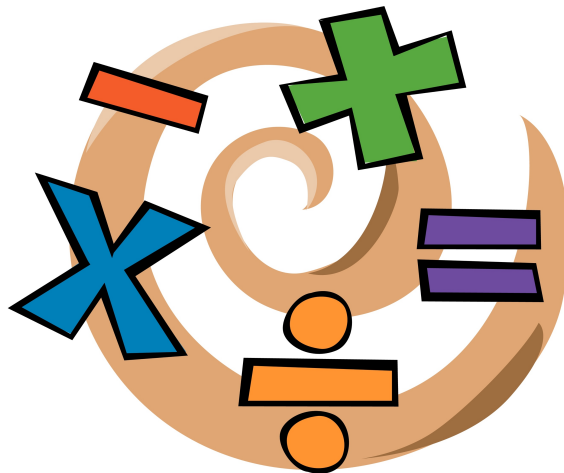# Mathematics

State • 2021

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO!

1. Solve for k: $3 \times 5 - 6 \times (1 + 2 \div k) = 6$

   (A) -6 (B) $-1.666...$ (C) 0.8 (D) 4 (E) 4.75

2. Trotter Turtle is loping at a speed of half a mile per hour. How long will it take Trotter to lope 20 feet? (nearest second)

   (A) 27 sec   (B) 2 sec   (C) 4 sec   (D) 30 sec   (E) 44 sec

3. Phil Hurrup has a pickup with a 25 gallon tank that averages 15 miles per gallon of gas. Phil spent $50.00 for gas at $2.32 per gallon to drive to his deer lease 600 miles away. How many miles was Phil able to drive before exhausting his $50.00 worth of gas? (nearest mile)

   (A) 539 mi   (B) 216 mi   (C) 375 mi   (D) 277 mi   (E) 323 mi

4.
```
         A              B              C              D              E
<---------------|---------------|---------------|---------------|---------------|--------------->
```
The distances between the hash marks (|) are equal. Find the midpoint of DE if
$A = 3\frac{7}{10}$ and $E = 17\frac{3}{5}$.

   (A) $15\frac{9}{10}$ (B) $15\frac{69}{80}$ (C) $15\frac{1}{2}$ (D) $15\frac{2}{5}$ (E) $15\frac{9}{40}$

5. Given the statements, $3x - 2 = k + 4$ and $k + 4 = 5$, which of the following reasons justifies the conclusion $3x - 2 = 5$?

   (A) symmetric property   (B) transitive property   (C) distributive property
   (D) multiplicative property of -1 (E) additive identity property

6. Simplify: $\frac{3n^2 - 27}{6 - n - n^2} \times \frac{4 - 2n}{3 - n} \div \frac{6}{3n + n^2}$

   (A) -36 (B) -1 (C) $-n^2 - 3n$ (D) $-\frac{3+n}{n}$ (E) $-(n+3)$

7. Let $(4x - 3)(ax + b)(cx + 1) = 4x^3 + dx^2 + ex + 6$, where a,b,c,d, and e are non-distinct integers. Find $a + b + c + d + e$.

   (A) -12 (B) -8 (C) -2 (D) 7 (E) 16

8. The Electric Eel Energy Company set new rates for electricity for 2021.

Base cost	first 500 kwh	next 500 kwh	above 1000 kwh	Delivery rate
$7.95	9.6¢ /kwh	9.0¢ /kwh	8.8¢ /kwh	3.2¢ /kwh

   If I. M. Shawked used 1400 kwh this month, what would be his total monthly bill, including the base cost, the usage cost, and the delivery cost? (tax not included)

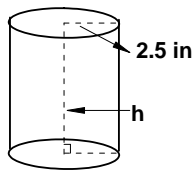   (A) $168.95   (B) $172.95   (C) $175.95   (D) $177.95   (E) $180.95

9. **The set of points on the line below are collinear with which of these points?**

