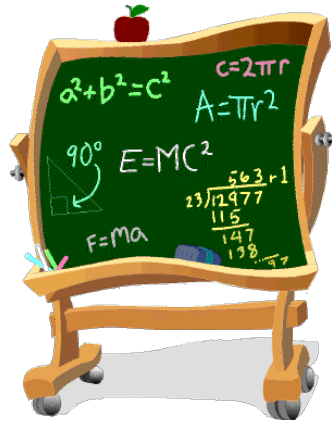




UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational B • 2024



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1. Denise drove to Lubbock and flew to Boise to spend some time with her cousins. A round-trip ticket cost \$815. She rented a car at the Boise Airport for \$37.95 per day plus \$0.25 per mile. She stayed for 10 days and drove a total of 225 miles while she was in Boise. Find the total cost of her trip.

- (A) \$1247.75 (B) \$1250.75 (C) \$1253.75 (D) \$1256.75 (E) \$1259.75

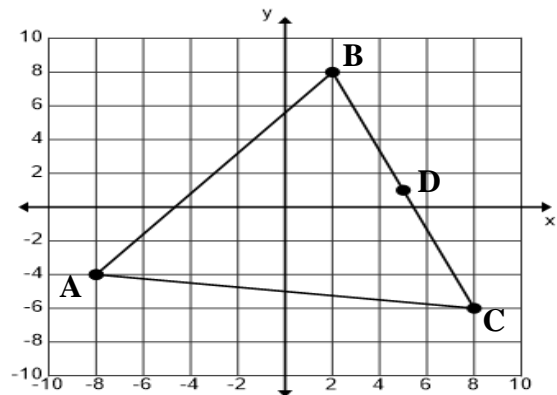
2. Julian is taking Calculus III this semester from Mrs. Edens. Sudan High School has a standard grading rubric. Forty percent of your semester grade is your homework average, 40% is your test average and 20% is your score on the semester exam. Julian has a 91 homework average. Julian's test scores are 93, 86, 84 and 91. An overall average of 90.00 or higher earns an A for the semester. What is the minimum score Julian can make on the final to earn an A?

- (A) 85 (B) 87 (C) 89 (D) 91 (E) 93

3-7. Consider $\triangle ABC$ shown on the right.

3. Point D is the midpoint of \overline{BC} . The y-intercept of \overline{AD} is the point $(0, b)$. $b = \underline{\hspace{2cm}}$.

- (A) $-\frac{14}{13}$ (B) -1 (C) $-\frac{12}{13}$
 (D) $-\frac{11}{13}$ (E) $-\frac{10}{13}$



Problems 3, 4, 5, 6, 7

4. Find the perimeter of $\triangle ADC$. (nearest tenth)

- (A) 37.1 (B) 37.3 (C) 37.5 (D) 37.7 (E) 37.9

5. Find the area of $\triangle ABD$. (nearest whole number)

- (A) 45 (B) 47 (C) 49 (D) 51 (E) 53

6. Find the measure of $\angle ABC$. (nearest tenth)

- (A) 63.0° (B) 63.3° (C) 63.6° (D) 63.9° (E) 64.2°

7. Find the length of the longest median of $\triangle ADC$. (nearest tenth)

- (A) 14.4 (B) 14.6 (C) 14.8 (D) 15.0 (E) 15.2

8. Riley went for a bike ride on Saturday. First, Riley left Sudan and headed north on Hwy 303 for 12 miles. Next, Riley headed east on Hwy 70 for 6 miles. Then, Riley headed south on Hwy 1055 for 16 miles. Finally, Riley headed straight back to Sudan on Hwy 84. If Riley averaged 25 mph, how long did the bike ride take? (nearest minute)

- (A) 1 hr 33 min (B) 1 hr 35 min (C) 1 hr 37 min (D) 1 hr 39 min (E) 1 hr 41 min

