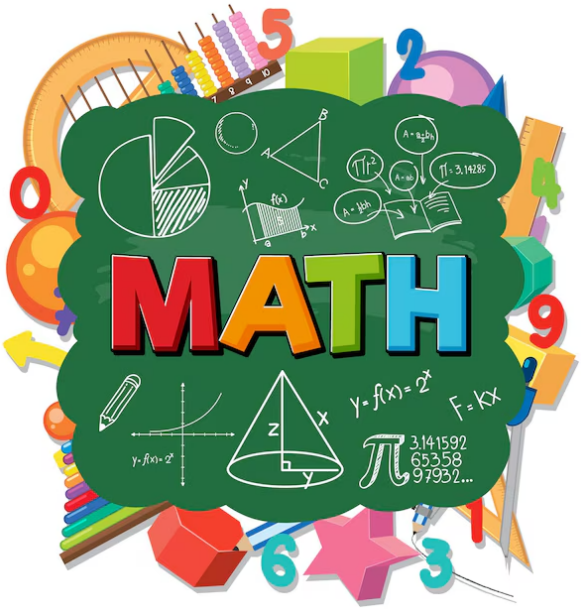
UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational B • 2025



DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO!

1. Moses got a $300 gift card from Academy Sports in O'Donnell. He bought some Nike shoes for $165.57, some Adidas shorts for $35.22, an Under Armor t-shirt for $29.95, and some socks for $12.95. The tax rate is 8.25%. How much is left on his gift card?

(A) $36.09 (B) $36.13 (C) $36.17 (D) $36.21 (E) $36.25

2. The total resistance R of three resistors connected in parallel is given by $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$. If $R = 48.65\ \Omega$, $R_1 = 120\ \Omega$ and $R_2 = 180\ \Omega$, then R_3 = _____ Ω. (nearest whole number)

(A) 90 (B) 120 (C) 150 (D) 180 (E) 210

3. Consider three consecutive odd numbers with a sum of 117. Find the product of these numbers.

(A) 59,155 (B) 59,157 (C) 59,159 (D) 59,161 (E) 59,163

4. The number of cats in small West Texas towns varies directly as the number of mice and inversely as the number of dogs. If Sundown has 84 cats, 56 dogs and 265 mice, how many cats will Sudan have if they have 48 dogs and 419 mice?

(A) 151 (B) 153 (C) 155 (D) 157 (E) 159

5. It is 108 miles from Childress to Canadian. Joan leaves Childress at 1:00 PM and begins running north on the highway toward Canadian at 11.2 mph. At 2:00 PM, Alberto leaves Canadian and begins running south on the same highway toward Childress at 12.2 mph. What time do they meet? (nearest minute)

(A) 6:06 PM (B) 6:08 PM (C) 6:10 PM (D) 6:12 PM (E) 6:14 PM

6. Running at full speed, a boat can travel 105 miles downstream in 3.5 hours, but it requires 7.5 hours to travel 105 miles upstream. How fast can the boat travel in still waters running at full speed? (nearest whole number)

(A) 19 mph (B) 20 mph (C) 21 mph (D) 22 mph (E) 23 mph

7. If s(x) is the slant asymptote of the graph of $h(x)=\frac{2x^2-6x+15}{x-1}$, then h(36) – s(36) = _____. (nearest tenth)

(A) 0.23 (B) 0.25 (C) 0.27 (D) 0.29 (E) 0.31

8. Addison is driving his 2025 Ford 150 Lightning pickup on Hwy 95 at a speed of 60 mph. Each of the tires has a radius of 17 inches. What is the rotational speed of the tires? (nearest whole number)

(A) 587 rpm (B) 590 rpm (C) 593 rpm (D) 596 rpm (E) 599 rpm

9. The manager of Millersview Grocers took a three-week vacation. The first week he was gone, Joey raised the price of lemons by 22%. The next week, he raised the price of lemons by another 28%. The third week, he raised the price by another 19%. When the manager returned, he told Joey to lower the price to the pre-vacation price. By what percent did he have to decrease the price? (nearest whole number)

(A) 46% (B) 48% (C) 50% (D) 52% (E) 54%

10. Thomas has only pennies and dimes in a jar on his desk. The mass of 1000 pennies is 2.500 kg and the mass of 1000 dimes is 2.268 kg. The total value of the coins is $131.00 and the total mass of the coins is 5.4716 kg. How many dimes does Thomas have?