    I. $(-17, 12)$ II. $(9, -10)$ III. $(7.5, -5.5)$
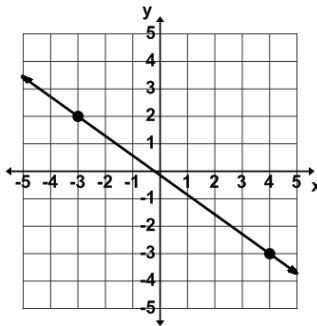
    (A) I only   (B) II only   (C) III only   (D) I & III   (E) I, II, & III

10. **The cylinder below holds at least a gallon of water. What would the minimum height be? (nearest tenth)**

    2.5 in

    h

    (A) 16.3 in   (B) 14.7 in   (C) 11.8 in   (D) 10.2 in   (E) 8.2 in

11. **Point P(2, 5) lies on the x-y plane. P is translated -2 units vertically and -5 units horizontally to point Q. Point Q is reflected across the line y = 2 to point R. Point R is reflected across the line x = 5 to point S. Find the distance from point P to point S. (nearest tenth)**

    (A) 10.0 units   (B) 6.4 units   (C) 10.2 units   (D) 7.8 units   (E) 11.7 units

12. **Given the sides of three triangles, which of these triangles has its orthocenter outside of the triangle?   I. 3, 3, 4   II. 3, 4, 5   III. 3, 4, 6**

    (A) I only   (B) II only   (C) III only   (D) I & II   (E) I & III

13. **Let $f(x) = x^2 - 3$ and $g(x) = 4x + 5$. Find $f\big(g(-2)\big) + g\big(f(2)\big)$.**

    (A) 15   (B) 12   (C) 9   (D) 6   (E) 3

14. **$(2C2_{16}) \times (8_{16}) - (4E4_{16}) = $ _____ $_{16}$.**

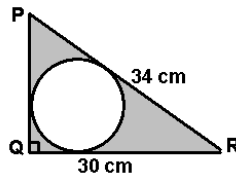   (A) 4396   (B) 2,574   (C) 5648   (D) 1C56   (E) 112C

15. **$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - ...$? (nearest hundredth)**

    (A) 1.05   (B) 0.63   (C) 0.79   (D) 0.39   (E) 1.57

16. **Given the sequence: $-12, a, -3, b, c, \frac{3}{8}, d, ...$. Find the sum of these seven terms.**

    (A) $-7\frac{5}{8}$ (B) $-7\frac{29}{64}$ (C) $-7\frac{7}{8}$ (D) $-8\frac{3}{64}$ (E) $-8\frac{1}{16}$

17. Given the circle inscribed in $\triangle$PQR, find the shaded area. (nearest cm^2)

P, 34 cm, Q, 30 cm, R

(A) 161 cm^2 (B) 139 cm^2 (C) 127 cm^2 (D) 117 cm^2 (E) 83 cm^2

18. The *Clickety Clack* and the *Choo Choo* leave the station on parallel tracks at the same time. *Clickety* travels 2 miles per hour faster than *Choo*. It takes *Choo* 15 minutes more time to reach the next station 300 miles away than *Clickety*. What was *Choo*'s rate of travel?

(A) 40 mph   (B) 42 mph   (C) 45 mph   (D) 48 mph   (E) 50 mph

19. Which of the following is a false statement?

(A) $\sin \theta = \cos (2\pi - \theta)$ (B) $\cos 3\theta = 4\cos^3\theta - 3\cos \theta$ (C) $\frac{1}{2}\sin 2\theta = \sin\theta\cos\theta$

(D) $\cos \theta = \sin (\frac{\pi}{2} - \theta)$ (E) $3\sin \theta = 4\sin^3\theta + \sin 3\theta$

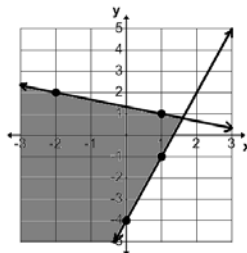20. If $\cos x \cos y = \frac{5}{8}$ and $\sin x \sin y = \frac{3}{8}$ then $\cos(x - y) = ?$

(A) $\cos \pi$ (B) $\cos \frac{5\pi}{12}$ (C) $\sin \frac{\pi}{12}$ (D) $\sin \frac{3\pi}{2}$ (E) $\sin \frac{5\pi}{2}$

21. Which type of conic is the graph of the equation $Ax^2 + By^2 + Cy + D = 1$, where A, B, C, and D are distinct integers and A, B, D > 0 and C < 0?