9. Donald can mow Oscar’s property by himself in 7.5 hours. On Saturday, Donald started mowing at 8:00. At 10:00, Jaysean came to help Donald and together they finished mowing the property at 12:00. How long would it take Jaysean to mow Oscar’s property by himself? (nearest minute)

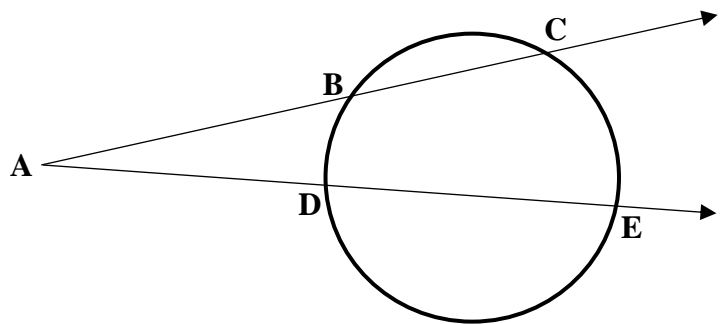
- (A) 238 min (B) 241 min (C) 251 min (D) 254 min (E) 257 min

10. The equation of state of a hypothetical ideal gas is given by $PV = nRT$, where P is the pressure in pascals, V is the volume in cubic meters, n is the number of moles, R is a constant equal to 8.31446, and T is the temperature in kelvins of the gas. Laynee has 2.24 moles of a gas at 333 K. If it has a volume of 0.042 m^3 , what is the pressure of the gas. (nearest whole number)

- (A) 147,659 Pa (B) 147,662 Pa (C) 147,665 Pa (D) 147,668 Pa (E) 147,671 Pa

11-12. Consider the circle shown with minor arcs CE and BD .

Given: $m\angle CE = 86^\circ$, $m\angle BD = 52^\circ$, $AB = 14$,
 $BC = 12$, and $AD = 13$.



Problems 11, 12

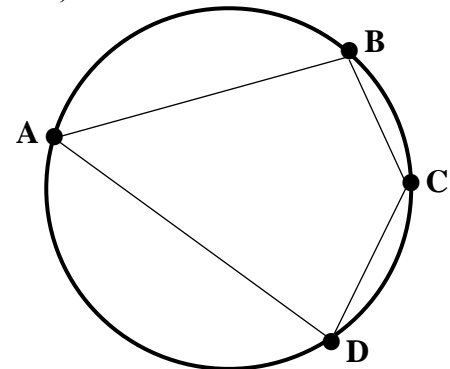
11. $m\angle CAE =$ _____. (nearest tenth)

- (A) 17.0°
 (B) 17.2°
 (C) 17.4°
 (D) 17.6°
 (E) 17.8°

12. $DE =$ _____. (nearest tenth)

- (A) 14.6 (B) 14.8 (C) 15.0 (D) 15.2 (E) 15.4

13-14. Consider the circle shown with $m\angle DAB = 62^\circ$, $m\angle ADC = 86^\circ$,
 and minor arc BC with $m\angle BC = 56^\circ$.



Problems 13, 14

13. $m\angle BCD =$ _____.

- (A) 116°
 (B) 117°
 (C) 118°
 (D) 119°
 (E) 120°

14. The measure of minor arc AD is _____. (nearest whole number)

- (A) 114° (B) 116° (C) 118° (D) 120° (E) 122°

15. A cone has a volume of 2654.65 and a diameter of 26. Find the total surface area of the cone. (nearest whole number)

- (A) 1330 (B) 1333 (C) 1336 (D) 1339 (E) 1342

16-17. Given: $\triangle ABC$ with $m\angle ABC = 90^\circ$. Point D lies on \overline{AC} such that $m\angle ADB = 90^\circ$.
 $AD = 10$ and $CD = 18$.

