loaves of bread and a gallon of milk?**

    (A) $9.61   (B) $8.72   (C) $8.23   (D) $7.83   (E) $6.94

15. If $a_1 = -1$, $a_2 = 1$, $a_3 = 3$, and $a_n = (a_{n-2})(a_{n-3}) - (a_{n-1})$, where $n \geq 4$, then a_6 equals:

(A) -19 (B) -9 (C) -4 (D) 7   (E) 10

16. Given that the set of natural numbers continue in the triangular pattern shown below, find the sum of the numbers in row 9 minus the median number in row 9.

$$
\begin{array}{cccccc}
 & & 1 & & & \text{(row 1)} \\
 & 3 & & 5 & & \text{(row 2)} \\
7 & & 9 & & 11 & \text{(row 3)} \\
13 & & 15 & & 17 & 19 \quad \text{(row 4)} \\
21 & 23 & & 25 & 27 & 29 \quad \text{(row 5)} \\
 & & \cdots & & & (\;\; \cdots \;\;)
\end{array}
$$

(A) 810   (B) 793   (C) 747   (D) 729   (E) 648

17. Polly Ticks is randomly selecting the order of the candidates to be listed on the ballot for an upcoming election. There are 3 male candidates and 2 female candidates. How many elements are in the successful sample space when computing the probability that the 2 female names are listed back to back?

(A) 60   (B) 48   (C) 24   (D) 8   (E) 4

18. The graph shown is the graph of which of the following equations.
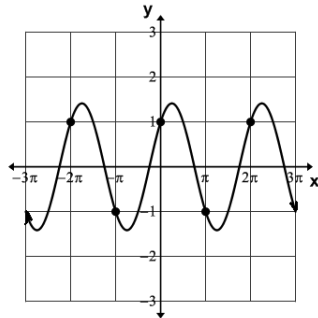
(A) $y = \sin(x)\cos(x)$ (B) $y = \sin(x) - \cos(x)$ (C) $y = \cos^2(x) - \sin^2(x)$
(D) $y = \sin^2(x) - \cos^2(x)$ (E) $y = \sin(x) + \cos(x)$

19. Simplify: $\dfrac{\cos\theta}{\sec\theta - \tan\theta}$

(A) $\csc\theta - 1$ (B) $\dfrac{1 - \sin\theta}{\sin\theta}$ (C) $\cot\theta$ (D) $\cos\theta + 1$ (E) $\sin\theta + 1$

20. How many elements are in $\left\{ x \,\middle|\, \sin^2(x) + \sin(x) - 6 = 0, x \in [0, 2\pi) \right\}$?

(A) 4   (B) 3   (C) 2   (D) 1   (E) 0

21. If $244_P = 164$ and $355_Q = 187$, then $244_Q + 355_P = ?$

(A) 339   (B) 351   (C) 367   (D) 452   (E) 599

22. The remainder when $f(x) = kx^3 + 3x^2 - 2x + 1$ is divided by $x - 2$ is 41. Find k.

    (A) 1   (B) 2   (C) 3   (D) 4   (E) 5

23. Ann, Bea, Cay, Dee, Eva, and Felecia are to sit together in a row. In how many ways can
    they do this if Dee and Eva insist on sitting next to each other, and Felecia sits in the first chair?

    (A) 24   (B) 48   (C) 72   (D) 120   (E) 240

24. Which of the following statements is/are true for $f(x) = |x - 3|$?
    I. f is continuous at x = 3   II. $f'\_(3)$ exists   III. $f'\_+(3)$ exists   IV. f is differentiable at x = 3

    (A) I only   (B) I & IV   (C) I, II, & III   (D) All of them   (E) none of them

25. Given the function $g(x) = x^3 - 3x^2 + 3$, at which of the following values of x is function g
    increasing?   I. -3 II. -1 III. 1   IV. 3

    (A) I & IV   (B) I & II   (C) I, II, & IV   (D) III only   (E) all of them

26. How many five-letter arrangements of the letters REGIONAL consisting of three consonants
    and two vowels can be formed if no letter is repeated?