(A) circle   (B) ellipse   (C) hyperbola   (D) parabola   (E) not a conic

22. The shaded area shown represents the solutions for which of the following pair of inequalities?

(A) $x + 3y \le 4$ (B) $x - 3y \le 4$ (C) $x + 3y \ge 4$ (D) $x + 3y \le 4$ (E) $x - 3y \le 4$
   $3x - y \ge 4$ $3x + y \le 4$ $3x - y \ge 4$ $3x - y \le 4$ $3x + y \ge 4$
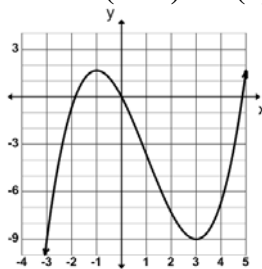
23. If $a_1 = 3, a_2 = 2, a_3 = -1$, and $a_n = [(a_{n-2}) - (a_{n-3})] \times (a_{n-1})$ for n $\ge$ 4, then a_6 equals:

(A) -6 (B) -3 (C) 1   (D) 4   (E) 24

24. Chip Picker selects two chips at random from a box containing five chips numbered 2, 3, 5, 7, and 11. The probability that the sum of the two chips is a prime number is $\frac{c}{t}$. Find t $-$ c.

(A) 11   (B) 7   (C) 5   (D) 3   (E) 2

25. The graph of h(x) is shown below. Find $h'(15.5)$ if h(1) = — 3.666... .

   (A) 205   (B) 205.75   (C) 206.25   (D) 207.5   (E) 212.25

26. The *Citizen* ship and the *Friend* ship are heading straight away from the same port along routes that make a 120° angle. The *Citizen* is traveling at 15 mph and the *Friend* is traveling at 25 mph. How fast are the ships moving apart when the *Citizen* is 5.8 miles from port and the *Friend* is 3.5 miles from port? (nearest whole mph)

   (A) 20 mph   (B) 21 mph   (C) 31 mph   (D) 34 mph   (E) 35 mph

27. Which of the following are considered to be happy numbers that are perfect or perfect numbers that are happy?   I. 6   II. 7   III. 28   IV. 496

   (A) I, III, & IV   (B) I & III   (C) II & III   (D) I & IV   (E) III & IV

28. Let U = {a, b, c, e, g, h, i, o, r, s, t, u} be a universal set. Let A = {a, c, u, t, e}, R = {r, i, g, h, t}, O = {o, b, t, u, s, e}, and C = (A ∪ R) ∩ (O). How many elements are in the complement of C?

   (A) 12   (B) 11   (C) 9   (D) 7   (E) 3

29. Mary D. Rapper bought a 5-yard spool of ribbon to wrap gifts. She used $\frac{1}{4}$ of the spool for one gift and 30% of the rest of the spool for a second gift. What was the length of the remaining ribbon on the spool?

   (A) 2 yds 1 ft 1.5 in   (B) 2 yds 2 ft 3 in   (C) 2 yds 1 ft 10.5 in   (D) 2 yds 9 in   (E) 1 yd 4.5 in

30. If y is inversely proportional to x + 2, and y = 13 when x = 2, find y when x = 1.25.

   (A) 16   (B) 20.8   (C) 8   (D) 41.6   (E) 10.5625

31. Let AB = 6". Find CF. (nearest tenth)
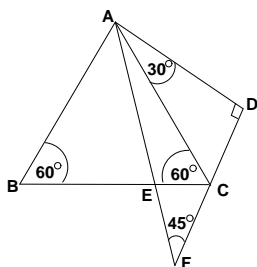
   (A) 2.4"   (B) 2.2"   (C) 1.4"   (D) 1.7"   (E) 3.7"

