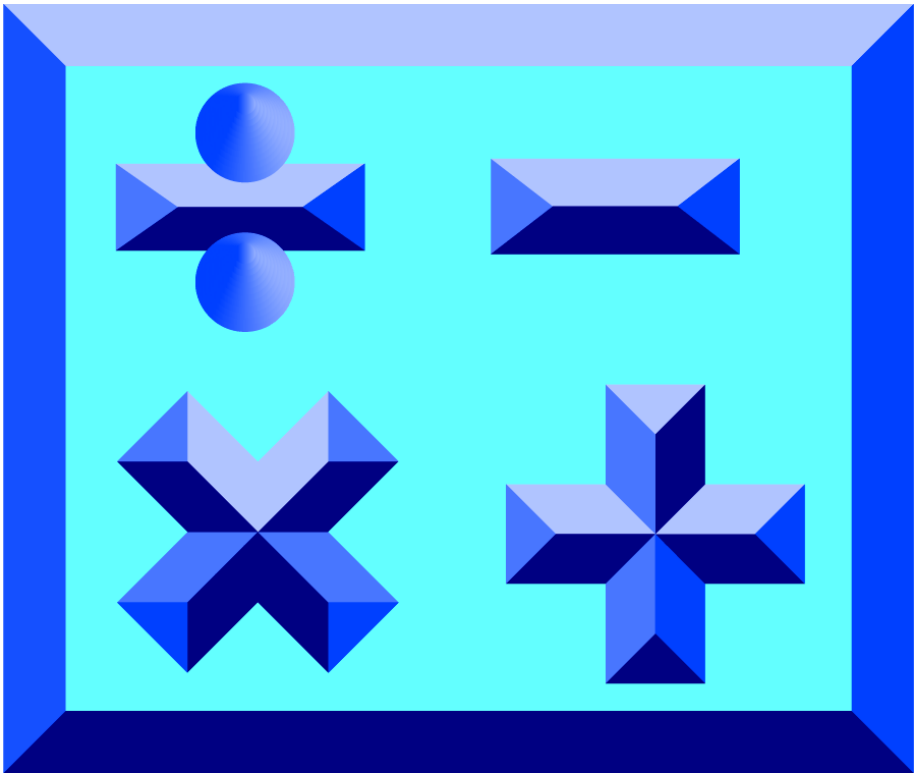
UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

District • 2018



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YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $1 - (1 + 2^3 - 5) \div 8 \times (1 - 3^2) + 1$

(A) -3 **(B)** 0 **(C)** 3.25 **(D)** 4 **(E)** 6

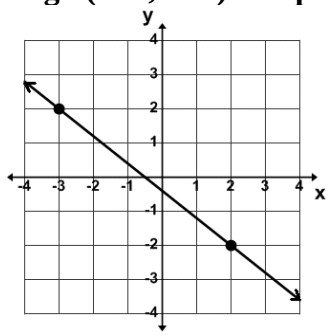
2. Three million nine hundred thousand nine hundred twenty is added to one million eight thousand three hundred twenty four. The sum is multiplied by eleven. The digits in the product are added together. What is the sum of the digits?

(A) 28 **(B)** 30 **(C)** 32 **(D)** 38 **(E)** 41

3. Les Tred is shopping for a new set of 4 tires at the local tire store. The regular price is $64.98. He can buy the 1st tire at the regular price. The 2nd tire is half off the regular price. The 3rd tire is discounted $33\frac{1}{3}$%. And, $10.98 is taken off the regular price for the 4th tire. What would it cost Les for the 4 tires before taxes? (nearest cent)

(A) $194.79 **(B)** $183.61 **(C)** $173.13 **(D)** $184.98 **(E)** $195.77

4. Find an equation of the line through (— 1, — 3) and perpendicular to the line shown.



(A) $4x + 5y = -7$ **(B)** $5x - 4y = 7$ **(C)** $4x + 5y = -11$
(D) $5x - 4y = -7$ **(E)** $5x + 4y = -17$

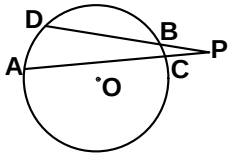
5. Simplify: $\left(\frac{2x^2 + 7x + 3}{x^2 - 9}\right)\left(\frac{x^2 - 3x}{2x^2 + 11x + 5}\right)$

(A) $\frac{x}{x+5}$ **(B)** $\frac{x+3}{x-4}$ **(C)** $x^2 + 5x$ **(D)** $\frac{4x+3}{11x-4}$ **(E)** $\frac{x^2}{x^2+5}$

6. Penni Les has 4 times as many dimes as nickels and half as many pennies as dimes. She has $4.70. How much would she have left if she spent all of her nickels?

