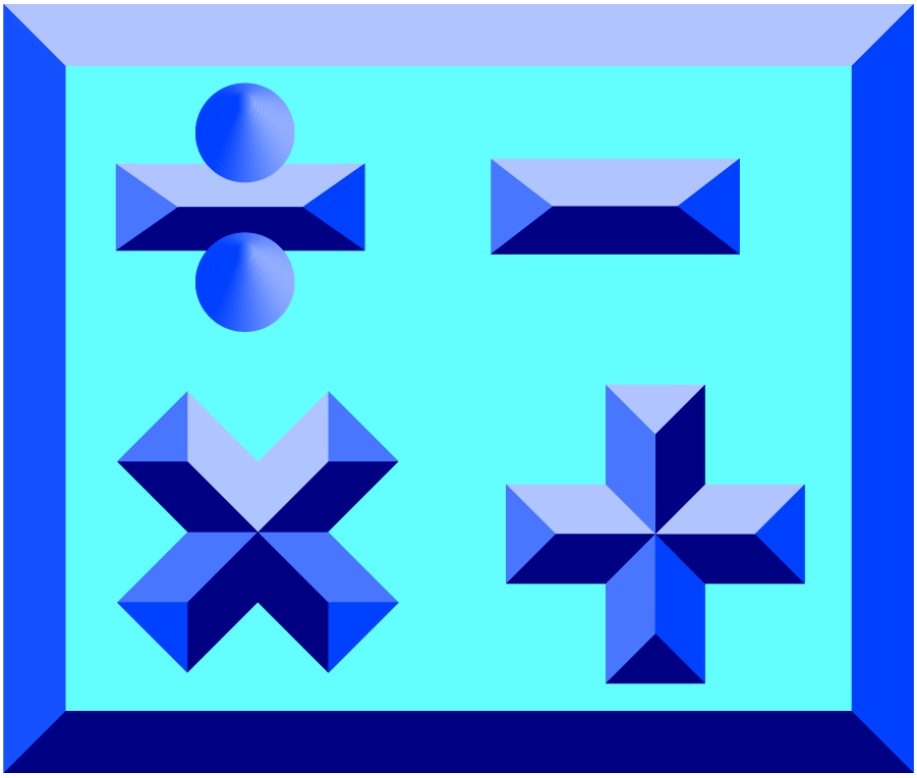
UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Region • 2018



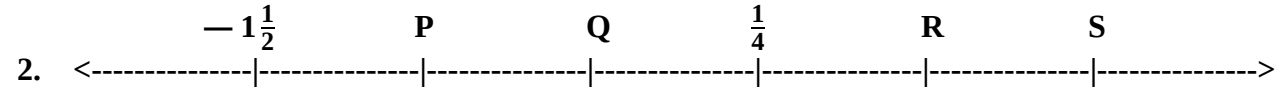
DO NOT TURN THIS PAGE UNTIL

YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $5! \div (4)^3 + (2 - 1)^{\frac{2}{3}} - 4 \times 5$

(A) -18.203125 **(B)** -17.125 **(C)** -9 **(D)** -5.625 **(E)** 35

2.



The distances between the hash marks (|) are equal. Find $P + Q + R - S$.

(A) $-3\frac{1}{12}$ **(B)** 1 **(C)** $-1\frac{1}{4}$ **(D)** $\frac{7}{12}$ **(E)** $-1\frac{5}{6}$

3. The universal set *U* = {r,e,g,i,o,n,a,l,s}. Subset A = {s,i,g,n,a,l} and subset B = {l,e,g,i,o,n}. How many elements are in the complement set of $A \cap B$?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

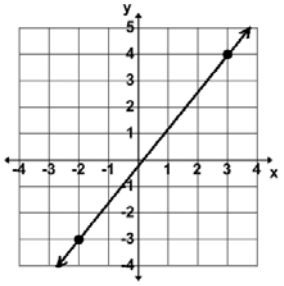
4. Little Richie bid on 4 old HP11C calculators at the EZPay auction site. He got two of the calculators each at half the original price, one at 20% off the original price, and got the last one for $40.00 less than the original price. How much did he pay for the 4 calculators if the original price of an HP11C was $89.99? (tax not included) (nearest cent)