32. **STEM Tech Academy offers 8 advanced math courses, 6 advance science courses, and 3 robotics courses. In how many ways can a six course semester schedule be set up consisting of 3 math courses, 2 science courses and 1 robotics course?**

    (A) 30,240   (B) 2,520   (C) 369   (D) 12,376   (E) 74

33. **Which of the following mathematicians would most likely be linked to the sequence**
    $\frac{1}{4} + \frac{3}{4} + 1 + 1\frac{3}{4} + 2\frac{3}{4} + \ldots + 7\frac{1}{4} + 11\frac{3}{4} + \ldots$?

    (A) Georg Cantor (B) Alan Turing (C) Mary Rudin (D) Zeno of Elea (E) Leonardo Bigollo

34. **The roots of $P(x) = x^3 + x^2 + kx + 8$ are integers. Find k.**

    (A) -10 (B) -4 (C) -2 (D) 10   (E) 14

35. **Solve for x: $\log_3(x-3) + \log_3(x-1) = \log_3(3-3x)$**

    (A) -3 (B) -1 (C) 1   (D) 2   (E) no solution

36. **Use following table to approximate a left hand Riemann sum of $\int f(x)$.**

x	0	2	3	4	9
f(x)	3	5	7	5	4

    (A) 16   (B) 19   (C) 43   (D) 47   (E) 96

37. **Rusty Yaht left his port and sailed his sloop 18 km on a bearing of 72° to Atoll. Then he sailed 24 km on a bearing of 336° to Eyland. He could have saved time by sailing straight to Eyland on a bearing of _____. (nearest degree)**

    (A) 9°   (B) 15°   (C) 39°   (D) 57°   (E) 345°

38. **Find the sum of all numbers c that satisfy the conclusions of the Mean Value Theorem for the function $f(x) = x^3 + x^2 - x$ on the interval $[-2, 1]$.**

    (A) $-\frac{1}{4}$ (B) $-\frac{1}{3}$ (C) $-\frac{1}{2}$ (D) $-\frac{2}{3}$ (E) -1

39. **Rollin Kubes rolls only one die. Assuming it is a fair die, what is the probability that the top face has 3 dots, 2 dots, or 1 dot showing, given that the number of dots showing is odd?**

    (A) $16\frac{2}{3}\%$ (B) 30%   (C) $33\frac{1}{3}\%$ (D) 60%   (E) $66\frac{2}{3}\%$

40. $333_8 + 222_4 + 111_2 = $ _____ $_{16}$.

    (A) 29A   (B) 2426   (C) 862C   (D) 10C   (E) 268

41. **Real numbers p, q, and r exist such that $p + q + r = 24$, $r^2 = p^2 + q^2$, and $pq = 12$, find the value of r.**

    (A) 12   (B) 11.5   (C) 10.5   (D) 8   (E) 5.75

42. The lengths of the sides of quadrilateral ABCD are given. If diagonal AC has integral length, find AC?

B
5"   8"
A
4"   C
D   15"
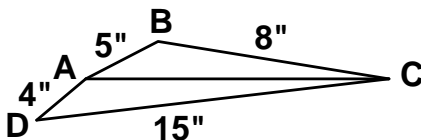
(A) 9"   (B) 12"   (C) 13"   (D) 19"   (E) 20"

43. A sine function, f(x), has a maximum value of 7, a minimum value of 3, a period of $\frac{2\pi}{3}$ and a horizontal phase shift of $\frac{\pi}{2}$. If f(0) = 3, then the value of $f(\frac{\pi}{5})$ is _____ . (nearest tenth)

