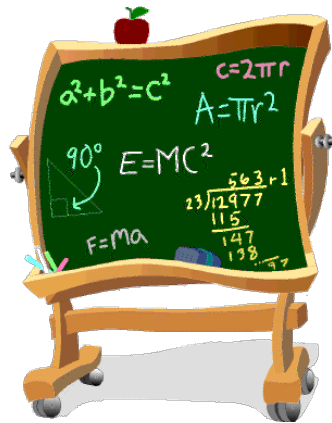




UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Region • 2024



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1. The rental fee from AAA Car Rentals includes a charge for each day the car is rented as well as a charge for each mile the car is driven. Tom rented a car for four days, drove 177 miles, and his bill was \$177.56. Diane rented a car for six days, drove 208 miles, and her bill was \$250.24. Find the charge for each mile driven.
- (A) \$0.24 (B) \$0.26 (C) \$0.28 (D) \$0.30 (E) \$0.32
- 2-5. Given points $A(-4,2)$, $B(6,8)$, $C(2,-4)$, $D(a,b)$ and $E(5,c)$.
2. If $\overrightarrow{AB} \perp \overrightarrow{CD}$ and $a = -10$, then $b = \underline{\hspace{2cm}}$.
- (A) 13 (B) 14 (C) 15 (D) 16 (E) 17
3. Find the perimeter of $\triangle ABC$. (nearest tenth)
- (A) 32.8 (B) 33.0 (C) 33.2 (D) 33.4 (E) 33.6
4. If \overrightarrow{AB} is parallel to \overrightarrow{CE} , then $c = \underline{\hspace{2cm}}$. (nearest tenth)
- (A) -2.2 (B) -2.0 (C) -1.8 (D) -1.6 (E) -1.4
5. If F is the midpoint of \overline{AB} and G is the midpoint of \overline{BC} , then $FG = \underline{\hspace{2cm}}$. (nearest tenth)
- (A) 3.6 (B) 3.8 (C) 4.0 (D) 4.2 (E) 4.4
6. Anthony left Seminole on his bike and headed west at 24 mph. After 90 minutes, he turned and headed north at 25 mph for two hours. Then he turned and headed east at 30 mph for 40 minutes. Then he stopped for lunch and calculated that he was miles from Seminole. (nearest tenth)
- (A) 51.7 (B) 51.9 (C) 52.1 (D) 52.3 (E) 52.5
7. The frequency of a vibrating string fixed at both ends varies directly as the square root of the tension the string is under and inversely as the length of the string. When the tension is 80 N and the length is 75 cm, the frequency is 512 Hz. What is the frequency if the tension is increased to 96 N and the length is decreased to 60 cm? (nearest whole number)
- (A) 689 Hz (B) 692 Hz (C) 695 Hz (D) 698 Hz (E) 701 Hz
8. The UIL Math Booster Club held a fundraiser to support the Salado math team. They flew in the Quebe Sisters to perform for the community. Adult tickets cost \$35 and student tickets cost \$20. If they sold 1110 tickets and raised \$34,020, how many adult tickets were sold?
- (A) 786 (B) 788 (C) 790 (D) 792 (E) 794
9. Grades for Mr. Cabaniss's calculus class are based on six tests. Grades on tests 1 through 5 count once and the grade on test 6 counts twice. Jason's grades on the first 5 tests are 89, 93, 91, 84 and 85. What is the lowest score he can make on test 6 and still have an average of 90.0 or higher?
- (A) 91 (B) 92 (C) 93 (D) 94 (E) 95

10. Austin is 218 miles from Argyle. At 9:00 AM, a van leaves Argyle heading toward Austin traveling at 55 mph. At 9:45 AM, a second van leaves Argyle traveling at 75 mph heading toward Austin. How far are the vans from Austin when the second van catches the first van? (nearest whole number)

- (A) 63 mi (B) 65 mi (C) 67 mi (D) 69 mi (E) 71 mi

11-12. Dennis's pool is rectangular in shape with a length of 24 feet, a width of 18 feet, and a constant depth of 4 feet. He uses two pipes to fill the pool. Each pipe can fill the pool by itself in 48 hours. The drain can empty the pool in 60 hours. The first pipe is turned on at noon on Monday. At 6:00 PM on Monday, the second pipe is turned on. At 10:00 PM on Monday, the drain is accidentally opened.

