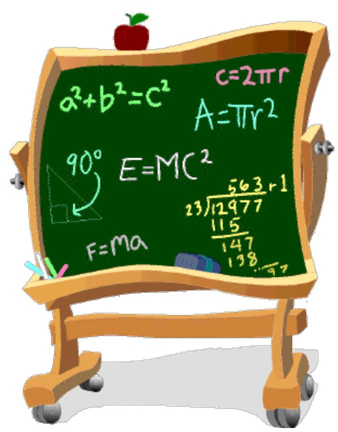
UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

State • 2024



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1. Consider the formula for a thin lens, $\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}$, where f is the focal length of the lens, d_o is the distance from the lens to the object, and d_i is the distance from the lens to the image. If the focal length of the lens is 24 cm and the distance from the lens to the object is 36 cm, find the distance from the lens to the image.

(A) 24 cm (B) 36 cm (C) 48 cm (D) 60 cm (E) 72 cm

2. During the tax-free weekend, Penelope, Sydney and Alexa went to Academy Sports and purchased some cross country attire. Penelope purchased 4 shirts, 3 shorts and some shoes. The shoes cost \$82 and she spent a total of \$196. Alexa purchased 6 shirts, 4 shorts and some shoes. The shoes cost \$98 and she spent a total of \$260. All shirts were the same price and all shorts were the same price. If Sydney purchased 2 shirts and 3 shorts, how much did she spend?

(A) \$80 (B) \$82 (C) \$84 (D) \$86 (E) \$88

3. Five times Rose's age is 2 more than Carrie's age. In 10 years, Arlene will be twice as old as Carrie. Two years ago, Arlene was 14 times as old as Rose. How old is Arlene?

(A) 84 (B) 86 (C) 88 (D) 90 (E) 92

4-6. Consider the points $A(-6, 2)$, $B(8, 4)$, $C(2, -6)$ and $D(-10, -4)$.

4. Find the distance from point A to the midpoint of $\overline{BC}$. (nearest tenth)

(A) 11.4 (B) 11.6 (C) 11.8 (D) 12.0 (E) 12.2

5. Given: $\overleftrightarrow{AC}$ is parallel to $\overleftrightarrow{DE}$. If the coordinates of point E are $(a, 2)$, then a = _________.

(A) −18 (B) −17 (C) −16 (D) −15 (E) −14

6. Given: $\overleftrightarrow{FG}$ is the perpendicular bisector of $\overleftrightarrow{AB}$. If the coordinates of point F are $(3, b)$, then b = _____.

(A) −14 (B) −13 (C) −12 (D) −11 (E) −10

7. Caleb took a three-day trip from Sanger to Aberdeen, a distance of 1383 miles. On day one, he drove 577 miles at an average speed of 62 mph. On day two, he drove 464 miles at an average speed of 72 mph. If the total driving time on the trip was 20 hr 48 min, what was his average speed on day three? (nearest tenth)

(A) 66.5 mph (B) 66.8 mph (C) 67.1 mph (D) 67.4 mph (E) 67.7 mph

8. Ronita flew from Portland to Dallas to visit her favorite sister. The round-trip ticket cost \$882. She rented a car for \$48 per day plus \$0.32 per mile. She drove a total of 448 miles during the 12 days she was there. She also spent \$366 on Texas Rangers shirts and hats. How much did the trip cost?

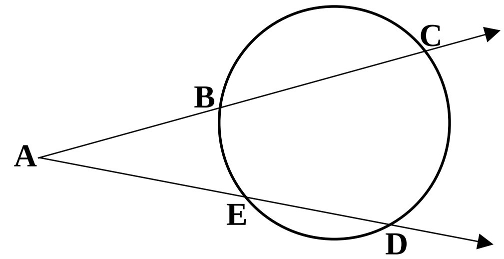
(A) \$1967.36 (B) \$1968.36 (C) \$1969.36 (D) \$1970.36 (E) \$1971.36

9. Consider an arithmetic sequence in which the fourth term is 37 and the eleventh term is 93. Find the sum of the first 16 terms.