    (A) 2,880   (B) 1,440   (C) 576   (D) 288   (E) 240

27. Given the Fibonacci characteristic sequence 2, p, q, 16, r, ... 107, 173, ..., find $p + q + r$.

    (A) 41   (B) 33   (C) 30   (D) 25   (E) 22

28. How many 4-digit odd numbers between 4,689 and 8,642 can be created using the digits 2, 3, 5,
    and 7? A digit cannot be used more than once in any of the 4-digit numbers.

    (A) 4   (B) 8   (C) 12   (D) 16   (E) 24

29. Which of the following points are collinear to $(-4, -4)$ and $(4, 2)$?
       I. $(-16, -13)$ II. $(-13, -16)$ III. $(11, 16)$ IV. $(12, 8)$

    (A) II & III   (B) I & IV   (C) II & IV   (D) I only   (E) I, II, & IV

30. Find the perimeter of the quadrilateral ABEF. (nearest tenth)
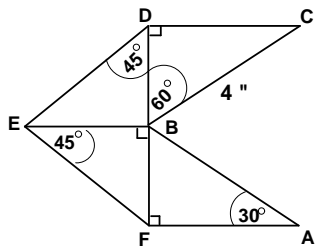
   (A) 10.3 in   (B) 12.6 in   (C) 14.3 in   (D) 10.5 in   (E) 12.3 in

31. If $x + y = -4$ and $xy = 3$ then $x^3 + y^3 = ?$

    (A) -343 (B) -37 (C) -28 (D) -21 (E) -1

32. Sumware is 70 miles due east of Noware. The bearing of Anyware is 15° from Noware and from Sumware is 290°. How much further is it from Anyware to Sumware than it is from Anyware to Noware? (nearest tenth)

    (A) 24.6 mi   (B) 26.2 mi   (C) 43.8 mi   (D) 45.9 mi   (E) 48.6 mi

33. Let $f(x) = \sin(x)$ and $g(x) = 2\cos(x)$. If (x, y) is the point where $f(x) = g(x)$ over the interval $0 \le x \le \pi$ radians, then $x + y = ?$ (nearest hundredth)

    (A) 1.98   (B) 2.00   (C) 2.01   (D) 2.03   (E) 2.05

34. The center of the graph of $2x^2 + y^2 + 8x - 8y - 48 = 0$ is (h, k). Find $h + k$.

    (A) -6 (B) -2 (C) 2   (D) 10   (E) 20

35. Let $f(x) = 7x - 1$ and $g(x) = 4x + 3$. Find $f(g(-x)) + g(f(x))$.

    (A) 21   (B) 20   (C) 19   (D) $56x + 19$ (E) $56x + 21$

36. The *Power Me Up* battery factory packages batteries in packs of 36. The actuaries' research data shows that the probability of at least one bad battery in a 36-pack is 12%. What are the odds that a randomly chosen 36-pack contains all good batteries?

    (A) $\frac{3}{25}$ (B) $\frac{3}{22}$ (C) $\frac{3}{1}$ (D) $\frac{1}{3}$ (E) $\frac{22}{3}$

37. $\frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \frac{1}{21} + ... + \frac{1}{120} = ?$

    (A) $\frac{6}{7}$ (B) $\frac{7}{8}$ (C) $\frac{9}{11}$ (D) $\frac{13}{15}$ (E) $\frac{15}{17}$

38. How many positive numbers less than 50 are considered to be "tetrahedral" numbers?

    (A) 9   (B) 8   (C) 7   (D) 6   (E) 5

39. The area (in square units) of the shaded regions below is:
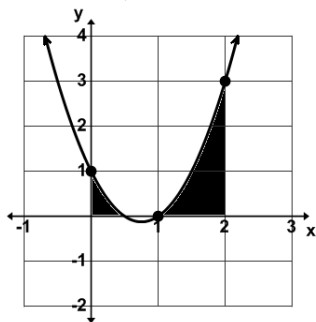
    (A) $1\frac{1}{4}$ (B) $1\frac{1}{6}$ (C) $1\frac{1}{3}$ (D) $1\frac{3}{8}$ (E) $1\frac{1}{2}$

40. Willie Pass is calculating his 6-weeks average. His daily grade average is 92, quiz average is 70 and test average is 75. The 6-weeks average is calculated using 25% of the daily average, 30% of the quiz average, and 45% of the test average. What is Willie's 6-weeks average?