11. How many gallons of water does the pool contain when full? (nearest whole number)

- (A) 12,926 (B) 12,930 (C) 12,934 (D) 12,938 (E) 12,942

12. What is the earliest time on Wednesday that the pool will be full?

- (A) 2:00 AM (B) 2:10 AM (C) 2:20 AM (D) 2:30 AM (E) 2:40 AM

13-14. Consider a circle with points A, B, C and D on the circle. Point O is the center of the circle. \overline{AC} intersects \overline{BD} at point O. $AB = 8$ and $AC = 16$.

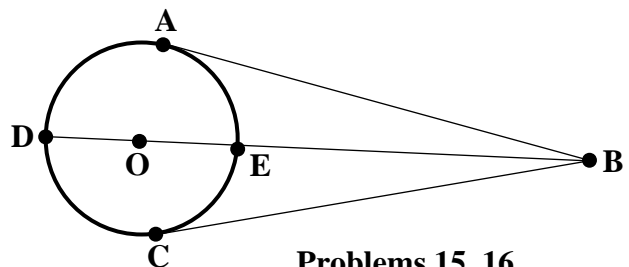
13. Find the area of $\triangle BOA$. (nearest tenth)

- (A) 26.9 (B) 27.1 (C) 27.3 (D) 27.5 (E) 27.7

14. Find the arc length of minor arc AD. (nearest tenth)

- (A) 16.2 (B) 16.4 (C) 16.6 (D) 16.8 (E) 17.0

15-16. Point O is the center of the circle.
 $DO = 12$ and $DB = 60$. \overline{BA} is tangent at point A and \overline{BC} is tangent at point C.



15. Find the area of $\triangle DCB$.
 (nearest whole number)

- (A) 337 (B) 340 (C) 343
 (D) 346 (E) 349

16. Find the area of the region inside $\triangle AOB$, but outside sector AOE. (nearest whole number)

- (A) 181 (B) 184 (C) 187 (D) 190 (E) 193

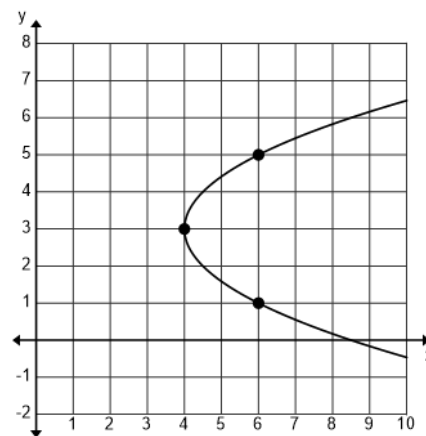
17. The measures of the angles of a triangle are in the extended ratio 4:5:6. Find the measure of the smallest angle.

- (A) 40° (B) 44° (C) 48° (D) 52° (E) 56°

18. Consider a right circular cylinder with a volume of 2714 cm^3 . If the diameter of each base is 12 cm, then the total surface area of the cylinder is _____ cm^2 . (nearest whole number)
- (A) 1131 (B) 1134 (C) 1137 (D) 1140 (E) 1143
19. Consider a regular hexagon with an inscribed circle and a circumscribed circle. If the area of the inscribed circle is 763 cm^2 , then the area of the circumscribed circle is _____ cm^2 . (nearest whole number)
- (A) 1008 (B) 1011 (C) 1014 (D) 1017 (E) 1020
20. If 7 is a zero of $3x^3 - kx^2 - 99x + 105 = 0$, then $k =$ _____.
- (A) 6 (B) 9 (C) 12 (D) 15 (E) 18
21. A boat leaves port at 1:00 PM and travels on a bearing of 236° at 24 knots. At 2:00 PM, a second boat leaves port and travels on a bearing of 24° at 20 knots. How far apart will the boats be at 6:00 PM? (1 knot = 1.15 mph) (nearest whole number)
- (A) 215 mi (B) 218 mi (C) 221 mi (D) 224 mi (E) 227 mi
22. Joe placed \$110,000 into an account at a bank in Brock that earns 5.06% annual interest compounded quarterly. On the same day, Joe placed \$110,000 into an account at a bank in Weatherford that earns 4.94% annual interest compounded monthly. At the end of 3 years, how much more money does he have in his account in Brock than he has in his account in Weatherford? (nearest dollar)
- (A) \$377 (B) \$379 (C) \$381 (D) \$383 (E) \$385
- 23-24. The time on my grandpa's old clock is exactly 6:09 PM.
23. The obtuse angle between the hour hand and the minute hand is _____ $^\circ$. (nearest tenth)
- (A) 129.4 (B) 130.5 (C) 131.6 (D) 132.7 (E) 133.8
24. How many minutes until the hour hand and the minute hand align for the second time? (nearest hundredth)
- (A) 86.45 min (B) 87.36 min (C) 88.27 min (D) 89.18 min (E) 90.09 min
25. If $f(x) = \frac{4x-5}{6x+7}$ and $h(x) = \frac{2-8x}{5+4x}$, then $(h^{-1} \circ f^{-1})(1) =$ _____.
- (A) -3 (B) -2 (C) -1 (D) 1 (E) 2
26. The elk population in Caribou County has been doubling every 6 years. If there were 112 elk in 2004, in what year should the number of elk reach 7168 if the population continues to double every 6 years?
- (A) 2038 (B) 2040 (C) 2042 (D) 2044 (E) 2046