(A) 1162 (B) 1164 (C) 1166 (D) 1168 (E) 1170

10. Consider the circle on the right. If AB = 14, BC = 18, and AE = 16, then DE = ________.

(A) 9
(B) 10
(C) 11
(D) 12
(E) 13



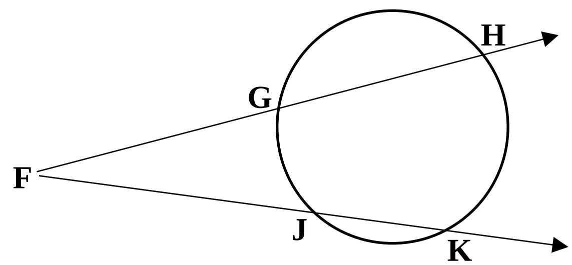
Problem 10

11. Consider equilateral triangle PQR with a circumscribed circle. If the area of the circle is 339, then the area of triangle PQR is ____________. (nearest whole number)

(A) 134 (B) 136 (C) 138 (D) 140 (E) 142

12. Consider the circle on the right. If the measure of minor arc $HK = 128°$ and the measure of $\angle GFJ = 33°$, then the measure of minor arc $GJ =$ ________.

(A) 56°
(B) 58°
(C) 60°
(D) 62°
(E) 64°



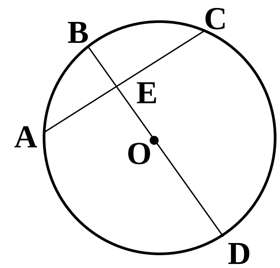
Problem 12

13. The total area of a cylinder with a radius of 14 cm is 3343 cm^2. The volume of the cylinder is __________ cm^3. (nearest whole number)

(A) 14,764 (B) 14,768 (C) 14,772 (D) 14,776 (E) 14,780

14. Consider the circle on the right with center O. Chord $\overline{AC}$ intersects diameter $\overline{BD}$ at point E. $\overline{AC} \perp \overline{BD}$, $BD = 18$, and $AC = 14$. BE = _______. (nearest tenth)

(A) 3.3 (B) 3.5 (C) 3.7
(D) 3.9 (E) 4.1



Problem 14

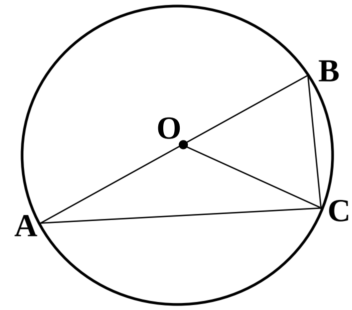
15. Given: $\triangle ABC$ is inscribed in a circle with $m\angle C = 90°$, AC = 7, and the perimeter of the triangle is 56. The area of the circle = ________. (nearest whole number)

(A) 491 (B) 494 (C) 497 (D) 500 (E) 503

16. Consider ΔABC with $m\angle ABC = 90°$. Point D lies on $\overline{AC}$ such that $m\angle ADB = 90°$. If AD = 6 and CD = 13, then the perimeter of ΔABC = ________. (nearest tenth)

(A) 44.5 (B) 44.8 (C) 45.1 (D) 45.4 (E) 45.7

17-18. The circle shown on the right has an area of 707.
The measure of $\angle BAC$ is 30°.
Point O is the center of the circle.



Problems 17, 18

17. Find the area of ΔAOC. (nearest tenth)

(A) 97.1 (B) 97.4 (C) 97.7
(D) 98.0 (E) 98.3

18. Find the area of the region bounded by chord $\overline{BC}$ and minor arc $\overset{\frown}{BC}$. (nearest tenth)

(A) 19.8 (B) 20.1 (C) 20.4 (D) 20.7 (E) 21.0

19-21. Given: ΔABC is similar to ΔDEF, AB = 36, BC = 39, AC = 42, and DF = 28.