(A) $0.50 **(B)** $0.70 **(C)** $4.00 **(D)** $4.20 **(E)** $4.50

7. Given the circle with center O shown with DP = 10 cm, BP = 3 cm, and AP = 12 cm. Find AC.



(A) 7.2 cm **(B)** 8.4 cm **(C)** 9.5 cm **(D)** 10.5 **(E)** 10.8

8. **Find the sum of the measure of an exterior angle of a regular pentagon, the measure of a central angle of a regular hexagon, and the measure of an interior angle of a regular heptagon. (nearest degree)**

(A) 261° **(B) 180°** **(C) 297°** **(D) 183°** **(E) 321°**

9. **Les Space increased the length of two opposite sides of a rectangle by 40%, and decreased the other two opposite sides by 10%. What percent of the area of the original rectangle is the area of the new rectangle?**

(A) 74% **(B) 50%** **(C) 230%** **(D) 4%** **(E) 126%**

10. **If $\frac{Ax+B}{4x+1} - \frac{2x+3}{3x-2} = \frac{7x^2-36x+5}{12x^2-5x-2}$, where A and B are constants, then A — B equals:**

(A) 1 **(B) 4** **(C) 5** **(D) 9** **(E) 10**

11. **The graph of $x^2 + y^2 - 10x + 12y + 57 = 0$ is a circle with a center (h, k) and a radius r. Find $h \times k - r$.**

(A) — 32 **(B) — 26** **(C) — 3** **(D) 46** **(E) 55**

12. **If $8^{(k+1)} = 16^{(k-1)}$, then $2^{(k)} = ?$**

(A) 512 **(B) 128** **(C) 64** **(D) 1,024** **(E) 4**

13. **Determine the range of $f(x) = 4\sin(3x - \pi) - 2$.**

(A) [— 2, 6] **(B) [4, — 2]** **(C) [— 1, 3]** **(D) [— 6, 2]** **(E) [3, — 1]**

14. **Which of the following is an identity for $\frac{\csc\theta - \cot\theta}{1 - \cos\theta}$?**

(A) $\csc\theta$ **(B) $\cot\theta$** **(C) $\cos\theta$** **(D) $\sec\theta$** **(E) $\tan\theta$**

15. **Given: $f(x) = 5\cos(2x - 1) - 3$. What quadrant(s) would the graph of f(x) be in if the amplitude is decreased by 2, the vertical displacement was increased by 2 and the phase shift was multiplied by 2?**

(A) I & II **(B) I & IV** **(C) II & III** **(D) III & IV** **(E) I, II, III, & IV**

16. **In the expansion of $(2x + 1)^6$, the sum of the coefficients of the 2nd, 3rd, 5th and 6th term is:**

(A) 2,688 **(B) 524** **(C) 672** **(D) 504** **(E) 1,344**

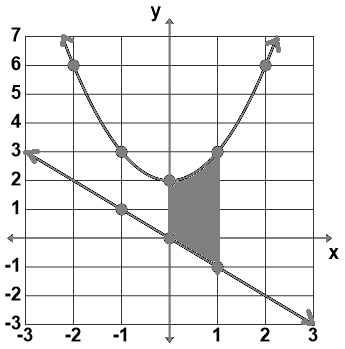
17. **Find $a + b + c + d$ given the Fibonacci characteristic sequence: 2, a, b, 12, c, d, 50, ...**

(A) 56 **(B) 57** **(C) 59** **(D) 62** **(E) 64**

18. The 4th term of a geometric sequence is $\frac{1}{8}$. The 7th term is $\frac{1}{64}$. Find the sum of the first 6 terms of this geometric sequence.