(A) 5.6   (B) 5.4   (C) 5.3   (D) 5.2   (E) 5.0

44. Find the sum of the values of a and b so that f(x) is continuous for all real values of x.

$$f(x) = \begin{cases} x - 3, & \text{if } x \geq 3 \\ \sqrt{a - x^2}, & \text{if } -3 < x < 3 \\ b - x, & \text{if } x \leq -3 \end{cases}$$
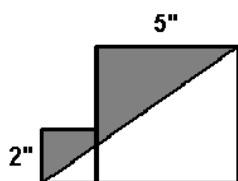
(A) −3   (B) 0   (C) 3   (D) 6   (E) 12

45. The sum of the y-values of all of the critical points of $f(x) = x^4 + 4x^3 - 2x^2 - 12x$ minus the sum of the y-values of all of the critical points of $f(x) = x^4 + 4x^3 - 2x^2 - 12x + 4$ is ?
(A) −12   (B) −11   (C) −4   (D) 2   (E) 12

46. Find the coefficients of the x^4 term when $\left(x^{\frac{1}{2}} - x^{\frac{2}{3}} \right)^7$ is expanded.

(A) −21   (B) −35   (C) 7   (D) 21   (E) 35

47. Two squares have dimensions as indicated in the drawing. What is the area of the shaded region?

5"
2"

(A) 10.5 in²   (B) 18.5 in²   (C) 11.5 in²   (D) 23.5 in²   (E) 17.5 in²

48. Poly Gawn has an irregular shaped quadrilateral garden. Two of the angles are equal. The third angle is equal to the sum of the two equal angles. The fourth angle is 60° less than twice the sum of the other three angles. How much bigger is the largest angle than the smallest angle?

(A) 220°   (B) 167.5°   (C) 150°   (D) 115°   (E) 185°

49. Points A, B, C, and D lie on a circle with center O, chord AC and chord BD are extended outside the circle intersecting at point P, BD = 5 cm, BP = 2 cm, and AP = 1.5 cm. Find AC.

   (A) $8\frac{1}{2}$ cm (B) $7\frac{5}{6}$ cm (C) $7\frac{1}{3}$ (D) $6\frac{2}{3}$ (E) $4\frac{1}{2}$

50. Sir Vayer staked out a plot of land using the layout below. How far was it from stake A to stake B? (nearest hm)

A ___ B
3.7 hm 113.4° 2.9 hm
91.2°
D ___ 5.2 hm ___ C
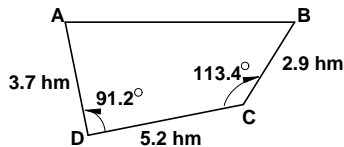
   (A) 5.9 hm (B) 6.1 hm (C) 6.3 hm (D) 6.5 hm (E) 6.7 hm

51. Let $f(x) = \sin(\frac{\pi}{2} - x)$ and $g(x) = x$. If (x, y) is the point where $f(x) = g(x)$ over the interval $0 \le x \le 2\pi$ radians, then $x + y = ?$ (nearest hundredth)

   (A) 1.40 (B) 1.44 (C) 1.48 (D) 1.53 (E) 1.57

52. The sequence $1 - \frac{9}{2} + \frac{81}{24} - \frac{729}{720} + \frac{6561}{40320} - ...$ simplifies to the decimal number $-0.ABCDEFG...$, where the letters represent digits. What digit does letter G represent?

   (A) 0 (B) 2 (C) 4 (D) 8 (E) 9

53. A 5" by 7" photo is being enlarged. How long will it take for the area of the original photo to be at least 5 times as big if the length and width are increasing at the rate of 2" per second? (nearest hundredth)

   (A) 3.65 sec (B) 3.63 sec (C) 3.41 sec (D) 3.36 sec (E) 2.50 sec

54. $(13121)^k \div 137$ has a remainder of 2 for which of these values of k?