19. Point G is the midpoint of $\overline{DF}$. EG = ________. (nearest tenth)

(A) 20.3 (B) 20.5 (C) 20.7 (D) 20.9 (E) 21.1

20. Point H lies on $\overline{AC}$ and ray $\overrightarrow{BH}$ bisects $\angle ABC$. AH = ________. (nearest hundredth)

(A) 20.13 (B) 20.16 (C) 20.19 (D) 20.22 (E) 20.25

21. The area of ΔBHC = _________. (nearest whole number)

(A) 326 (B) 329 (C) 332 (D) 335 (E) 338

22. Rachel accepted a job with a salary of $95,000 the first year. During the next 19 years, she was given a 6% raise each year. Find the total compensation she received over the 20-year period. (nearest dollar)

(A) $3,494,628 (B) $3,494,631 (C) $3,494,634 (D) $3,494,637 (E) $3,494,640

23. Given: $\sin(u) = -\frac{24}{25}$ and $\cos(v) = -\frac{3}{5}$. Both u and v are in quadrant III. Evaluate $\sec(u - v)$.

(A) $\frac{121}{117}$ (B) $\frac{41}{39}$ (C) $\frac{125}{117}$ (D) $\frac{127}{117}$ (E) $\frac{44}{39}$

24. Consider the sequence 2, 5, 9, 14, 20, 27, 35, …. The sum of the first 24 terms is _________.

(A) 2896 (B) 2900 (C) 2904 (D) 2908 (E) 2912

25. Audrey invested \$100,000 for 4 years. If the interest was compounded monthly rather than quarterly, she would have made \$345.93 more. What was the annual interest rate? (nearest hundredth)

(A) 4.22% (B) 5.33% (C) 6.44% (D) 7.55% (E) 8.66%

26. Assume July temperatures vary sinusoidally in Denali National Park with a low of 48° at 4:00 AM and a high of 68° at 4:00 PM. The number N of brown bears that are visible from Keith's campsite is given by $N(t) = (T - 46°)$, $48° \le T \le 68°$, where $N(t) =$ the number of brown bears visible at time t and T is the temperature. How many brown bears are visible from Keith's campsite at 12:00 PM?

(A) 16 (B) 17 (C) 18 (D) 19 (E) 20

27. The circle $(x-6)^2 + (y-12)^2 = 20$ is tangent to the circle $x^2 + y^2 = 80$. The common internal tangent is a line with x-intercept $(a, 0)$ and y-intercept $(0, b)$. $a + b =$ ________. (nearest whole number)

(A) 26 (B) 28 (C) 30 (D) 32 (E) 34

28. Justin obtained a sample of radioactive plutonium 234 at 5:00 AM on Wednesday. Only 1.510 g remained at 5:00 AM on Thursday and only 0.501 g remained at 7:00 PM on Thursday. Find the amount of plutonium Justin originally obtained. (nearest thousandth)

(A) 10.008 g (B) 10.082 g (C) 10.156 g (D) 10.230 g (E) 10.304 g

29-30. Consider the graph of a parabola with vertex $V(2, -6)$. Points $P(0, -4)$ and $Q(0, -8)$ both lie on the graph of the parabola.

29. The equation of the directrix of the graph of the parabola is $x =$ __________.

(A) $\frac{17}{8}$ (B) $\frac{9}{4}$ (C) $\frac{5}{2}$ (D) 3 (E) 4

30. Point $T(a, 0)$ lies on the graph of the parabola and point $F(e, f)$ is the focus of the graph of the parabola. FT = __________. (nearest tenth)

(A) 18.5 (B) 18.7 (C) 18.9 (D) 19.1 (E) 19.3

31. Computer World in Big Timber, Montana currently has 20 computers in stock. Fifteen have 16 GB RAM and five have 8 GB RAM. If Rancher Rob randomly selects four computers to purchase, what is the probability that at least two of the computers have 16 GB RAM? (nearest thousandth)

(A) 0.940 (B) 0.947 (C) 0.954 (D) 0.961 (E) 0.968

32. Consider $\overleftrightarrow{AB}$ such that every point on $\overleftrightarrow{AB}$ is the same distance from point $P(-6, 4)$ as the distance from point $Q(8, -2)$. If point $R(13, c)$ lies on $\overleftrightarrow{AB}$, then c = ________. (nearest tenth)