16. Find the perimeter of $\triangle ABC$. (nearest tenth)

- (A) 65.6 (B) 66.0 (C) 66.4 (D) 66.8 (E) 67.2

17. Find the area of $\triangle BDC$. (nearest whole number)

- (A) 119 (B) 121 (C) 123 (D) 125 (E) 127

18-19. Given: $\triangle DEF$ with $DE = 16$ and $EF = 24$. Point G lies on \overline{DF} such that $FG = 16$ and \overline{EG} bisects $\angle DEF$.

18. $DG =$ _____.

- (A) $\frac{28}{3}$ (B) $\frac{29}{3}$ (C) 10 (D) $\frac{31}{3}$ (E) $\frac{32}{3}$

19. $m\angle GEF =$ _____ (nearest tenth)

- (A) 40.5° (B) 40.8° (C) 41.1° (D) 41.4° (E) 41.7°

20. The sum of the measures of the interior angles of an icosagon (20 sides) is _____.

- (A) 3240° (B) 3312° (C) 3384° (D) 3456° (E) 3600°

21-22. Given: The length of a side of a regular icosagon is 8. The icosagon is inscribed in a circle.

21. The area of the icosagon is _____ (nearest tenth)

- (A) 2012.0 (B) 2014.1 (C) 2016.2 (D) 2018.3 (E) 2020.4

22. The area of the circle is _____ (nearest tenth)

- (A) 2048.7 (B) 2050.8 (C) 2052.9 (D) 2054.0 (E) 2056.1

23. A hawk is perched at the edge of the roof of the Three Rivers State bank. The hawk spots a mouse at an angle of depression of 37° on the ground below. The mouse is located 100 feet from the base of the bank. How tall is the Three Rivers State bank? (nearest inch)

- (A) 74 ft 2 in (B) 74 ft 3 in (C) 75 ft 4 in (D) 76 ft 5 in (E) 77 ft 6 in

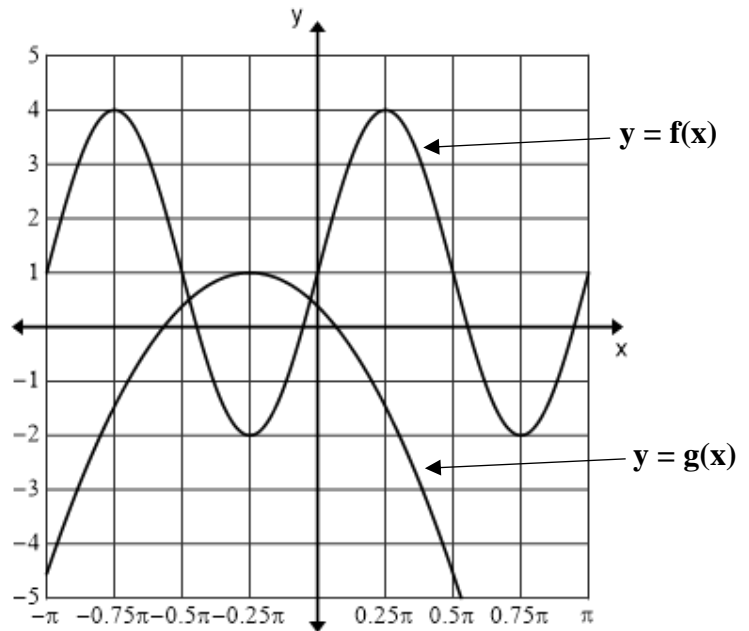
24. Given: $f^{-1}(x) = \frac{7x+b}{-2x+3}$ and $f^{-1}(1) = 12$. Evaluate $f(-3)$.