(A) 1190 (B) 1195 (C) 1200 (D) 1205 (E) 1210

11-15. Consider ΔABC with vertices $A(-8,-2)$, $B(2,6)$ and $C(8,-8)$. Point D lies on $\overline{AC}$ such that ray $\overrightarrow{BD}$ bisects $\angle ABC$. Point E is the midpoint of $\overline{AB}$.

11. Find the perimeter of ΔECA. (nearest tenth)

(A) 38.1 (B) 38.4 (C) 38.7 (D) 39.0 (E) 39.3

12. ΔECB is a/an _____ triangle.

(A) equilateral (B) isosceles (C) right (D) acute (E) obtuse

13. AD = _____. (nearest tenth)

(A) 7.8 (B) 8.1 (C) 8.4 (D) 8.7 (E) 9.0

14. BD = _____. (nearest tenth)

(A) 10.9 (B) 11.1 (C) 11.3 (D) 11.5 (E) 11.7

15. The area of ΔBAD = _____. (nearest tenth)

(A) 41.7 (B) 42.0 (C) 42.3 (D) 42.6 (E) 42.9

16. The volume of a cone with a diameter of 12.8 cm is 755 cm^3. The total area of the cone is _____ cm^2. (nearest whole number)

(A) 496 (B) 499 (C) 502 (D) 505 (E) 508

17. Consider an equilateral triangle inscribed in a circle. If the area of the triangle is 73.18 cm^2, then the area of the circle is _____ cm^2. (nearest whole number)

(A) 165 (B) 168 (C) 171 (D) 174 (E) 177

18. The area of a rectangle is 209.6 cm^2. A diagonal of the rectangle is twice as long as the width.

The length of the rectangle is _____ cm. (nearest whole number)

(A) 18 (B) 19 (C) 20 (D) 21 (E) 22

19-20. Point O is the center of a circle with a radius of 17 inches. Point O is 8 inches from chord $\overline{AB}$.

19. AB = _____. (nearest tenth)

(A) 29.8 (B) 30.0 (C) 30.2 (D) 30.4 (E) 30.6

20. The area of the region between minor arc AB and chord $\overline{AB}$ equals _____. (nearest whole number)

(A) 176 (B) 180 (C) 184 (D) 188 (E) 192

21. Find the range of $f(x) = \sqrt{6 - x - x^2}$.

(A) (0, 2.5] (B) [0, 2.5] (C) [0, 2.5) (D) (0, 2.5) (E) $(0, \infty)$

22. If $f(x) = 2x^2 + 3x - 5$ and $g(x) = x + 1$, then $(f \circ g)(2) =$ _____.

(A) 18 (B) 19 (C) 20 (D) 21 (E) 22

23. The Knippa State Bank is the tallest building in town. A hungry hawk is perched on the edge of the roof. Sophia's bunny escaped from her back yard and was resting in the grass 318 feet from the base of the bank. If the angle of elevation from the bunny to the hawk is 37°, how tall is the bank? (nearest whole number)

(A) 234 ft (B) 237 ft (C) 240 ft (D) 243 ft (E) 246 ft

24. The seven members of the Grand Saline math team were seated in a circle around a large circular table along with their coach in math practice. If there were eight seats available, how many distinct seating arrangements were possible?

(A) 24 (B) 120 (C) 720 (D) 5040 (E) 40320

25. A regular nonagon is inscribed in a circle. If the area of the circle is 107.425, what is the area of the nonagon? (nearest whole number)

(A) 95 (B) 97 (C) 99 (D) 101 (E) 103

26. Consider a geometric sequence in which the 1st term is 8 and the 4th term is $-\frac{27}{8}$. Find the sum of the first 10 terms of the sequence. (nearest thousandth)

(A) 4.299 (B) 4.302 (C) 4.307 (D) 4.310 (E) 4.314

27. Consider the graph of $f(x)=5+3\csc\left(\frac{2\pi x}{3}-\frac{\pi}{6}\right)$. The phase shift is _____ to the right.

(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{\pi}{6}$ **(E)** $\frac{\pi}{3}$

28. The Nazareth Swiftettes have advanced to 31 state tournaments and have won the state tournament 25 times. Coach Lombard had 2 posts, 6 wings and 4 guards on his 1984 team. If his starting lineup always consisted of one post, two wings and two guards, how many starting lineups were possible?