(A) $201.98 **(B)** $211.97 **(C)** $157.98 **(D)** $152.98 **(E)** $215.97

5. If $(4x - 1)(3x + 2)(5 - x) = ax^3 + bx^2 + cx + d$ **then** $a + b + c + d =$ _____.

(A) 78 **(B)** 70 **(C)** 60 **(D)** 40 **(E)** 6

6. Find an equation of the line shown.



(A) $5x - 7y = 11$ **(B)** $7x - 5y = 1$ **(C)** $x - y = 1$ **(D)** $7x - 5y = 13$ **(E)** $x - 5y = 13$

7. A line contains the points (− 3, 5) and (6, − 1). The line intersects the y-axis at y = ?

(A) -2 **(B)** 0 **(C)** 3 **(D)** 3.5 **(E)** 5.5

8. The line $4x - 2y = 7$ **is perpendicular to the line** $ax + 2y = 3$**. The point of intersection is (p, q). Find** $p + q$.

(A) -1.5 **(B)** -1 **(C)** 2 **(D)** 2.5 **(E)** 3

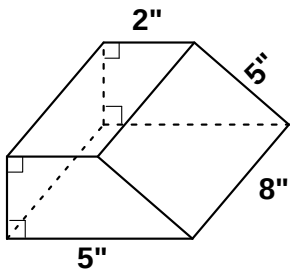
9. How many integers are NOT in the solution set for $|2x - 5| \geq 3$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

10. Papa John is sitting on one side of a seesaw and his son and daughter are sitting on the other side. He weighs 200 lbs, his son weighs 75 lbs, and his daughter weighs 50 lbs. How far from the fulcrum will Papa John have to sit if his son is sitting 5 feet from the fulcrum and his daughter is sitting 4 feet from the fulcrum in order to balance the seesaw?

(A) 2' 10.5" **(B) 2' 9.75"** **(C) 2' 9"** **(D) 2' 7.5"** **(E) 2' 5"**

11. Find the volume of the trapezoidal prism shown. (nearest cu. in)



(A) 112 cu. in **(B) 160 cu. in** **(C) 106 cu. in** **(D) 80 cu. in** **(E) 200 cu. in**

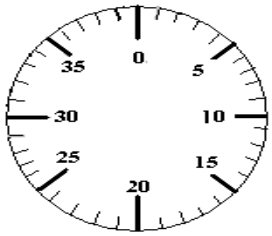
12. Find $a + b + c + d$ given the Fibonacci characteristic sequence: a, b, 9, c, d, 31, ...

(A) 31 **(B) 40** **(C) 50** **(D) 62** **(E) 80**

13. $\frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \frac{1}{21} + \ldots + \frac{1}{120} + \frac{1}{136} = ?$

(A) $\frac{15}{17}$ **(B)** $\frac{7}{8}$ **(C)** $\frac{13}{15}$ **(D)** $\frac{8}{9}$ **(E)** $\frac{9}{10}$

14. A (right) - B (left) - C (right) is the combination needed to open the safe with the dial shown below. How many distinct combinations exist if A is a positive multiple of 4, B is a factor of 38, and C is a nonnegative power of 3.



(A) 180 **(B) 144** **(C) 17** **(D) 108** **(E) 200**

15. If two dice are tossed, what are the odds that the sum of the faces is 7 or an 11?

(A) $\frac{1}{2}$ **(B)** $\frac{7}{36}$ **(C)** $\frac{2}{7}$ **(D)** $\frac{7}{29}$ **(E)** $\frac{2}{9}$

16. Which of the following is/are Aryabhata's contribution(s)?

I. concept of sine called a "half chord"
II. a place value system using letters to represent numbers
III. an early approximation of pi

(A) I only **(B) I & II** **(C) II & III** **(D) I, II, & III** **(E) none of them**

17. How many positive proper fractions in lowest terms have a denominator of 18?

(A) 32 (B) 5 (C) 6 (D) 7 (E) 9

18. If the roots of $2x^3 + bx^2 + cx + d = 0$ are -2, 1, and 4, then $b + c + d$ equals:

(A) -2 (B) -1 (C) 1 (D) 3 (E) 5

19. Les Area cut off 30% of the length of his rectangle and 40% off the width of his rectangle. What percent of the area of his original rectangle is the area of his new rectangle?