- (A) -15 (B) -14 (C) -13 (D) -12 (E) -11

25. At the 2023 graduation ceremony for Rankin High School, 6 chairs were placed in the front row for the honor graduates. If Madison insisted on sitting next to Jaxon, how many different seating arrangements for the front row were possible?
- (A) 80 (B) 240 (C) 400 (D) 560 (E) 720
26. Consider the circle $x^2 + y^2 + ax + by + c = 0$. The center of the circle is the point $(-4, 6)$ and the diameter is 18. $a + b + c =$ _____.
- (A) -35 (B) -33 (C) -31 (D) -29 (E) -27
27. Find the number that is $\frac{3}{4}$ of the way from $-2\frac{1}{3}$ to $5\frac{5}{6}$.
- (A) $\frac{87}{24}$ (B) $\frac{11}{3}$ (C) $\frac{89}{24}$ (D) $\frac{15}{4}$ (E) $\frac{91}{24}$
28. The equation of the line that is equidistant from the points $(-5, 9)$ and $(7, -6)$ is $8x + by + c = 0$. $b + c =$ _____.
- (A) -7 (B) -5 (C) -3 (D) -1 (E) 1
29. Find the number of ways 10 charms can be arranged on a charm bracelet.
- (A) 40,320 (B) 181,440 (C) 362,880 (D) 1,814,400 (E) 3,628,800
30. Consider the parabola with a vertex at $(3, 4)$ and a focus at $(5, 4)$. If the point $(11, b)$ lies on the graph of the parabola and $b > 0$, then $b =$ _____.
- (A) 10 (B) 11 (C) 12 (D) 13 (E) 14
- 31-32. The vertices of an ellipse are $(2, -4)$ and $(10, -4)$. The length of the minor axis is 6.
31. The eccentricity of the ellipse is _____. (nearest hundredth)
- (A) 0.64 (B) 0.66 (C) 0.68 (D) 0.70 (E) 0.72
32. Find the distance from the center of the ellipse to the line $y = .75x + 6$. (nearest tenth)
- (A) 10.7 (B) 11.0 (C) 11.3 (D) 11.6 (E) 11.9
33. My clock shows that it is exactly 3:00. How long will I have to wait until both hands (hour hand and minute hand) of my clock point in the same direction for the second time? (nearest second)
- (A) 81 min 46 sec (B) 81 min 49 sec (C) 81 min 52 sec (D) 81 min 55 sec (E) 81 min 58 sec

34. A population of Goliath Birdeater spiders is increasing exponentially in Gaines County. Anthony introduced a population of n Birdeaters at $t = 0$. At $t = 60$ days, the population reached 72 Birdeaters. At $t = 150$ days, the population reached 374 Birdeaters. Find n .
- (A) 18 (B) 20 (C) 22 (D) 24 (E) 26
35. A ball is dropped from a height of 200 feet. On each bounce, the ball rebounds three-fifths of the distance it fell. How far does the ball rebound on the 12th bounce? (nearest inch)
- (A) 4 (B) 5 (C) 6 (D) 7 (E) 8
- 36-37. The equation of a conic is $2x^2 + 6xy - y^2 + 4x + 6y + 12 = 0$.
36. Classify the conic.
- (A) circle (B) parabola (C) ellipse (D) hyperbola (E) line
37. The angle of rotation is _____. (nearest tenth)
- (A) 30.5° (B) 30.8° (C) 31.1° (D) 31.4° (E) 31.7°
38. Three times the complement of angle A exceeds the supplement of angle A by 18° . Find the measure of angle A.
- (A) 36° (B) 38° (C) 40° (D) 42° (E) 44°

39-43. Consider the graph shown on the right. The equation of $y = f(x)$ is of the form $f(x) = c + a\sin(bx)$.



Problems 39, 40, 41, 42, 43

39. $a + b + c =$ _____.