(A) 48 (B) 86 (C) 180 (D) 8,240 (E) 34,560

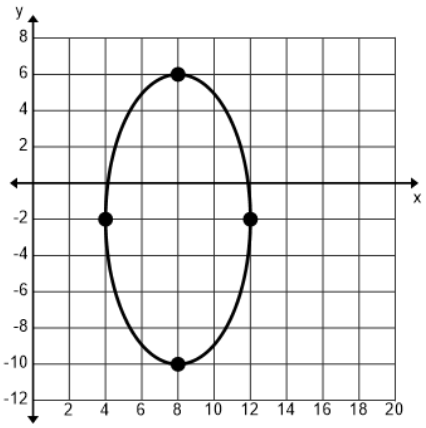
29. Consider the circle $x^2+y^2+ax+by+c=0$. The center of the circle is the point $(6,-2)$ and the diameter is 22. $a+b+c=$ _____.

(A) –89 (B) –87 (C) –85 (D) –83 (E) –81

30. Find the eccentricity of the ellipse. (nearest thousandth)

(A) 0.844
(B) 0.855
(C) 0.866
(D) 0.877
(E) 0.888

31. The parametric equations that produce the ellipse are $x(t)=a\cos(t)+b$ and $y(t)=c\sin(t)+d$, $-2\pi\le t\le 2\pi$. $a+b=$ _____.



Problems 30, 31, 32

(A) –6 (B) –4 (C) 6
(D) 10 (E) 12

32. Find the distance from the center of the ellipse to the line $y=.25x+12$. (nearest tenth)

(A) 15.5 (B) 15.8 (C) 16.1 (D) 16.4 (E) 16.7

33. The graph of the polar equation $r=2+4\cos\theta$ is a _____.

(A) cardioid
(B) four petaled rose curve
(C) circle of radius 4
(D) limacon with an inner loop
(E) dimpled limacon

34. When you get into a Ferris Wheel car at the bottom of the ride, you are 2 feet off the ground. When the ride starts and the Ferris Wheel begins rotating, it takes 15 seconds to reach the highest point of the ride where you are 82 feet off the ground. Ivan gets into a car at the bottom and the Ferris Wheel rotates for 36 seconds and stops. How far above the ground is Ivan? (nearest inch)

(A) 29 ft 8 in (B) 30 ft 1 in (C) 30 ft 6 in (D) 30 ft 11 in (E) 31 ft 4 in

35-36. Ship A leaves port at 12:00 PM and travels on a bearing of 90° at 24 mph. Ship B leaves port at 1:00 PM and travels on a bearing of 180° at 20 mph. Ship C leaves port at 2:00 PM and travels on a bearing of 270° at 16 mph.

35. How far apart are ship A and ship B at 6:00 PM? (nearest whole number)

(A) 172 mi (B) 175 mi (C) 178 mi (D) 181 mi (E) 184 mi

36. At what time will ship A and ship C be 300 mi apart? (nearest minute)

(A) 8:15 PM (B) 8:18 PM (C) 8:21 PM (D) 8:24 PM (E) 8:27 PM

37. A population of Giant Flying Joro Spiders was discovered on June 15th, 2022 on the Newberry Ranch near Seminole. Female Joro spiders are yellow with legs that can grow up to four inches long. The original population included 114 adult females. On June 15th, 2024, the population reached 998 adult females. Assume exponential growth and predict the population on June 15th, 2029.

(A) 226,293 (B) 226,297 (C) 226,301 (D) 226,305 (E) 226,309

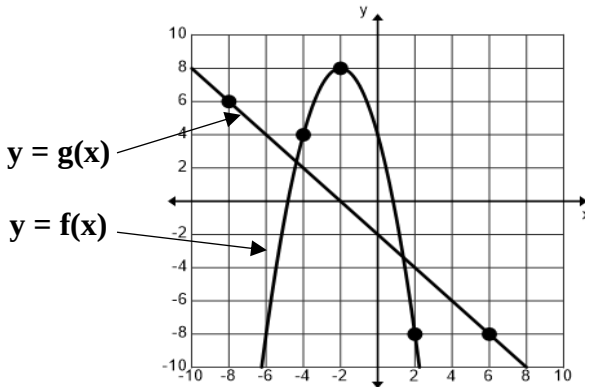
38. Find the acute angle between the line $2x - y = 4$ and the line $3x + 7y = 9$. (nearest tenth)