27. Find the sum of the first ten terms of the sequence $24, 18, 13\frac{1}{2}, 10\frac{1}{8}, 7\frac{19}{32}, \dots$ (nearest tenth)
- (A) 90.0 (B) 90.2 (C) 90.4 (D) 90.6 (E) 90.8
28. A hungry hawk that is perched on top of a 78-ft-tall pole spots a mouse on the ground. The angle of depression from the hawk to the mouse is 20° . The mouse begins moving directly toward the pole at 1.00 ft/s. Exactly 80 seconds later, the mouse stops. What is the angle of depression from the hawk to the mouse at this point? (nearest tenth)
- (A) 30.1° (B) 30.4° (C) 30.7° (D) 31.0° (E) 31.3°
29. Given: The equation of an ellipse is $\frac{(x-3)^2}{a^2} + \frac{(y+2)^2}{b^2} = 1$, $a+b=20$, a and b are integers. If the area of the ellipse is 311, then the eccentricity of the ellipse is _____. (nearest thousandth)
- (A) 0.575 (B) 0.586 (C) 0.597 (D) 0.608 (E) 0.619
30. The graph of $x^2 - 6xy + 25y^2 + 4x - 12 = 0$ is a/an _____.
- (A) ellipse (B) hyperbola (C) parabola (D) circle (E) line
31. Assume that the life expectancy for men living in American Falls is 77.5 years and for women it is 83.6 years. If the total average is 80.9 years, what is the number of women divided by the number of men? (nearest hundredth)
- (A) 1.20 (B) 1.23 (C) 1.26 (D) 1.29 (E) 1.32
32. Consider the function $f(x) = 6 - 4\csc 2\left(x - \frac{\pi}{4}\right)$. Which of the following are true?
- I. The range of $f(x)$ is $(-\infty, 2] \cup [10, \infty)$.
- II. The domain of $f(x)$ is all $x \neq \frac{\pi}{4} + \frac{n\pi}{2}$, where n is an integer.
- III. The period of $f(x)$ is 2π .
- IV. The graph of $f(x)$ is symmetric about the y -axis.
- (A) II, III only (B) I, IV only (C) I, II, IV only
(D) I, II, III only (E) I, II, III, IV
33. Assume that the temperature on February 16th at Steve's cabin near Anchorage varies sinusoidally with a low of 15° at 6:00 AM and a high of 27° at 6:00 PM. The temperature is equal to or above 18° on February 16th for _____ hours. (nearest tenth)
- (A) 15.4 (B) 15.6 (C) 15.8 (D) 16.0 (E) 16.2

34. Consider the graph of the ellipse represented by the parametric equations $x = 4\cos\theta - 6$ and $y = 2\sin\theta + 4$. The distance between the foci is _____. (nearest tenth)
- (A) 6.9 (B) 7.1 (C) 7.3 (D) 7.5 (E) 7.7
35. The graph of the polar equation $r = 6 - 6\sin\theta$ is a _____.
- (A) circle (B) lemniscate (C) cardioid
(D) rose curve with 6 petals (E) rose curve with 12 petals
36. Consider the vectors $\mathbf{u} = \langle 4, n, -3 \rangle$ and $\mathbf{v} = \langle -2, n+1, -5 \rangle$. If the angle between the vectors is 44.735° , then $n =$ _____. ($n > 0$) (nearest whole number)
- (A) 4 (B) 5 (C) 6 (D) 7 (E) 8
37. Montana Institute of Technology admitted 240 freshmen last fall. Of those, 86 took E&M, 90 took DE, 108 took Cal III, 18 took E&M and DE but not Cal III, 16 took DE and Cal III but not E&M, 12 took all 3, and 34 did not take any of these courses. How many took E&M, but not DE or Cal III?
- (A) 32 (B) 34 (C) 36 (D) 38 (E) 40
38. Carter invented a new card game consisting of 64 cards. There are 15 red cards, 15 green cards, 15 blue cards, 15 yellow cards, and 4 wild cards. Each player is randomly dealt a 6-card hand. What is the probability that a hand will contain exactly 2 wild cards? (nearest thousandth)
- (A) 0.036 (B) 0.039 (C) 0.042 (D) 0.045 (E) 0.048
- 39-41. Consider the parabola shown on the right and the function $f(x) = x - 3$.
39. The equation of the directrix is $x =$ _____.
- (A) 1 (B) 2 (C) 3
(D) 3.5 (E) 3.75
40. If the point $(a, 11)$ lies on the graph of the parabola, then $a =$ _____.
- (A) 28 (B) 30 (C) 32
(D) 34 (E) 36
41. The area bounded by the graph of the parabola and the graph of $y = f(x)$ is _____. (nearest tenth)
- (A) 7.3 (B) 7.5 (C) 7.7 (D) 7.9 (E) 8.1