- (A) 2
- (B) 3
- (C) 4
- (D) 5
- (E) 6

40. The graph of $g(x) = -\left(x + \frac{\pi}{4}\right)^2 + 1$ intersects the graph of $f(x)$ at points E and F. $EF =$ _____.
(nearest hundredth)

- (A) 1.35
- (B) 1.37
- (C) 1.39
- (D) 1.41
- (E) 1.43

41. The x-intercept of the line tangent to the graph of $y = g(x)$ when $x = \frac{\pi}{6}$ is the point $(j, 0)$. $j =$ _____.
(nearest hundredth)

- (A) 0.21
- (B) 0.23
- (C) 0.25
- (D) 0.27
- (E) 0.29

42. Find the area bounded by the graphs of $y = f(x)$ and $y = g(x)$. (nearest tenth)

- (A) 2.5
- (B) 2.7
- (C) 2.9
- (D) 3.1
- (E) 3.3

43. Find the volume generated when the region bounded by the graphs of $y = f(x)$ and $y = g(x)$ is revolved about the line $x = -\pi$. (nearest whole number)

- (A) 36
- (B) 37
- (C) 38
- (D) 39
- (E) 40

44. The derivative of the function $f(x)$ is given by $f'(x) = -\frac{\pi}{x} \cdot \sin(0.5x - \pi)$. If $f(x)$ is continuous for all positive real numbers, then the maximum value of $f(x)$ for $2\pi \leq x \leq 4\pi$ is _____.

- (A) $f(2\pi)$
- (B) $f(2.5\pi)$
- (C) $f(3\pi)$
- (D) $f(3.5\pi)$
- (E) $f(4\pi)$

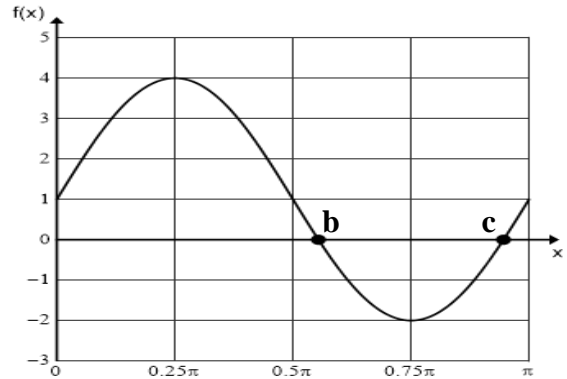
45. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x^2 + y$ with the initial condition $f(0) = 3$. Find the approximation for $f(-1)$ obtained using Euler's method with two equal steps starting at $x = 0$. (nearest thousandth)

- (A) 0.625
- (B) 0.636
- (C) 0.647
- (D) 0.658
- (E) 0.669

46. Given: $\frac{dy}{dx} = 2x(3-y)$ and $y = -1$ when $x = 0$. Find the value of y when $x = 2$. (nearest tenth)

- (A) 2.1 (B) 2.3 (C) 2.5 (D) 2.7 (E) 2.9

47. Consider the graph of $f(x)$ shown on the right. Arrange in order from least to greatest.



Problem 47

- I. $\int_0^b f(x)dx$ II. $\int_0^c f(x)dx$
 III. $\int_0^c |f(x)|dx$ IV. $\int_b^c f(x)dx$

- (A) I, II, III, IV (B) I, IV, III, II
 (C) IV, II, I, III (D) IV, II, III, I
 (E) III, IV, I, II

48-49. Consider the polar curve $r = 3 + 3\sin(\theta)$.

48. Find the area of the region in the fourth quadrant bounded by the graph of this polar curve and the x -axis. (nearest tenth)

- (A) 1.4 (B) 1.6 (C) 1.8 (D) 2.0 (E) 2.2

49. The y -intercept of the line tangent to the graph of this polar curve at $\theta = \frac{7\pi}{4}$ is the point $(0, b)$.