(A) 29.0 (B) 30.1 (C) 31.2 (D) 32.3 (E) 33.4

33. Ship A leaves port at 1:00 PM and travels at an average speed of 18 mph on a bearing of 144°. Ship B leaves port at 3:00 PM and travels at an average speed of 24 mph on a bearing of 284°. At what time will the ships be 155 miles apart? (nearest minute)

(A) 6:01 PM (B) 6:04 PM (C) 6:07 PM (D) 6:10 PM (E) 6:13 PM

34. Consider an ellipse such that for any point $P(e, f)$ that lies on the ellipse, the distance from P to the point $(2, 4)$ plus the distance from P to the point $(14, 4)$ equals 40. If the equation of the ellipse is $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$, then b = ________. (nearest tenth)

(A) 18.3 (B) 18.5 (C) 18.7 (D) 18.9 (E) 19.1

35. The graph of $4x^2+5xy+2y^2-16=0$ is an ellipse in which the axes have been rotated _______°. (nearest whole number)

(A) 28 (B) 30 (C) 32 (D) 34 (E) 36

36. The graph of the parametric equations $x=4\sec\theta+3$ and $y=3\tan\theta-2$ is a hyperbola. The asymptote with positive slope has an x-intercept of $(e, 0)$. e = _________. (nearest tenth)

(A) 5.3 (B) 5.5 (C) 5.7 (D) 5.9 (E) 6.1

37. The graph of the polar equation $r=\frac{4}{3+2\sin\theta}$ is an ellipse centered at the point $P(a, b)$. a + b = _________. (nearest tenth)

(A) –1.8 (B) –1.7 (C) –1.6 (D) –1.5 (E) –1.4

38. Find the distance from the point $Q(2, 3, 5)$ to the plane $x-2y+3z=6$. (nearest tenth)

(A) 1.1 (B) 1.3 (C) 1.5 (D) 1.7 (E) 1.9

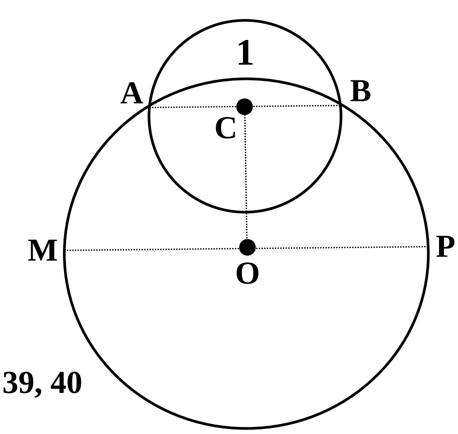
39-40. The radius of the large circle with center O is 10 and the radius of the small circle with center C is 6. The crescent-shaped region (1) is called a lune. $\overline{CO}\perp\overline{AB}$ and $\overline{CO}\perp\overline{MP}$.

39. The perimeter of the lune is _______. (nearest tenth)

(A) 31.1 (B) 31.4 (C) 31.7
(D) 32.0 (E) 32.3

40. The area of the lune is _________. (nearest tenth)

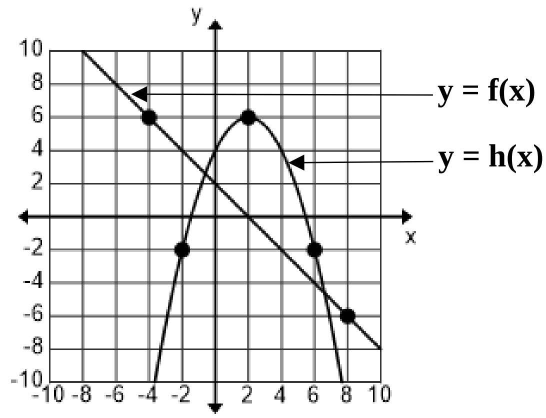
(A) 39.4 (B) 39.6 (C) 39.8
(D) 40.0 (E) 40.2



Problems 39, 40

41. Find the area of the region bounded by the graphs of $y = h(x)$ and $y = f(x)$. (nearest tenth)

(A) 30.6 (B) 30.9 (C) 31.2
(D) 31.5 (E) 31.8



Problems 41, 42, 43, 44

42. Find the volume of the solid formed when the region bounded by the graphs of $y = h(x)$ and $y = f(x)$ is revolved about the line $y = -10$. (nearest tenth)