    (A) 81.4   (B) 80.25   (C) 79   (D) 77.85   (E) 77.75

41. The sum of two numbers is 86. If four times the larger number is decreased by the smaller number, the result is 144. Find the product of the two numbers?

    (A) 1,849   (B) 1,848   (C) 1,845   (D) 1,833   (E) 1,840

42. Penni Les has a coin purse containing dimes and quarters. The ratio of dimes to quarters is 8 to 3 and the number of dimes exceeds the number of quarters by 35. What is the monetary value of the coins in Penni's coin purse?

    (A) $12.45   (B) $9.75   (C) $9.30   (D) $11.75   (E) $10.85

43. The measure of a central angle of a regular pentadecagon is?

    (A) 30   (B) 24   (C) 18   (D) 15   (E) 9

44. Point P(3, — 2) lies on the x-y plane. P is rotated $90°$ clockwise about the origin to Point Q. Point Q is translated vertically $+ 4$ units to point R. Point R is reflected across the line $y = 2 — x$ to point S. Point S is _____ to point T(— 1, 4). (Fill in the blank.)

    (A) translated vertically $+ 1$ units   (B) rotated $90°$ counterclockwise about the origin
    (C) reflected across the x-axis   (D) translated horizontally — 2 units
    (E) translated vertically — 1 units

45. Find the domain of $f(x) = \frac{\sqrt{8 - 4x}}{2x^2 - 7x + 5}$.

    (A) $x \in R \big| x \leq 2$ (B) $x \in R \big| x \leq 2, x \neq 0.5, 5$ (C) $x \in R \big| x \leq 2, — 5, — 0.5$
    (D) $x \in R \big| x \leq 2, x \neq 1$ (E) $x \in R \big| x \leq 2, x \neq — 2.5, — 1$

46. If $f(x) = \frac{2x - 5}{3 - 4x}$, $f^{-1}(6) =$ _____. (nearest tenth)

    (A) — .3   (B) — .1   (C) 0.2   (D) 0.6   (E) 0.9

47. If the shadow of a building increases by 10 meters when the angle of elevation of the sun rays decreases from $70°$ to $60°$, what is the height of the building? (nearest meter)

    (A) 17 meters   (B) 27 meters   (C) 35 meters   (D) 41 meters   (E) 47 meters

48. Which of the following is closest to the length of the segment bounded by the parametric equations: $x = 2t — 3$, $y = 3t + 2$, $0 \leq t \leq 1$.

    (A) 8.1   (B) 7.3   (C) 5.0   (D) 3.6   (E) 2.2

49. Find the average value of the function f defined by $f(x) = x^3 + 1$ on the interval $[-2, 2]$.

   (A) 8   (B) 4   (C) 2   (D) 1   (E) 0

50. Given the function $f(x) = 2\cos(x) + 1$, find the slope of the secant line between $x = \frac{3\pi}{2}$ and $x = 2\pi$. (nearest tenth)

   (A) 0.4   (B) 0.8   (C) 1.3   (D) 2.5   (E) no slope

51. Polly Ticks is randomly selecting the order of the candidates to be listed on the ballot for an upcoming election. There are 3 male candidates and 2 female candidates. What is the probability that all three male candidates are listed back to back to back, given that the third candidate on the ballot is a male?

   (A) 20%   (B) 25%   (C) 30%   (D) 40%   (E) 50%

52. For the general population, IQ scores are normally distributed with a *mean* of 100 and a *standard deviation* of 15. Approximately, what percent of the population have IQ scores above 115?

   (A) 15%   (B) 16%   (C) 30%   (D) 32%   (E) 34%

53. Let g, g + 2, and g + 8 be the first three terms of a geometric sequence. What is the sum of the first four terms of this sequence?