(A) 12% (B) 58% (C) 88% (D) 35% (E) 42%

20. If $\frac{x+5}{Ax+B} - \frac{2x-3}{4x+1} = \frac{2x^2+36x-13}{4x^2-23x-6}$, where A and B are constants, then $A + B$ equals:

(A) -5 (B) -3 (C) 1 (D) 6 (E) 7

21. Which of the following is a looped limacon?

(A) $r = 2 + \cos(\theta)$ (B) $r = 1.5 + \cos(\theta)$ (C) $r = 1 + \cos(\theta)$
(D) $r = 0.5 + \cos(\theta)$ (E) $r = \cos(\theta)$

22. Determine the range of $f(x) = 3\sin(2x + 1) - 4$.

(A) $[-7, -1]$ (B) $[-6.5, -2.5]$ (C) $[-8, -2]$ (D) $[-6, 0]$ (E) $[-8, 1]$

23. Find $P + Q$ if $\begin{bmatrix} -1 & 3 \\ 6 & -10 \end{bmatrix} \cdot \begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

24. Let $f(x) = 3x^2 - 4x - 5$ and $g(x) = 2x^2 + x$. Find $g'(f'(-1))$

(A) -39 (B) -22 (C) 7 (D) 10 (E) 210

25. The function $f(x) = x^3 - 6x^2 + 12x - 7$ has an inflection point at (x, y). Find $x + y$.

(A) -2 (B) -1 (C) 1 (D) 3 (E) 4

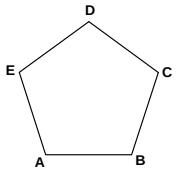
26. Let set P = {2, 1, 3, 4, 7}, set Q = {1, 3, 6, 10, 15}, and set R = {2, 3, 4, 5, 6, 7}. Exactly one digit is chosen from each set. What is the probability of picking at least one prime number? (nearest %)

(A) 50% (B) 53% (C) 75% (D) 83% (E) 89%

27. The ordered pair (x, y) are the fangs of the vampire number, 1395. Find $x + y$.

(A) 18 (B) 36 (C) 54 (D) 72 (E) 108

28. Given the regular pentagon shown, find the length of a side with AC + AD + BE + BD + CE = 72". (nearest tenth)



(A) 8.9" (B) 4.7" (C) 6.4" (D) 9.1" (E) 8.1"

29. Lotta Bucks had a pack of one-dollar bills. She put 25% of them in her safe for future spending. Then she put 20% of what was left in an envelope to send to her brother. Then she gave her sister five one-dollar bills. How many dollar bills did she put in the envelope for her brother if she had 7 one-dollar bills left?

(A) 3 (B) 5 (C) 7 (D) 12 (E) 15

30. Tu Yung is three years younger than her cousin Soh Ohd. In two years the sum of their ages will be twenty-one. What will Tu Yung's age be in five years?

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

31. Point P (3, 3) lies on the x-y plane. P is reflected across the origin to point Q. Point Q is translated vertically + 4 units to point R. Point R is reflected across the y-axis to point S. Point S is translated horizontally — 4 units to point T (x, y). Find x + y

(A) — 14 (B) — 8 (C) 0 (D) 6 (E) 12

32. A large pipe can fill an empty tank in 3 hours. A smaller pipe can fill the same empty tank in 5 hours. The drain pipe in the tank can empty the full tank in 4 hours. How long would it take to fill the tank if both of the fill pipes and the drain tank are open? (nearest minute)

(A) 1 hr 3 min (B) 2 hrs 2 min (C) 3 hrs 3 min (D) 3 hrs 32 min (E) 4 hrs

33. The sum of the digits of a certain 3-digit number is 17. The sum of the squares of the digits is 109. The absolute value difference between the 3-digit number and 495 is the 3-digit number written in reverse. How many of the digits are divisible by 3?

(A) 0 (B) 1 (C) 2 (D) 3 (E) not enough information

34. Mary Goround rode the Ferris Wheel at the county fair. When Mary gets on the cart, the wheel is 3 ft off the ground. The diameter of the wheel is 40 ft. The ride makes 5 revolutions over a 2 minute time span. How high off the ground was Mary's cart after 40 seconds? (nearest foot)

(A) 40 ft (B) 31 ft (C) 30 ft (D) 35 ft (E) 33 ft

35. Betty Luzes invested some of her money in some mutual funds. Her investment earned 3% the first year, lost 6% the second year, and lost 5% the third year. What was her mean loss over the 3 year period? (nearest tenth)

(A) 2.1% (B) 2.4% (C) 2.7% (D) 2.8% (E) 3.1%

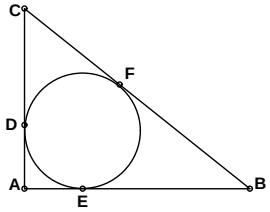
36. Rancher O. K. Corale needs to fence in a rectangular pen for his horses and mules. The pen is divided by a fence to separate the horses from the mules into two equal smaller pens. Find the maximum area of the pen that he can fence in if he has 300 feet of fencing. (nearest ft^2)

(A) 5,625 ft^2 (B) 3,750 ft^2 (C) 4,500 ft^2 (D) 3,450 ft^2 (E) 1,875 ft^2

37. Eight girls, twelve boys, four men coaches and six women coaches will be attending the Teckie math camp. A special council consisting of four girls, three boys, and two coaches, one man and one women will be formed. How many different councils are possible?