(A) 2233.1 (B) 2244.2 (C) 2255.3
(D) 2266.4 (E) 2277.5

43. Find the volume of the solid whose base is the region bounded by the graphs of $y = h(x)$ and $y = f(x)$ and whose cross sections perpendicular to the x-axis are semicircles. (nearest tenth)

(A) 63.5 (B) 63.8 (C) 64.1 (D) 64.4 (E) 64.7

44. If the arc length of the graph of $y = h(x)$ on the interval $[a, 7.4]$ is 24.51, and $a < 0$, then a = _______. (nearest tenth)

(A) –2.2 (B) –2.0 (C) –1.8 (D) –1.6 (E) –1.4

45-46. A particle is moving along the x-axis so that at any time t, in seconds, the acceleration of the particle is given by $a(t) = 1 + 8\cos(t),\ t \geq 0$, where $a(t)$ is the acceleration in cm/s^2. At t = 0, the particle's position is at x = 3 cm and the particle's velocity is 2 cm/s to the right. Consider the path of the particle from t = 0 to t = 6 seconds. (radians)

45. The position of the particle at t = 4 seconds is at x = ________ cm. (nearest tenth)

(A) 32.2 (B) 32.5 (C) 32.8 (D) 33.1 (E) 33.4

46. The maximum speed of the particle when it is traveling to the left is _________ cm/s. (nearest hundredth)

(A) 1.35 (B) 1.46 (C) 1.57 (D) 1.68 (E) 1.79

47. The number of fire ants in Mr. Garcia's backyard is given by a differentiable function f, where f(t) is the number of fire ants present and t is measured in weeks. The number of fire ants is increasing according to the equation $\frac{df}{dt} = kf$, where k is a constant. At t = 0, the number of fire ants is 450 and is increasing at the rate of 150 fire ants per week. Find the expected number of fire ants at t = 6 weeks. (nearest whole number)

(A) 3303 (B) 3314 (C) 3325 (D) 3336 (E) 3347

t (min)	0	13	22	34	48
r(t) (in/min)	0.098	0.074	0.061	0.048	0.035

48-49. Rancher Rob has an elk ranch near Driggs. He stores rainwater in a large cylindrical tank which has a radius of 4 feet and a height of 3 feet. The top of the tank has been removed so that rain can fill the tank and the elk can drink from the tank. The depth of the water in the tank was 2 feet when a storm blew in and it began raining. The rate at which the depth of the water in the tank is increasing is shown for various values of t in the table above.

48. Use the table above to estimate the increase in the depth of the water in the tank from t = 0 to t = 48 minutes. Use a right Riemann sum, RRAM, with four subintervals. (nearest hundredth)

(A) 2.55 in (B) 2.58 in (C) 2.61 in (D) 2.64 in (E) 2.67 in

49. Rob developed a mathematical model for the rate at which the depth of the water in the tank is increasing. His model is the function r where $r(t)=0.098(0.979)^t$, $0\le t\le 48$, and where $r(t)$ is measured in inches per minute and t is measured in minutes. Find the amount of water in the tank at t = 48 minutes using Rob's model. (nearest gallon)

(A) 835 gal (B) 838 gal (C) 841 gal (D) 844 gal (E) 847 gal

50. Given: $y^2+5x^2y^3+x^4=37$. Evaluate $\frac{dy}{dx}$ when x = 2. (nearest hundredth)

(A) –0.96 (B) –0.93 (C) –0.90 (D) –0.87 (E) –0.84

51. Let f be a function with third derivative $(2x+8)^{\frac{2}{3}}$. The coefficient of x^4 in the Taylor series for f about x = 0 is ___________.