(A) $\frac{63}{64}$ **(B)** $\frac{31}{32}$ **(C)** $1\frac{63}{64}$ **(D)** $1\frac{15}{16}$ **(E)** $1\frac{31}{32}$

19. Find the area of the shaded region.



(A) 2.666... **(B)** 2.75 **(C)** 2.8333... **(D)** 2.875 **(E)** 3

20. Let $f(x) = \frac{x^2 - 4x - 5}{x + 1}$. A *removable discontinuity* exists at x = ?

(A) -4 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 5

21. Let $f(x) = 3x^2 - 4x - 5$ and $g(x) = 4x + 5$. Find $g(f'(1))$

(A) 13 **(B)** 2 **(C)** -3 **(D)** -19 **(E)** 27

22. Al Fahbett randomly selected a letter from the set {L, E, T, T, E, R}. What are the odds that he selected E?

(A) $\frac{1}{2}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{1}$ **(D)** $\frac{1}{8}$ **(E)** $\frac{1}{3}$

23. The Millersview Dunkers have 4 centers, 6 guards, and 7 forwards. How many different teams consisting of 1 guard, 2 forwards, and 2 centers could be formed?

(A) 6,188 **(B)** 756 **(C)** 1,237 **(D)** 126 **(E)** 1,260

24. Given the equation: $4^x = 7$. Which of the following mathematicians would be the best one to ask for help to solve for x?

(A) Aryabhata **(B)** Charles Babbage **(C)** John Napier **(D)** George Boole **(E)** Alan Turing

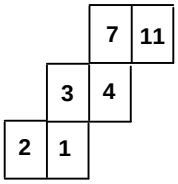
25. If $x^2 - 3x + b = (x + a)(x - 7)$, where a and b are integers then $a + b =$ _____.

(A) -34 **(B)** -28 **(C)** -24 **(D)** 32 **(E)** 36

26. If $x + y = -3$ and $xy = 6$ then $x^3 + y^3 = ?$

(A) -19 **(B)** -18 **(C)** 21 **(D)** 27 **(E)** 33

27. Poly Gawn folds the net shown into a cube. She lets the face with the 11 on it be the base of the cube. What is the sum of the numbers on the lateral faces?



(A) 10 (B) 11 (C) 13 (D) 14 (E) 16

28. How many non-negative proper fractions in lowest terms have a denominator of 24?

(A) 12 (B) 11 (C) 10 (D) 9 (E) 8

29. If $a_1 = -3$, $a_2 = -2$, $a_3 = 1$ and $a_n = [(a_{n-1}) + (a_{n-3})] \times (a_{n-2})$ for $n \geq 4$, then a_6 equals:

(A) 32 (B) 12 (C) 6 (D) 4 (E) 2

30. Find the sum of the x-values in $\{x \mid \cos(2x) + \sin(x) = 0, x \in [0, 2\pi)\}$? (nearest tenth)

(A) 12.6 (B) 11.0 (C) 9.4 (D) 7.3 (E) 5.2

31. How many asymptotes does $f(x) = \frac{x^2+4x+3}{x+2}$ have?

(A) 4 (B) 3 (C) 2 (D) 1 (E) 0

32. A function, $g(x) = x^2 + bx + c$, exists such that $g(1) = 2$ and $g(2) + g(3) = 7$. Find $g(-4)$.

(A) $20\frac{3}{4}$ (B) $31\frac{2}{3}$ (C) 21 (D) 25 (E) $30\frac{1}{3}$

33. Given the function $f(x) = 2\cos(x) + 1$, find the slope of the secant line between $x = 0$ and $x = \frac{\pi}{2}$.

(A) 0 (B) -2π (C) $\frac{2}{\pi}$ (D) $-\frac{4}{\pi}$ (E) no slope

34. Find the 20^{th} tetrahedral number.

(A) 35 (B) 210 (C) 1,330 (D) 1,540 (E) 1,771

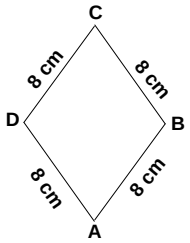
35. The fraction $\frac{11}{14}$ base 6 can be written as which of the following decimals in base 6?

(A) $0.333..._6$ (B) $0.1444..._6$ (C) $0.3222..._6$ (D) $0.1414..._6$ (E) $0.4111..._6$

36. Let $424_b + 332_b - 241_b = 1020_b$. Find 113_b in base 10.

(A) 45 (B) 33 (C) 59 (D) 81 (E) 41

37. Find the area of the rhombus shown given that AC — BD = 2 cm,



(A) 32 cm^2 (B) 15.75 cm^2 (C) 64 cm^2 (D) 72 cm^2 (E) 63 cm^2

38. The sum of the digits of a certain three-digit number is 18. The sum of the hundreds digit and the tens digit is 10. And, the tens digit is one less than half the units digit. How many of the digits are prime numbers?

