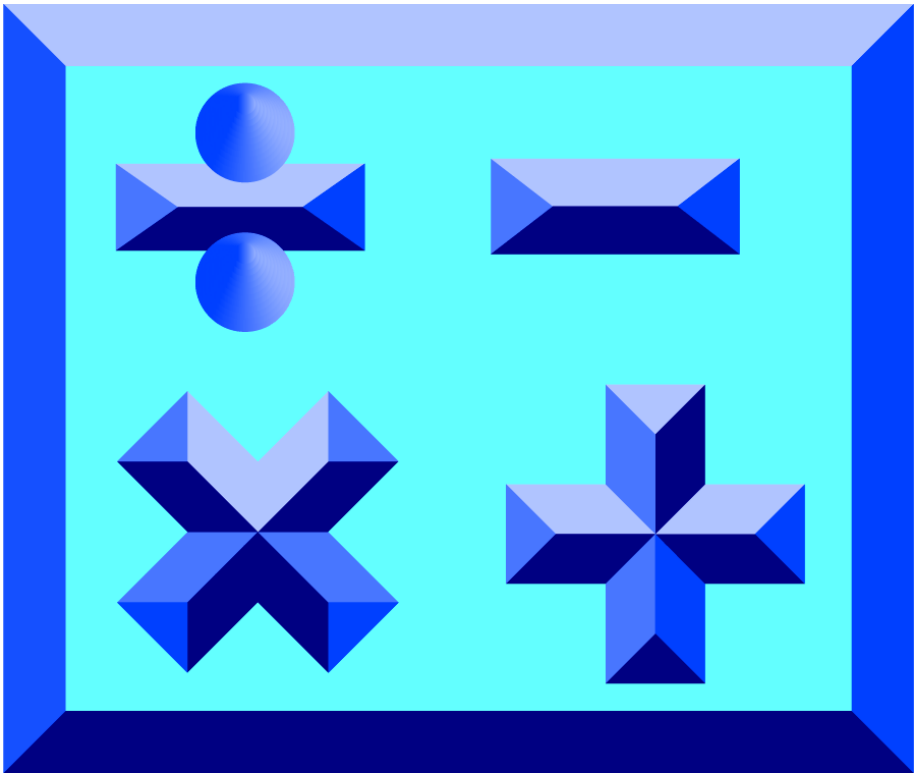
UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational A • 2018



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YOU ARE INSTRUCTED TO DO SO!

1. **Evaluate:** $4! \times (4)^{-2} + (4^2)^{\frac{1}{4}} - 4 \div 2$

   (A) -1.25 **(B)** -0.25 **(C)** 1.5 **(D)** 3.5 **(E)** 6

2. **Lotta Cash received a $50.00 gift card for graduation. She went shopping at the *Cheap Shoppe*. She bought 2 pair of shorts at $7.99 each, 3 pair of flip-flop sandals at $4.50 each, a bottle of suntan lotion at $8.25, a sun hat at $9.89, and 2 bottles of water at 75¢ each. She got 15% off for using a gift card instead of a credit card. How much does she have left on her gift card if the tax rate was 7.5%?**

   (A) $8.25 **(B)** $5.12 **(C)** $4.35 **(D)** $3.68 **(E)** $2.80

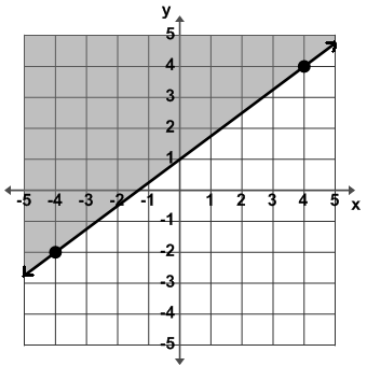
3. **If 45% of A is $4\frac{1}{5}$ of B, then B is what per cent of A?**

   (A) 3% **(B)** $4\frac{2}{7}\%$ **(C)** $7\frac{6}{7}\%$ **(D)** $9\frac{3}{7}\%$ **(E)** $10\frac{5}{7}\%$

4. **$2 \times 4 \times 8 = 8 \times 8 = 64$ and $2 \times 4 \times 8 = 2 \times 32 = 64$ are examples of the ? property of equality.**

   (A) associative **(B)** commutative **(C)** distributive **(D)** identity **(E)** inverse

5. **Which of the inequalities is best represented by the graph below?**



   (A) $4x - 3y \geq 4$ **(B)** $3x - 4y \geq -4$ **(C)** $x + 4y \leq 4$
   (D) $3x - 4y \leq -4$ **(E)** $x - 4y \leq -4$