(A) $\frac{1}{72}$ (B) $\frac{1}{36}$ (C) $\frac{1}{24}$ (D) $\frac{1}{12}$ (E) $\frac{1}{6}$

52. Find the area of the region lying between the inner and outer loops of the polar graph of $r=1-2\cos(\theta)$. (nearest hundredth)

(A) 8.31 (B) 8.34 (C) 8.37 (D) 8.40 (E) 8.43

53. The distribution of the amount of water in a 20 oz bottle of Olney Natural Springs Water is approximately normal with a mean of 20 oz and a standard deviation of 0.45 oz. Approximately what proportion of bottles have less than 19 oz? (nearest thousandth)

(A) 0.013 (B) 0.016 (C) 0.019 (D) 0.022 (E) 0.025

54. Assume that the length of a fully grown Lesser Bandicoot Rat has a roughly normal distribution with a mean of 36 cm and a standard deviation of 1.5 cm. Find the interquartile range of this distribution. (nearest tenth)

(A) 1.8 cm (B) 2.0 cm (C) 2.2 cm (D) 2.4 cm (E) 2.6 cm

55. A survey planned to determine how much personal debt people in the 25 to 34 age group have due to home mortgages, car loans and credit cards. Of the following, which is the minimum number of people in this age group that researchers should plan to survey to be within \$1,000 of the true mean with 90% confidence? A previous study found that the standard deviation of the personal debt of people in this age group was \$15,500.

(A) 449 (B) 550 (C) 651 (D) 752 (E) 853

Student	1	2	3	4	5	6	7	8	9
Test 1	82	88	77	81	90	74	83	85	82
Test 2	84	91	78	85	93	76	84	88	86

56-57. Professor Satterfield randomly selected 9 of his statistics students to participate in a small study. He wanted to see if offering a group study session with a T.A. the night before major tests would improve their scores. Assume test 1 and test 2 were of equal difficulty. Students were not offered a study session before test 1, but they were required to attend a study session before test 2. Results are shown in the table above. When evaluating the results of the study, the null hypothesis was H_o : The study session had no effect on test scores. The alternative hypothesis was H_a : The study session improved test scores. The significance level was $\alpha = 0.10$.

56. Using an appropriate test, he should reject H_o if the test statistic is greater than ________. (nearest thousandth)

(A) 1.286 (B) 1.397 (C) 1.508 (D) 1.619 (E) 1.730

57. The P-value obtained from using the appropriate test was _________. (nearest hundred-thousandth)

(A) 0.00007 (B) 0.00010 (C) 0.00013 (D) 0.00016 (E) 0.00019

58. A Lottery ticket cost \$10. In the Lottery, six numbers are randomly chosen without repetition from the numbers 1 to 40. If you select all 6 numbers, you win \$10,000,000. If you only select 5 of the 6, you win \$100,000. If you only select 4 of the 6, you win \$100. Find the expected value of a lottery ticket.

(A) –\$2.02 (B) –\$1.98 (C) –\$1.94 (D) –\$1.90 (E) –\$1.86

59-60. Professor Stat randomly selected 50 students at ISU for a study. He collected information about their parents. When he analyzed the data, he noticed that a strong positive linear relationship exists between a student's final grade (FG) in English 101 and the college grade average (CA) of the student's mother. The results of computing a LSRL from the data were:
FG mean = 88, FG standard deviation = 4, CA mean = 92, CA standard deviation = 3, r^2 = 0.81.

59. Find the predicted final grade of a student whose mother had a CA of 98. (nearest whole number)

(A) 92 (B) 93 (C) 94 (D) 95 (E) 96

60. His analysis predicts that for each increase of one point in a mother's college grade average, there is a corresponding increase of ________ points in a student's final grade. (nearest tenth)

(A) 1.2 (B) 1.4 (C) 1.6 (D) 1.8 (E) 2.0

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University Interscholastic League
MATHEMATICS CONTEST
HS ● State ● 2024
Answer Key

1. E	21. E	41. C
2. C	22. B	42. E
3. B	23. C	43. B
4. A	24. B	44. C
5. C	25. E	45. A
6. D	26. B	46. A
7. E	27. C	47. C
8. A	28. A	48. B
9. D	29. C	49. D
10. D	30. A	50. E
11. D	31. E	51. B
12. D	32. A	52. B
13. E	33. B	53. A
14. A	34. E	54. B
15. A	35. D	55. C
16. D	36. C	56. B
17. B	37. C	57. A
18. C	38. B	58. E
19. C	39. C	59. D
20. B	40. E	60. A