   (A) 27   (B) 35   (C) 40   (D) 45   (E) 53

54. Find the slope of the tangent line to the curve whose equation is $r = 1 - \cos \theta$ at the point $(1 - \frac{\sqrt{2}}{2}, \frac{\pi}{4})$. (nearest tenth)

   (A) 1.4   (B) 1.7   (C) 2.4   (D) 2.9   (E) 3.4

55. Find distance, d, in order to balance the beam. (nearest cm)
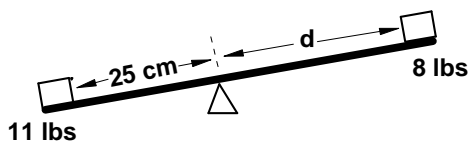
   (A) 35 cm   (B) 34 cm   (C) 28 cm   (D) 19 cm   (E) 18 cm

56. Find the sum of the values of a and b so that f(x) is continuous for all real values of x.

$$f(x) = \begin{cases} x + a, & \text{if } x > 4 \\ 3 - |x - 1|, & \text{if } -1 \le x \le 4 \\ b, & \text{if } x < -1 \end{cases}$$

   (A) -3 (B) -1 (C) 1   (D) 4   (E) 5

57. Suppose $\triangle P'Q'R' = D_O, -\frac{2}{3}(\triangle PQR)$ and the coordinate of Q$'$ is (x, y). Find x $+$ y.
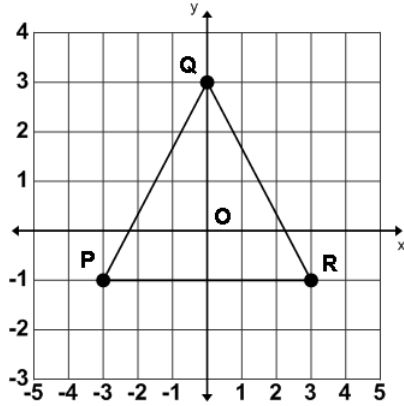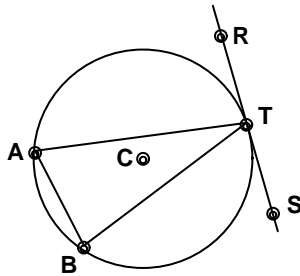

(A) -2 (B) $-1\frac{1}{3}$ (C) $-\frac{2}{3}$ (D) 1   (E) $1\frac{2}{3}$

58. Given the harmonic sequence $\frac{3}{11}, \frac{3}{17}, \frac{3}{23}, \frac{3}{29}, \dots$, which of the following would be an element of this sequence?

(A) $\frac{3}{209}$ (B) $\frac{3}{205}$ (C) $\frac{3}{199}$ (D) $\frac{3}{193}$ (E) $\frac{3}{181}$

59. Given: $\triangle$ ABT is inscribed in the circle with center C; RS is tangent to the circle at point T; m$\angle$ATB is 28°; and m$\angle$BAT is 71°. Find m$\angle$ATR.

(A) 71°   (B) 78°   (C) 81°   (D) 90°   (E) 98°

60. Two cardboard squares with side lengths 4" and 3" overlap with the corner of the smaller square at the center of the larger square. If the overlap portions are removed what is the difference between the remaining areas?
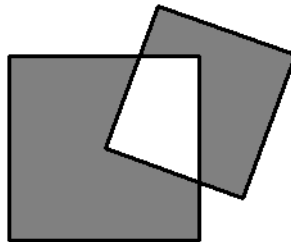
(A) 7 in^2 (B) 6 in^2 (C) 5 in^2 (D) 1 in^2 (E) can be determined

University Interscholastic League
MATHEMATICS CONTEST
HS ● Regional ● 2021
Answer Key

1. A	21. C	41. E
2. C	22. D	42. E
3. D	23. B	43. B
4. A	24. C	44. D
5. C	25. C	45. D
6. B	26. A	46. E
7. D	27. A	47. E
8. C	28. B	48. D
9. C	29. B	49. D
10. A	30. E	50. C
11. D	31. C	51. E
12. C	32. C	52. B
13. D	33. B	53. C
14. B	34. C	54. C
15. A	35. C	55. B
16. E	36. E	56. A
17. B	37. B	57. A
18. E	38. E	58. A
19. E	39. D	59. C
20. E	40. E	60. A