(A) 83.3° (B) 84.4° (C) 85.5° (D) 86.6° (E) 87.7°

39. Amarillo Slim is playing 5 card poker with a well shuffled, standard 52-card deck. What is the probability that he will be dealt a full house? An example of a full house is 3 jacks and 2 queens. (nearest hundred-thousandth)

(A) 0.00111 (B) 0.00122 (C) 0.00133 (D) 0.00144 (E) 0.00155

40-41. The graphs of $y = f(x)$ and $y = g(x)$ are shown on the right.



Problems 40, 41

40. The focus of the graph of $y = f(x)$ is the point $(-2, b)$. $b =$ _____.

(A) 6 (B) 7 (C) $\frac{15}{2}$ (D) $\frac{31}{4}$ (E) $\frac{63}{8}$

41. Find the slope of the line normal to the graph of $y = f(x)$ at the point $P(-4, 4)$.

(A) $-\frac{5}{4}$ (B) -1 (C) $-\frac{3}{4}$ (D) $-\frac{1}{2}$ (E) $-\frac{1}{4}$

42. A spherical balloon is losing helium at a constant rate of 4π cm^3 per minute while maintaining its spherical shape. When the radius of the sphere is 20 cm, the surface area of the sphere is decreasing at a rate of _____ cm^2 per minute. (nearest hundredth)

(A) 1.26 (B) 1.37 (C) 1.48 (D) 1.59 (E) 1.70

43. Find the dimensions of the rectangle with the largest area that can be inscribed in a semicircle of radius 8. The area of this rectangle is _____. (nearest whole number)

(A) 64 (B) 66 (C) 68 (D) 70 (E) 72

44-45. Consider the graph of $f(x) = 0.0105x^3 - 0.2x + 3$.

44. Find the number c in the interval $(-4, 9)$ that satisfies the mean value theorem for $f(x)$. (nearest tenth)

(A) 4.3 (B) 4.5 (C) 4.7 (D) 4.9 (E) 5.1

45. Find the average value of $f(x)$ over the interval $[-5, 10]$. (nearest tenth)

(A) 4.1 (B) 4.3 (C) 4.5 (D) 4.7 (E) 4.9

46. Air pressure decreases continually with the height above sea level at a rate proportional to the pressure at that height. Desarae's barometer reads 101.3 kP at sea level and 89.415 kP at 1000 m. At what height above sea level will her barometer read exactly one-half the reading at sea level? (nearest whole number)

(A) 5510 m (B) 5521 m (C) 5532 m (D) 5543 m (E) 5554 m

47-48. A particle is traveling along the x-axis. At t = 0, the position of the particle is at x = 6 cm. The velocity of the particle is given by $v(t) = t^3 - 8t^2 + 3t + 12,\ 0 \le t \le 10$.

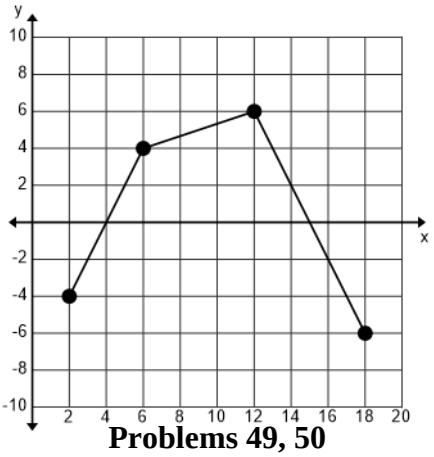
47. The position of the particle when the acceleration of the particle is 15 cm/s^2 to the right is at x = _____ cm. (nearest negative integer)

(A) −120 cm (B) −114 cm (C) −108 cm (D) −102 cm (E) −96 cm

48. Find the total distance traveled by the particle on the time interval $[0, 10]$. (nearest whole number)

(A) 415 cm (B) 424 cm (C) 433 cm (D) 442 cm (E) 451 cm

49-50. The graph of the piecewise function h(x) is shown on the right.