$b = \underline{\hspace{2cm}}$. (nearest hundredth)

- (A) -0.45 (B) -0.42 (C) -0.39 (D) -0.36 (E) -0.33

50. The radius of a sphere is increasing at a rate of 9.2 inches per minute. What is the volume of the sphere at the instant that the volume is increasing at 462 cubic inches per minute? (nearest tenth)

- (A) 33.1 in^3 (B) 33.3 in^3 (C) 33.5 in^3 (D) 33.7 in^3 (E) 33.9 in^3

51. Find the radius of convergence for the series $\sum_{n=1}^{\infty} \frac{(x+3)^n}{n \cdot 4^{n+1}}$.

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

52. Ron just purchased a certified used car. The salesman told him that in the coming year, the probability of a transmission issue is 0.03, the probability of an alternator issue is 0.02, and the probability of a braking issue is 0.06. If these issues occur independently of each other, what is the probability that the car will not have one of these issues next year? (nearest thousandth)

- (A) 0.861 (B) 0.872 (C) 0.883 (D) 0.894 (E) 0.905

Tire A	44,000	48,500	52,200	39,000	47,800	50,600	49,700
Tire B	51,000	50,200	47,900	55,600	53,000	50,200	49,800

Ron claims that tires made by company B will outlast tires made by company A. Seven tires from each company were tested independently and the miles were recorded. Use the table above for problems 53-55.

53. Find the sum of the mean, median and mode of the Tire B results. (nearest whole number)

- (A) 151,488 (B) 151,492 (C) 151,496 (D) 151,500 (E) 151,554

54. Find the IQR of the Tire A results.

- (A) 6,400 (B) 6,500 (C) 6,600 (D) 6,700 (E) 6,800

55. Assuming that all conditions for inference are met, an appropriate test was conducted to see if company B tires do last longer than company A tires based on the data. The p-value of the test was _____. (nearest thousandth)

- (A) 0.033 (B) 0.044 (C) 0.055 (D) 0.066 (E) 0.077

56-57. Assume that the distribution of the heights of high school senior boys in Allen High School is approximately normal with a mean of 70 inches and a standard deviation of 5.5 inches.

56. Mark is 5 ft 6 in tall. What percentile does that place Mark at?

- (A) 17th (B) 19th (C) 21st (D) 23rd (E) 25th

57. What is the IQR for the heights of senior boys in Allen High School? (nearest hundredth)

- (A) 7.30 (B) 7.33 (C) 7.36 (D) 7.39 (E) 7.42

Pies	Apple	Cherry	Chocolate	Lemon	Blueberry
# of students	26	28	24	12	10

Caroline believes that the students at Argyle High School like all five kinds of pies that are sold in the cafeteria equally. She randomly selected 100 students and asked them to identify their favorite type of pie. The results are in the table above. Use the table above for problems 58 and 59.

58. Caroline assumed that all conditions for inference were met and she conducted an appropriate test. The p-value of the test was _____. (nearest thousandth)

- (A) 0.007 (B) 0.010 (C) 0.013 (D) 0.016 (E) 0.019

59. The blueberry cell contributed _____ to the chi-square statistic. (nearest hundredth)

- (A) 4.22 (B) 4.44 (C) 4.66 (D) 4.88 (E) 5.00

60. Sarah rolls a fair die 10 times. What is the probability that she will get a 5 at least 3 times? (nearest thousandth)

- (A) 0.217 (B) 0.221 (C) 0.225 (D) 0.229 (E) 0.232

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**University Interscholastic League
MATHEMATICS CONTEST
HS • Invitational B • 2024
Answer Key**

1. B	21. E	41. C
2. D	22. D	42. B
3. C	23. C	43. E
4. D	24. B	44. A
5. E	25. B	45. A
6. A	26. B	46. E
7. B	27. E	47. C
8. D	28. C	48. B
9. E	29. B	49. D
10. C	30. C	50. C
11. A	31. B	51. D
12. C	32. D	52. D
13. C	33. B	53. D
14. D	34. D	54. C
15. E	35. B	55. B
16. E	36. D	56. D
17. B	37. E	57. E
18. E	38. A	58. A
19. A	39. E	59. E
20. A	40. D	60. C