6. **If $2(3 + 5) = 16$ and $16 = 4^2$ then $2(3 + 5) = 4^2$. Which of the following properties does this example illustrate ?**

   (A) associative **(B)** commutative **(C)** distributive **(D)** symmetric **(E)** transitive

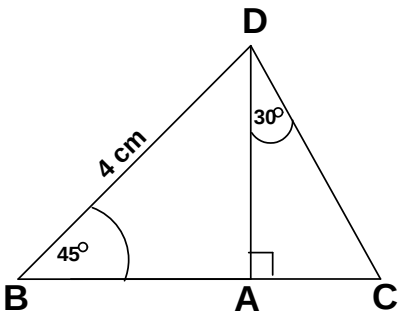
7. **Simplify:** $\left(\frac{2x^2 - 7x + 5}{4x^2 + 8x - 12}\right) \div \left(\frac{4x^2 - 8x - 5}{2x^2 + 7x + 3}\right)$

   (A) $\frac{1}{4}$ **(B)** $\frac{x+3}{4(x-1)}$ **(C)** $\frac{2x+1}{2x-5}$ **(D)** $\frac{2x+5}{4(2x-1)}$ **(E)** $\frac{4(x+3)}{x-1}$

8. **If $4x^2 - x + c = (ax + b)(x + 1)$ then $a + b + c =$ _____.**

   (A) -4 **(B)** -6 **(C)** 1 **(D)** 5 **(E)** 14

9. Find the perimeter of $\triangle BCD$. (nearest tenth).



(A) 11.7 cm (B) 12.5 cm (C) 13.8 cm (D) 10.1 cm (E) 8.9 cm

10. The line $y = mx + b$ contains the point $(-5, -2)$ and has a slope of $-\frac{3}{4}$. The y-intercept is:

(A) $(0, 2\frac{1}{3})$ (B) $(0, 7\frac{2}{3})$ (C) $(0, \frac{3}{4})$ (D) $(0, -1\frac{3}{4})$ (E) $(0, -5\frac{3}{4})$

11. The circles $(x-3)^2 + (y+1)^2 = 16$ and $(x-4)^2 + (y-2)^2 = 9$ intersect in two points. The slope of the line through the two points of intersection is:

(A) $\frac{3}{4}$ (B) $\frac{9}{16}$ (C) $-\frac{1}{7}$ (D) $-\frac{1}{3}$ (E) -3

12. A rectangular swimming pool is twice as long as it is wide and has a 10 foot-wide concrete border around it. If the border has an area of 2800 sq. ft., find the perimeter of the pool.

(A) 210 ft (B) 240 ft (C) 280 ft (D) 300 ft (E) 320 ft

13. If $27^{(k)} = 9^{(k+1)}$, then $3^{(k+2)} = ?$

(A) 243 (B) 81 (C) 27 (D) 9 (E) 3

14. Let $f(x) = x - 2$, $g(x) = 2x - 1$, $h(x) = 3x$, and $g(f(x)) + f(h(x)) = -4$. Find x.

(A) 1 (B) $-2\frac{1}{5}$ (C) $-\frac{1}{5}$ (D) $1\frac{2}{5}$ (E) $\frac{3}{5}$

15. Which of the following functions does not have an inverse function?

(A) $y = 2x - 4$ (B) $y = \frac{1}{4}x + 2$ (C) $y = -x^2 + 4$ (D) $y = \ln(x+4) - 2$ (E) $y = \sqrt{2x - 4}$

16. Phil Dewallit got a $20.00 allowance for mowing his parent's lawn this week. They agreed to increase his previous week' s allowance 80¢ each week for the next 24 weeks. Phil decides to put half of his allowance in his piggy bank each week. How much will he have in the bank at the end of the 25 week period?

(A) $370.00 (B) $351.00 (C) $333.25 (D) $266.50 (E) $257.50

17. Determine the range of f(x) = 2 — 4cos(x + 3).

(A) [— 4, 4] (B) [— 2, 4] (C) [— 2, 6] (D) [— 4, 2] (E) [4, 12]

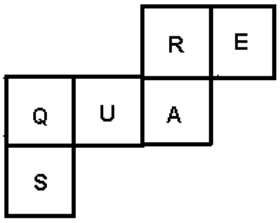
18. $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$ **is equivalent to:**

(A) 1 **(B)** $\frac{1}{\sin\theta}$ **(C)** $\frac{\cos\theta}{2\sin\theta}$ **(D) 2sec** θ **(E) 2csc** θ