Problems 49, 50

49. $\int_2^{18} h(x)dx =$ _____. (nearest whole number)

(A) 26 (B) 28
(C) 30 (D) 32
(E) 34

50. $\int_2^{18} h'(x)dx =$ _____. (nearest integer)

(A) −2 (B) 0 (C) 2
(D) 4 (E) 6

51. The graph of $y = f(x)$ passes through the point $(1, 6)$ and satisfies the differential equation $\frac{dy}{dx} = \frac{3x^2 - 2}{y}$. Solve the differential equation and find the value of b if the point $(2, b)$ lies on the graph of $y = f(x)$. (nearest tenth)

(A) 6.6 (B) 6.8 (C) 7.0 (D) 7.2 (E) 7.4

52. Find the radius of convergence of $\sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+1}}{(3n+1)!}$.

(A) $\frac{1}{3}$ (B) 1 (C) 3 (D) 9 (E) ∞

53. Assume 60% of the boys at NCHS play a varsity sport and 50% of those obtain a college degree. Only 30% of the other boys obtain a college degree. Given that a randomly selected graduate of NCHS has a college degree, what is the probability that he played a varsity sport in high school? (nearest hundredth)

(A) 0.65 (B) 0.67 (C) 0.69 (D) 0.71 (E) 0.73

Time (hr)	0	1	2	3.5	5.5	9
Score	120	144	158	200	238	324

54-55. Dominic recorded his mathematics test scores and the number of hours he practiced the week of each test. The results are in the table above. He plotted the data and decided there was a linear relationship between time spent practicing and the resulting test score on Saturday.

54. The regression equation predicts an increase of _____ points for each additional hour he practices. (nearest tenth)

(A) 22.0 (B) 22.3 (C) 22.6 (D) 22.9 (E) 23.1

55. The residual for 3.5 hours of practice is _____. (nearest tenth)

(A) 2.7 (B) 2.9 (C) 3.1 (D) 3.3 (E) 3.5

5-year	46	52	56	48	58	64	48	44
3-year	50	42	46	52	56	54	38	48

56-57. Ted claims that his 5-year batteries will last longer than his 3-year batteries. Eight batteries of each type are tested independently and the number of months each battery lasted is recorded.

56. If a 5-year battery costs $425, find the average cost per month for a 5-year battery.

(A) $8.11 (B) $8.14 (C) $8.17 (D) $8.20 (E) $8.23

57. Assume the assumptions for inference have been met and conduct an appropriate test. The p-value of this test is _____. (nearest hundredth)

(A) 0.05 (B) 0.07 (C) 0.09 (D) 0.11 (E) 0.13

Day	Sun	Mon	Tues	Wed	Thur	Fri	Sat
# C.E.	48	52	38	42	36	46	53

58-59. The number of patients who came to the emergency room at the Patton Springs Medical Center with cardiac events last year is shown in the table. Jet analyzed the data. He also performed a Chi-Square GOF Test with the following null hypothesis.
H_0: ER visits at the Medical Center are evenly distributed across the days of the week.

58. The IQR of the data is _____. (nearest whole number)

(A) 13 (B) 14 (C) 15 (D) 16 (E) 17

59. The Chi-Square statistic is _____. (nearest tenth)

(A) 5.8 (B) 6.0 (C) 6.2 (D) 6.4 (E) 6.6

60. The Thursday cell contributed _____ to the Chi-Square statistic. (nearest tenth)

(A) 1.4 (B) 1.6 (C) 1.8 (D) 2.0 (E) 2.2

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University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitational B ● 2025
Answer Key

1. D	**21. B**	**41. E**
2. C	**22. E**	**42. A**
3. E	**23. C**	**43. A**
4. C	**24. D**	**44. B**
5. B	**25. C**	**45. A**
6. D	**26. E**	**46. E**
7. E	**27. A**	**47. A**
8. C	**28. C**	**48. E**
9. A	**29. A**	**49. C**
10. C	**30. C**	**50. A**
11. B	**31. E**	**51. B**
12. D	**32. A**	**52. E**
13. A	**33. D**	**53. D**
14. B	**34. A**	**54. C**
15. E	**35. B**	**55. A**
16. D	**36. B**	**56. C**
17. E	**37. D**	**57. E**
18. B	**38. D**	**58. B**
19. B	**39. D**	**59. A**
20. E	**40. D**	**60. C**