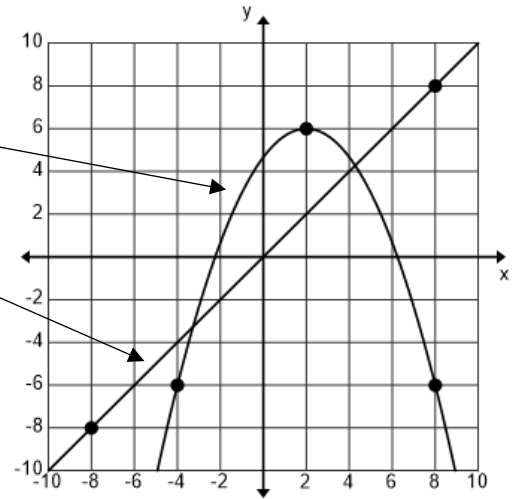
Problems 39, 40, 41

42. The line tangent to $y = h(x)$ has a slope of -1.6 when $x =$ _____. (nearest tenth)

- (A) 4.4
- (B) 4.6
- (C) 4.8
- (D) 5.0
- (E) 5.2

43. The volume of the solid generated by revolving the region bounded by the graphs of $y = h(x)$ and $y = g(x)$ about the line $y = b$ is 961.39. If $b < 0$, then $b =$ _____. (nearest whole number)

- (A) -8 (B) -7 (C) -6
- (D) -5 (E) -4



Problems 42, 43

44. Rancher Rob is designing a jogging track on his property. It will consist of a rectangle with a semicircle on each end. The perimeter of the track will be 600 meters. If Rob wants to maximize the rectangular area, what is this maximum value? (nearest whole number)

- (A) 14,308 m² (B) 14,312 m² (C) 14,316 m² (D) 14,320 m² (E) 14,324 m²

45. Let $f(x) = \frac{3}{8}x^3 + 2x - 1$. Find the value of $(f^{-1})'(6)$. (nearest thousandth)

- (A) 0.154 (B) 0.165 (C) 0.176 (D) 0.187 (E) 0.198

46-47. In April, researchers from Texas A&M found that the Lake Marvin trout population was zero. They believe the lake can support no more than 5000 trout. On May 1st, 1000 trout were released into the lake. They believe that the rate of increase of the population of trout after t weeks can be modeled by the logistic differential equation $\frac{dP}{dt} = 0.00003P(5000 - P)$.

46. How long after May 1st will the trout population be growing the fastest? (nearest tenth)

- (A) 8.8 weeks (B) 9.0 weeks (C) 9.2 weeks (D) 9.4 weeks (E) 9.6 weeks

47. The model derived from solving the differential equation predicts the trout population will reach _____ trout 25 weeks after the lake was stocked. (nearest whole number)