19. Captain Ed Inberg went sailing on Lake Falcon. He sailed his scow from the dock 8 km on a bearing of 40°. Then he changed course and sailed 5 km on a bearing of 120°. Then he decided to return to the dock. What bearing will Captain Ed have to sail to go straight back to the dock? (nearest degree)

(A) 249° (B) 231° (C) 219° (D) 151° (E) 111°

20. Paulie Gone folds the net shown into a cube. What letter will be on the opposite face of face E?



(A) U (B) S (C) R (D) Q (E) A

21. In the expansion of $(3x - 2)^5$**, the sum of the coefficients of the 3rd and the 4th term is:**

(A) 1,320 (B) 360 (C) — 1,520 (D) 480 (E) 1,800

22. Find a + b + c + d given the Fibonacci characteristic sequence: 3, a, b, 17, c, d, 71, ...

(A) 179 (B) 91 (C) 159 (D) 88 (E) 105

23. $\sum_{k=1}^{3} (-1)^k (kx - (k+1)y - k) = ?$

(A) $-6x + 9y + 6$ **(B)** $2x - 3y - 2$ **(C)** $-(2x - 3y - 2)$
(D) $6x - 3y + 2$ **(E)** $6x - 9y - 6$

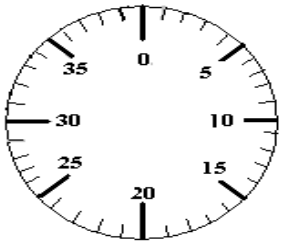
24. Sameer, Anisha, and Ian worked a total of 125 problems on the number sense test at the math camp. Sameer worked 28% of the total problems, Anisha worked 40 less problems than Ian did. What percent of the problems did Ian work?

(A) 36% (B) 48% (C) 52% (D) 65% (E) 68%

25. Find the area of the region bounded by the graphs of $x = 4 - y^2$ **and** $x = 4 - 4y$**.**

(A) $10\frac{2}{3}$ **(B) 10 (C)** $9\frac{2}{3}$ **(D)** $9\frac{1}{3}$ **(E) 9**

26. P-Q-R is the combination needed to open the safe with the combination dial shown below. How many distinct combinations exist if P is a triangular number, Q is a square number greater than 0, R is a pentagonal number.



(A) 240 (B) 225 (C) 175 (D) 128 (E) 19

27. If $f'(x) = 3x^2 - 5$ and $f(-1) = 4$, find $f(1)$.

(A) -5 (B) -4 (C) -3 (D) -2 (E) -1

28. Max Space has a rectangular sheet of cardboard that is 4 feet by 6 feet. He is going to cut out a 5 inch square from each of the four corners, then fold up the sides, tape edges, and make a rectangular box without a top. What is the volume of the box? (nearest tenth)

(A) 9.7 cu. ft. (B) 8.3 cu. ft. (C) 6.8 cu. ft. (D) 6 cu. ft. (E) 3.8 cu. ft.

29. Roland Bones rolls a pair of dice. What are the odds that the sum of top faces he rolls is a 7 or an 11?

(A) $\frac{2}{9}$ (B) $\frac{1}{8}$ (C) $\frac{1}{9}$ (D) $\frac{1}{17}$ (E) $\frac{2}{7}$

30. Which of the following mathematicians is known for developing a "machine" that uses a system of rules, states, and transitions used to decide a language or to solve mathematical functions? It is a powerful tool used in computer science and code breaking.

(A) Eratosthenes (B) Charles Babbage (C) John Napier
(D) George Boole (E) Alan Turing

31. Arnie has a bag with 3 white golf balls and 2 yellow golf balls. Jack has a bag with 4 yellow golf balls and 2 white golf balls. Tiger picks a bag and a ball at random. The probability that the ball will be white is: (nearest whole percent)

(A) 47% (B) 10% (C) 23% (D) 45% (E) 20%

32. Find $a + b + c + d$ given the arithmetic sequence: $-11, a, b, c, 3, d, ...$

(A) 6.5 (B) 3.25 (C) -2.25 (D) -2.5 (E) -5.5

33. Let $f(x) = ax^3 - bx + 3$ where a and b are integers. If $f(2) = -4$, then $f(-2) = ?$

(A) 4 (B) 7 (C) 10 (D) -3 (E) -4

34. Mr. White's 'bath tub mat' pattern table consists of 19 columns and 12 rows. Only 7 rows are shown. Determine the sum of the numbers in the 8^{th} row.