   (A) 138 (B) 134 (C) 133 (D) 131 (E) 130

55. Find the displacement of a particle traveling along the curve, $c(t) = 4^x$, from $t = 1$ to $t = 4.3$. (nearest tenth)

   (A) 277.1 (B) 279.2 (C) 381.4 (D) 384.0 (E) 480.3

56. The circle shown is tangent to all sides of the square and has a radius shown. What are the odds that an arrow hits in the square but outside the circle? (nearest tenth)

14 cm
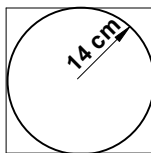
   (A) 27.3% (B) 21.5% (C) 20.0% (D) 15.5% (E) 14.0%

57. A bag contains white balls and red balls. If two balls are randomly removed, the probability that they are both white is $\frac{1}{3}$. If three are randomly removed, the probability that they are all white is $\frac{1}{6}$. How many red balls are in the bag?

(A) 4 (B) 5 (C) 6 (D) 8 (E) 10

58. All 15 residents of Millersview were asked to rate something from 1 to 5 where 1 is the worst and 5 is the best. The responses are distributed as shown in the chart. Find the sum of the mean, the standard deviation, and the variance. (nearest tenth)

Response	1	2	3	4	5
Frequency	4	5	3	1	2

(A) 5.5 (B) 2.9 (C) 4.8 (D) 5.4 (E) 5.9

59. Mr. White's 'bath tub mat' pattern table consists of 19 columns and 12 rows. Only 7 rows are shown. The sum of the numbers in column 16 row 10 and column 15 row 11 is?

1				1				2				3				5		
		2				3				5				8				
	3				5				8				13				21	
	5				8			13				21				34		
8				13			21				34				55			
		21				34				55				89				
	34				55				89				144				233

(A) 1,595 (B) 1,974 (C) 2,584 (D) 2,843 (E) 3,571

60. Mr. White wrote 11,127 math problems for UIL and TMSCA over the last 19 years. Twenty-two of those problems were fill in the blank problems with each having only one correct answer. All of the other problems were multiple choice problems containing one correct answer and four distracters. The total number of correct answers and distracters Mr. White had to come up with over the 19 year period is?

(A) 55,635 (B) 55,613 (C) 55,569 (D) 55,566 (E) 55,547

University Interscholastic League
MATHEMATICS CONTEST
HS ● State ● 2021
Answer Key

1. D	21. B	41. B
2. A	22. D	42. B
3. E	23. A	43. A
4. B	24. B	44. D
5. B	25. C	45. A
6. C	26. D	46. B
7. A	27. E	47. C
8. E	28. C	48. E
9. D	29. C	49. B
10. C	30. A	50. D
11. E	31. B	51. C
12. C	32. B	52. C
13. A	33. E	53. B
14. E	34. A	54. A
15. C	35. E	55. D
16. E	36. C	56. A
17. C	37. B	57. A
18. D	38. D	58. A
19. A	39. E	59. C
20. E	40. D	60. E

University Interscholastic League
MATHEMATICS CONTEST

Final _____ _____

2nd _____ _____

1st _____ _____

Score   **Initials**

Contestant # _____

Conference _____

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

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53. _____

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55. _____

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58. _____

59. _____

60. _____

Larry White - Mathematics Contest Director - texasmath@centex.net

    **As you know, this will be the last update of my 'Mathematics Test Corrections and Comments'
page for this 20-21 season and my last as the UIL Mathematics Contest Director.
Mr. McCurdy will be taking over as the contest director for the 21-22 season. I will be his
assistant and will work with him in helping to make this transition as smooth as possible. I
believe he will be a great director and will bring a freshness of ideas mixed in with his unique
style of problem writing. I thank all of you, past and present, for giving me the opportunity
these past 19 years to be part of your academic growth through UIL competition. It has been a
great joy and my pleasure to have served you in the best ways I could. I hope I have enhanced
your love for mathematics and hope I have shown you the glorious paths in life you can choose
to take. I have been deeply blessed.
This has been a most unusual season. I encourage everyone to let the UIL staff know
how thankful you are for all of the work it took them to put together a season amongst all of the
surrounding difficulties. Finding ways to hold district, regional, and state competitions was not
an easy task, to say the least. It would have been easier to cancel the season again as they had to
last year, but that is not how UIL reacts to hardships. Kudos to all the UIL staff and all the
coaches and hub workers for all their hard and tireless work to provide for our most valuable
assets; our students and their competive spirit.
I would like to congratulate all of the students for working through all of the issues facing us
this year and continuing to grow academically through UIL competitions. The knowledge and
skills gained through the UIL experience is everlasting and can never be taken away from you.
I would like to congratulate all 125 of the math students who made to state and a special
congratulations to those state championship individuals and those state championship teams. I
missed not getting to see you all and I sure missed not getting to put your medals around your
necks. Please don't forget to thank your parents, thank your coaches, thank your schools, and,
most importantly, thank HIM. I hope you all continue to find a few minutes each day in your life
to walk with, talk with, and give thanks to your creator. Without HIM the path is lonely and
bleak. I am looking forward to seeing you all in the Fall.**