- (A) 4549 (B) 4556 (C) 4563 (D) 4570 (E) 4577

x	2	6	11	14	18
$f(x)$	3	7	9	10	11

48. Use the values in the table above to approximate the value of $\int_2^{18} f(x)dx$ using the trapezoidal approximation method with four subintervals. (nearest tenth)
- (A) 129.4 (B) 130.5 (C) 131.6 (D) 132.7 (E) 133.8
49. Consider the region M in the first quadrant bounded by the graph of $y_1 = -x + 8$, $y_2 = x \cdot \sin(.25x)$, and the y-axis. The vertical line $x = c$ divides M into two regions of equal area. $c =$ _____. (radians) (nearest hundredth)
- (A) 1.32 (B) 1.34 (C) 1.36 (D) 1.38 (E) 1.40
- 50-51. Consider the graph of a rose curve given by $r = 4\sin(3\theta)$.
50. Find the area of one leaf of the rose curve. (nearest tenth)
- (A) 3.6 (B) 3.8 (C) 4.0 (D) 4.2 (E) 4.4
51. Find the perimeter of one leaf of the rose curve. (nearest tenth)
- (A) 8.5 (B) 8.7 (C) 8.9 (D) 9.1 (E) 9.3
52. Consider the curve $5y^2 - 4xy + 2x^3 - 3y = 14$. Find the equation of the line tangent to the curve at the point $(2, c)$, $c > 1$. The y-intercept of the tangent line is the point $(0, b)$. $b =$ _____. (nearest tenth)
- (A) 4.4 (B) 4.7 (C) 5.0 (D) 5.3 (E) 5.6
53. Suppose 25% of the students at Latexo plan to attend Harvard. Of those, 40% plan to major in mathematics. Of the 75% who do not plan to attend Harvard, 20% plan to major in mathematics. Given that a randomly selected student plans to major in math, what is the probability that this student plans to attend Harvard? (nearest hundredth)
- (A) 0.30 (B) 0.35 (C) 0.40 (D) 0.45 (E) 0.50
54. According to the Idaho Insurance Agency, the state accident rate for 18-year-old males was 0.125 accidents for every driver during 2020. A random sample of 120 of the 18-year-old males in Boise found that 18 of them had been in an accident in 2020. If the accident rate for males in Boise is the same as the accident rate statewide, what is the probability of getting a sample whose accident rate is 0.150 or greater? (nearest hundredth)
- (A) 0.20 (B) 0.23 (C) 0.26 (D) 0.29 (E) 0.32

Week	1	2	3	4	5	6
# Pushups	12	15	19	25	27	30

55-57. Mr. Cantu decided to start an exercise program in January. On Fridays, he finishes his workout by seeing how many pushups he can do. The results from the first 6 weeks are shown in the table above. Mr. Cantu analyzed the data by calculating a LSRL.

55. The absolute value of the residual for week 4 is _____. (nearest hundredth)

- (A) 1.34 (B) 1.45 (C) 1.56 (D) 1.67 (E) 1.78

56. Based on the LSRL, how many push ups does Mr. Canto expect to do at the end of his workout on week 9?

- (A) 38 (B) 40 (C) 42 (D) 44 (E) 46

57. Calculate the standard deviation of the residuals to find the approximate size of a typical prediction error. (nearest hundredth)

- (A) 0.97 (B) 1.01 (C) 1.05 (D) 1.09 (E) 1.13

58. The mean score on the first exam in Professor Stat's class was 82. The scores on the first exam were approximately normal. If Justin's score of 89 placed him at the 90th percentile, what was the standard deviation of the scores on the first exam? (nearest hundredth)

- (A) 5.15 (B) 5.46 (C) 5.77 (D) 6.08 (E) 6.39

	N	Mean	Standard Deviation
BMI Male	88	27.7	3.8
BMI Female	96	26.3	3.6

59. The table above shows the BMI (Body Mass Index) of two independent samples of 70-year-old people in Idaho. The first sample consisted of 88 randomly selected males and the second sample consisted of 96 randomly selected females. It is thought that the BMI of 70-year-old males in Idaho is significantly higher than the BMI of 70-year-old females in Idaho. Professor Stat used an appropriate test to analyze the data. The P-value of the test is _____. (nearest thousandth)

- (A) 0.006 (B) 0.009 (C) 0.012 (D) 0.015 (E) 0.018

60. It is known that 28% of the seniors at Pokie High enroll in ISU after graduating from high school. If a group of 18 seniors is randomly selected, what is the probability that at least 6 of them will enroll at Idaho State? (nearest hundredth)

- (A) 0.37 (B) 0.39 (C) 0.41 (D) 0.43 (E) 0.45

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**University Interscholastic League
MATHEMATICS CONTEST
HS • Region • 2024
Answer Key**

1. C	21. C	41. B
2. D	22. A	42. A
3. A	23. B	43. E
4. A	24. D	44. E
5. D	25. B	45. A
6. E	26. B	46. C
7. E	27. D	47. D
8. B	28. A	48. B
9. D	29. A	49. B
10. A	30. A	50. D
11. A	31. C	51. C
12. C	32. C	52. E
13. E	33. D	53. C
14. D	34. A	54. A
15. E	35. C	55. E
16. B	36. C	56. C
17. C	37. C	57. C
18. A	38. B	58. B
19. D	39. D	59. A
20. B	40. E	60. B