1				1				2				3				5		
			2				3				5				8			
		3				5				8				13				21
	5				8				13				21				34	
8				13				21				34				55		
			21				34				55				89			
		34				55				89				144				233

(A) 932 (B) 665 (C) 864 (D) 521 (E) 898

35. Coach Ball has 22 students in his PE class. 9 of the students play football, 10 play basketball, 5 play tennis and basketball but not football, 5 play basketball and football but not tennis, and 2 play tennis only. How many students do not play any of these 3 sports?

(A) 1 (B) 3 (C) 5 (D) 6 (E) not enough information

36. I. Cee and U. Saul used a 2 in. x 12 in. x 16 ft. board to make a teeter-totter with the center being on a fulcrum. Cee weighs 85 pounds and is sitting 8 feet from the center of the teeter-totter. Saul weighs 100 pounds and is sitting on the opposite end. How far from the center should Saul sit if the teeter-totter has a slope of zero? (nearest inch)

(A) 4' 1" (B) 5' 8" (C) 6' 10" (D) 7' 9" (E) 9' 5"

37. Twenty-five seniors took the state math test last year. Fifteen of them were boys and ten were girls. All of them had an equal chance to win one of the top three medals. What was the probability that one girl and two boys won one of the top three medals? (nearest whole percent)

(A) 21% (B) 25% (C) 42% (D) 29% (E) 46%

38. If the probability that a student in a Statistics class studies for an exam is 70%, and the probability that a student who studies passes the test is 85%, then the probability that a student both studies and passes the test is: (nearest whole percent)

(A) 75 % (B) 60% (C) 55 % (D) 50% (E) 45%

39. Given: $\triangle ABE$ is similar to $\triangle DON$; $\angle A \cong \angle N$; $\angle B \cong \angle D$; AB = 30 cm; DN = 24 cm; and NO = 16 cm. Find AE.

(A) 45 cm (B) 12.8 cm (C) 22 cm (D) 16 cm (E) 20 cm

40. If $\log_6(16) - \log_6(4x) = \log_6(x+2)$, then x equals _____.

(A) $\sqrt{5}+1$ (B) 2 (C) $1\frac{1}{2}$ (D) $\sqrt{5}-1$ (E) $\frac{2}{3}$

41. $\triangle ABC$ is a scalene triangle. Point P lies on segment AB such that segment CP is the altitude of the triangle, $m\angle CBP = 65°$, AP = 12", BP = 15". Find $m\angle ACP$. (nearest degree)

(A) 15 ° **(B)** 20 ° **(C)** 25 ° **(D)** 32 ° **(E)** 35 °

42. Given: $f(x) = 2 - 4\sin(x + 3)$. What quadrant(s) would the graph of f(x) be in if the amplitude is cut in half, the vertical displacement is decreased by 5 and the phase shift is increased by 1?

(A) I & II **(B)** I & IV **(C)** II & III **(D)** III & IV **(E)** I, II, III, & IV

43. The harmonic mean of the real roots of $3x^3 + 2x^2 + 5x + 4 = 0$ is _____.

(A) $-0.333...$ **(B)** -0.8 **(C)** $-1.333...$ **(D)** -2.4 **(E)** -3

44. Let $g(x) = 3x^2 - 2x + 1$. Find k if $g(k - 1) - g(k) = 11$.

(A) $2\frac{2}{3}$ **(B)** $\frac{3}{5}$ **(C)** 0 **(D)** $-\frac{2}{3}$ **(E)** -1

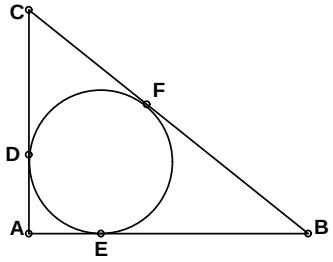
45. Let $\frac{1}{x} + \frac{1}{y} = 1$. Find $D_x y$.

(A) $\frac{y-1}{1-x}$ **(B)** $\frac{x-1}{1-y}$ **(C)** $\frac{1-y}{x}$ **(D)** $\frac{x}{y-1}$ **(E)** $\frac{y-1}{x-1}$

46. What is the instantaneous rate of change at x = 2 of the function f given by $f(x) = \frac{x^2-2}{x-1}$

(A) -2 **(B)** 0.1666... **(C)** 0.5 **(D)** 2 **(E)** 6

47. Find the radius of the circle inscribed in $\triangle ABC$ with AC = 3", AB = 4", and BC = 5".



(A) $\frac{1}{2}$ " **(B)** $\frac{2}{3}$ " **(C)** 1" **(D)** 2" **(E)** 6"

48. Let $f(x) = \begin{cases} 3 + x & \text{if } x \leq 1 \\ 3 - x & \text{if } 1 < x \end{cases}$. Which of the following is/are true?