(A) 0 (B) 1 (C) 2 (D) 3 (E) not enough information

39. The average of Seymore Anser's first five quiz grades is 87. He made 75 on his sixth quiz. What does he have to make on his seventh quiz to have a quiz average of 84?

(A) 68 (B) 75 (C) 78 (D) 85 (E) 88

40. Jose Canyusee is standing 20 feet from the base of a flag pole. The angle of depression from his eyes to the base of the pole is 16°. The angle of elevation from his eyes to the top of the pole is 60°. What is the height of the flag pole? (nearest foot)

(A) 104 ft (B) 81 ft (C) 40 ft (D) 52 ft (E) 80 ft

41. Every morning, Johnny Jogger covers 9 miles on a trail near Lake Ray Roberts. He walks the first 3 miles at a speed of 4 mph, he runs the next 3 miles at a speed of 7 mph, and he jogs the last 3 miles at a speed of 5 mph. Find the mean speed for his 9 mile trek. (nearest tenth)

(A) 5.1 mph (B) 5.2 mph (C) 5.3 mph (D) 5.4 mph (E) 5.5 mph

42. Twenty class 2A seniors took the TMSCA State math test this year. Twelve of them were boys and eight were girls. All of them had an equal chance to win one of the top three trophies. What was the probability that all three of the top trophies were won by girls? (nearest whole percent)

(A) 5% (B) 7% (C) 13% (D) 19% (E) 25%

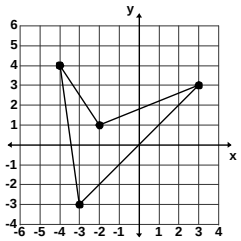
43. Cookie Baykur baked chocolate chip cookies, peanut butter cookies, raisin cookies, and snickerdoodles. She put six cookies per zip lock bag to sell at the bake sale. How many different bags of 6 cookies could she make?

(A) 126 (B) 24 (C) 696 (D) 90 (E) 84

44. How many distinct combinations exist for a 4-digit combination padlock so that the first digit is a prime number, the second digit is a factor of 10, the third digit is a positive Fibonacci number, and the fourth digit is divisible by 5?

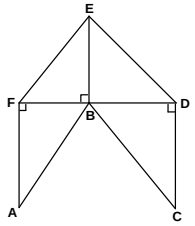
(A) 14 (B) 60 (C) 80 (D) 120 (E) 160

45. Rene Dezkartez drew the quadrilateral shown, whose vertices are integers. What is the area of Rene's quadrilateral?



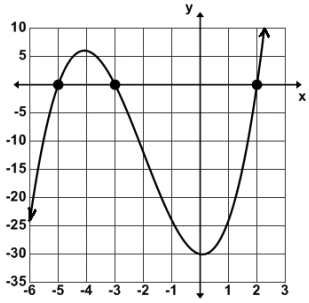
(A) 15 units^2 (B) 14.5 units^2 (C) 14 units^2 (D) 13.5 units^2 (E) 13 units^2

46. Given: m∠BED = 45°, m∠ABF = 30°, m∠EFB = 60°, m∠BCD = 45°,and EF = 4". Find the perimeter of pentagon ABDEF. (nearest tenth).



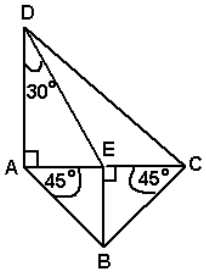
(A) 12.0" (B) 12.9" (C) 15.8" (D) 20.2" (E) 24.2"

47. The graph of $f(x) = Ax^3 + Bx^2 + Cx + D$ is shown here. Find $A + B + C + D$.



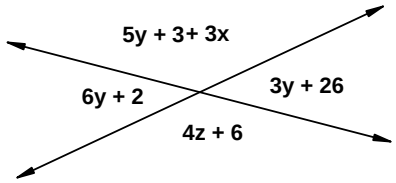
(A) 37 (B) 6 (C) — 23 (D) — 24 (E) — 37

48. Find the perimeter of the quadrilateral ABCD if AE = 4". (nearest inch)



(A) 24 in (B) 25 in (C) 29 in (D) 31 in (E) 32 in

49. Find the sum of x, y, and z, given the degree measures of the angles shown.



(A) 29 (B) 50 (C) 68 (D) 70 (E) 80

50. Write this expression as a simplified proper fraction. $0 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{5 + \frac{1}{7}}}}$