UIL Test Comments — 2020-21

*** NOTE: See **Off on a Tangent** below for information on workshops, Student Activity Conferences,
   and test discussions *******

SAC - - - > No errors, corrections, or comments reported at this time. (Release dates: 10/1/20)

A - - - > # 53 needs an ' = ' sign after f(x)   (Release dates: 1/8/21 - 2/6/21)

B - - - > # 35 needs a '5' between 'of ' and 'fish '   (Release dates: 2/12/21 - 3/13/21)

District - - - > #23 -- the units in the answer choices should have been meters not feet.
             #35 -- should have said 'integers' not 'digits'   (Release dates: 3/22/21 - 3/27/21)

Regional - - - > No errors, corrections, or comments reported at this time. (Release dates: 4/16/21 - 4/17/21)

State - - - > No errors, corrections, or comments reported at this time. (Release date: 4/29/21 - 5/1/21)

TMSCA Test Comments — 2020-21 (tests I write for TMSCA)

 # 6 - - - >   No errors, corrections, or comments reported at this time. (Release date: 12/07/19)

 # 13 - - - >   No errors, corrections, or comments reported at this time. (Release date: 3/07/20)

State - - - >   No errors, corrections, or comments reported at this time. (Release date: 3/21/20)

Off on a Tangent

Workshops and/or Presentations I will be doing:
1. UIL Capitol Conference, Austin - ~~June 23-24, 2020~~. Cancelled --- Virtual
   See the UIL Academic website for two recorded Zoom sessions and multiple downloads
   Keep an eye out on the UIL Academic Website for information of the upcoming 2021 virtual Capitol Conference.

2. *What's Your 11th Problem* - Math Camp at Texas Tech University, Lubbock on ~~July 13-18, 2020~~.
   Cancelled --- future camps have not been addressed at this time ---
   For more information contact Jack Barton at jack.barton@ttu.edu or 806-742-2350.
3. Student Activity Conferences: **The 2020 conferences will be virtual.**

   --- This Year in Number Sense and Mathematics: News -Updates- Hot Topics
   (prerecorded session to be posted on the UIL Academic website on Oct. 1)
   --- Number Sense Problem Solving
   (prerecorded session to be posted on the UIL Academic website on Oct. 29)
   --- Mathematics Problem Solving
   (prerecorded session to be posted on the UIL Academic website on Oct. 29)
   --- Number Sense and Math - Coaches Chat
   (live zoom session at 4:00 pm Wednesday, Nov. 4 --- register in advance)

Resources Update

T**he mathematics contest is a curriculum based contest. The best resources for the contest are the courses and textbooks adopted by the state of Texas.**

Test Discussions

The district, regional, and state tests will be created based on the problems from these 6 tests:
2020SAC, TMSCA 6, UIL A, UIL B, TMSCA 13, & TMSCA STATE.
If a concept is addressed by a problem(s) from these 6 practice tests, then other types of problems in that concept area can appear on the district, regional, and state tests.

*** Not applicable for this year *** STATE MEET PROOFERS AND GRADERS