1. $\lim_{x \to 1^+} f(x)$ exists 2. $\lim_{x \to 1^-} f(x)$ exists 3. f(x) is continuos

(A) none of these **(B)** 1 & 2 but not 3 **(C)** 1 only **(D)** 2 only **(E)** 1, 2, & 3

49. Which of the following pairs of numbers are considered to be 'fangs' of a 'vampire' number?
I. (35, 41) II. (21, 87) III. (72, 27) IV. (51, 63)

(A) I & III (B) I, II, & III (C) II & IV (D) I & II (E) I only

50. Let $4022_b - k_b = 1665_b$, where k_b is a four digit number. Find k_b in base 10.

(A) 949 (B) 1,117 (C) 1,263 (D) 2,066 (E) 2,135

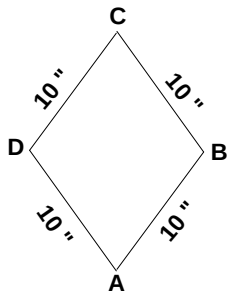
51. 7,158,AB3 ÷ 9 has a remainder of 7. Find A + B.

(A) 3 (B) 6 (C) 9 (D) 10 (E) 12

52. How many ordered pairs of positive integers (a, b) with $a + b \leq 50$, satisfy the equation: $(a + b^{-1}) \div (a^{-1} + b) = 13$.

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

53. Find the area of the rhombus shown given that AC — BD = 4",



(A) 100 in^2 (B) 98 in^2 (C) 96 in^2 (D) 94 in^2 (E) 92 in^2

54. If $x < y$ and $x < 0$, which of the following is never greater than any of the others?

(A) $x + y$ (B) $x - y$ (C) $x + |y|$ (D) $x - |y|$ (E) $-|x + y|$

55. The *Hole-In-One* golf shop has periodic sales given by the function $G(m) = 5 + 5\cos((\frac{\pi}{3})(m + 3))$ where m is the number of months and G(m) is the number of golf sets sold. If the store opened on Jan. 1, when did the maximum sales first occur?

(A) 3 months (B) 4 months (C) 5 months (D) 6 months (E) 7 months

56. A *square-free semiprime* is a composite number that is the product of two different primes. How many composite numbers less than 20 are considered *square-free semiprimes*?

(A) 8 (B) 6 (C) 4 (D) 2 (E) 0

57. The function f(x) = $\begin{cases} nx^3 - x & \text{if } x \leq 1 \\ mx^2 + 5 & \text{if } 1 < x \end{cases}$ is differentiable everywhere. Find n.

(A) 13 **(B) -17** **(C) -14** **(D) -11** **(E) -9**

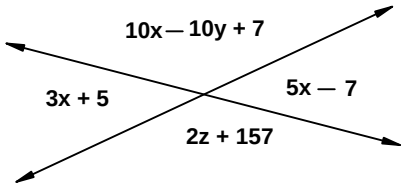
58. In how many ways can the letters of the word 'DIVIDE' be arranged in such a way that the vowels always come together?

(A) 18 **(B) 36** **(C) 72** **(D) 144** **(E) 180**

59. Given the sequence, $\frac{7}{(1\times1+1)} - \frac{7}{(2\times2-1)} + \frac{7}{(3\times3+1)} - \frac{7}{(5\times5-1)} + \frac{7}{(8\times8+1)} - \ldots$, find the digit in the ten-thousandths place.

(A) 6 **(B) 5** **(C) 4** **(D) 2** **(E) 1**

60. Find the sum of x, y, and z, given the degree measures of the angles shown.



(A) -3 **(B) -1** **(C) 0** **(D) 12** **(E) 15**

University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitation A ● 2018
Answer Key

1. C	**21. B**	**41. B**
2. B	**22. D**	**42. D**
3. E	**23. C**	**43. D**
4. A	**24. C**	**44. E**
5. D	**25. A**	**45. A**
6. E	**26. A**	**46. D**
7. A	**27. B**	**47. C**
8. B	**28. C**	**48. B**
9. A	**29. E**	**49. D**
10. E	**30. E**	**50. B**
11. D	**31. A**	**51. D**
12. B	**32. E**	**52. B**
13. B	**33. C**	**53. C**
14. E	**34. E**	**54. D**
15. C	**35. D**	**55. A**
16. A	**36. C**	**56. C**
17. C	**37. E**	**57. D**
18. E	**38. B**	**58. B**
19. A	**39. E**	**59. C**
20. A	**40. D**	**60. A**