(A) $\frac{37}{86}$ **(B)** $\frac{115}{266}$ **(C)** $\frac{3}{7}$ **(D)** $\frac{58}{133}$ **(E)** $\frac{36}{115}$

51. $\left\{(x, y) \middle| x,y \in \{\text{Integers}\}, -7 \leq x \leq 11, \text{and} -11 \leq y \leq 7\right\}$ **is the solution set of $3x - 2y = 5$. How many such ordered pairs exist?**

(A) 5 (B) 6 (C) 7 (D) 10 (E) 11

52. How many ordered pairs of positive integers (a, b) with $a + b \leq 91$, satisfy the equation: $(a + b^{-1}) \div (a^{-1} + b) = 19$.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

53. Points P (0, 5), Q(4, — 7), R(7, — 3), and S(x, y) are the coordinates of the vertices of a parallelogram, where S(x,y) is in quadrant II. Find $x + y$.

(A) — 2 (B) — 1 (C) 2 (D) 4 (E) 6

54. Let function f = {(1, 1), (3, 4), (2, 5)} and function g = {(3, 1), (1, 3), (2, 2)}. Which of the following is a member of the function f o g ?

(A) (1, 1) (B) (3, 3) (C) (2, 1) (D) (3, 1) (E) (3, 4)

55. P, Q, & R are the real roots of $x^3 + Bx^2 + Cx + D = 0$. The harmonic mean of P, Q, & R is — 9 and C is — 16. Find D.

(A) — 72 (B) — 48 (C) — 37 (D) — 32 (E) — 25

56. Let f(x) = $\begin{cases} x^2 + 1 & \text{if } x \leq 1 \\ 2x & \text{if } x > 1 \end{cases}$ **, for all real numbers x. Which of the following must be true?**

I. f(x) is continuous everywhere.
II. f(x) is differentiable everywhere
III. f(x) has a local minimum at x = 1

(A) I only (B) I and II only (C) II and III only (D) I and III only (E) I, II, and III

57. Which of the following polar equations will produce the graph of a lemniscate that is symmetric to the polar axis?

(A) $r^2 = 2\sin(4\theta)$ **(B)** $r = 2\cos(\theta)$ **(C)** $r^2 = \sin(\theta)$ **(D)** $r = 4\cos(\theta)$ **(E)** $r^2 = 4\cos(2\theta)$

58. Pennie Flipper is going to toss a fair penny 6 times. What is the probability that she will get at least two tails? (nearest whole percent)

(A) 67% **(B) 64%** **(C) 89%** **(D) 28%** **(E) 25%**

59. Given that the set of natural numbers continue in the triangular pattern shown below, find the sum of the numbers in row 10.

			1				(row 1)
		2	3	4			(row 2)
	5	6	7	8	9		(row 3)
10	11	12	13	14	15	16	(row 4)
			...				(...)

(A) 1,729 **(B) 2,030** **(C) 1,638** **(D) 1,748** **(E) 1820**

60. Let $f(x) = (2x - 1)^2$. The tangent to f(x) at (x, y) is perpendicular to $x = 4 - 2y$. Find $x + y$.

(A) 3.25 **(B) 2.375** **(C) 2** **(D) 1** **(E) 0**

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University Interscholastic League
MATHEMATICS CONTEST
HS ● District ● 2018
Answer Key

1. E	**21. A**	**41. A**
2. A	**22. A**	**42. A**
3. A	**23. B**	**43. E**
4. B	**24. C**	**44. D**
5. A	**25. C**	**45. B**
6. D	**26. D**	**46. C**
7. C	**27. D**	**47. D**
8. A	**28. E**	**48. C**
9. E	**29. B**	**49. C**
10. D	**30. B**	**50. B**
11. A	**31. C**	**51. B**
12. B	**32. E**	**52. D**
13. D	**33. D**	**53. A**
14. A	**34. D**	**54. D**
15. E	**35. E**	**55. B**
16. D	**36. B**	**56. B**
17. D	**37. E**	**57. E**
18. E	**38. C**	**58. C**
19. C	**39. C**	**59. A**
20. B	**40. C**	**60. D**