(A) 300 (B) 30,800 (C) 693,000 (D) 2,700 (E) 369,600

38. The circle shown is inscribed in △ABC. The radius of the circle is 29.5 cm and the perimeter of △ABC is 310 cm. Find the area of △ABC. (nearest cm^2)

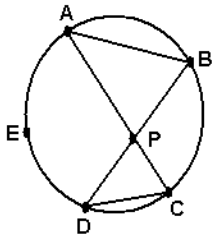


(A) 2,286 cm^2 (B) 3,048 cm^2 (C) 4,573 cm^2 (D) 6,097 cm^2 (E) not enough information

39. Find the digit in the units place of the integer 7^{654}.

(A) 1 (B) 3 (C) 7 (D) 9 (E) cannot be determined

40. $\overline{AB}$, $\overline{AC}$, $\overline{BD}$, and $\overline{CD}$ are chords of circle O and point E lies on circle O. Find $m\overset{\frown}{AED}$ given $m\angle DPC = 80°$ and $m\angle PAB = 30°$.



(A) 160° (B) 130° (C) 150° (D) 140 ° (E) 110°

41. Solve for x: $\log_3(x + 10) = \log_3(x - 2) + \log_3(x)$.

(A) — 2 (B) — 1 (C) 2 (D) 3 (E) 5

42. Let f(x) = 1 — x and g(x) = 2x — 3. Find $f\Big(g\big(-f(x)\big)\Big)$.

(A) — 2x (B) 2x — 6 (C) 2x + 6 (D) 6 (E) 6 — 2x

43. If the following pattern continues, find the sum of the 3rd term and the 19th term in row 20.

1	row 0
1 1	row 1
1 2 1	row 2
1 3 3 1	row 3
1 4 6 4 1	row 4
1 5 10 10 5 1	row 5
...	...

(A) 272 (B) 306 (C) 342 (D) 380 (E) 420

44. Which of the following is an identity for $\frac{\cos(\theta)}{1 - \sin(\theta)} - \tan(\theta)$

(A) $\csc^2(\theta)$ (B) $\cot(\theta)$ (C) $\tan^2(\theta)$ (D) $\sec(\theta)$ (E) $\sec^2(\theta)$

45. Let f(x) = $|2x + 1| - |x - 3|$. Find the minimum value of f(x).

(A) -4 (B) -3.5 (C) -0.5 (D) 2.5 (E) 3

46. The area (in square units) of the region bounded by $x = y^2 - 2$ and $y = -x$ is:

(A) $2\frac{2}{3}$ (B) $3\frac{1}{4}$ (C) $3\frac{2}{3}$ (D) $4\frac{1}{3}$ (E) $4\frac{1}{2}$

47. A number N in base 5 is 159 in base 10. If $34P_5 + 2Q3_5 = N$ in base 5, where P and Q are digits, then P + Q is:

(A) 5 (B) 4 (C) 3 (D) 2 (E) 1

48. The ratio of the measures of the non-adjacent angles of a quadrilateral is 2:1. The measure of the smallest acute angle is 55°. The sum of the two acute angles is 120°. Find the measure of the largest obtuse angle.

(A) 135° (B) 130° (C) 125° (D) 120° (E) 115°

49. Which of the following equations can be obtained from the graph of the parent function y = cos(x) by applying a vertical stretch of — 2 units, a vertical shift of — 3 units, and a phase shift of 5? y = :

(A) $-3\cos(x - 5) - 2$ (B) $2\cos(x + 5) + 3$ (C) $-2\cos(x + 5) - 3$
(D) $5\cos(x + 2) - 3$ (E) $-2\cos(x - 5) - 3$

50. How many composite numbers less than or equal to 55 and divisible by 5 are considered to be *square-free semiprimes*?

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

51. Let A + B = 24 and A × B = 40. Find A — B, where A < B.

(A) $4 - \sqrt{26}$ (B) $-4\sqrt{26}$ (C) 24 (D) 12 (E) $2 - \sqrt{26}$

52. Let $f(x) = \begin{cases} nx^4 + 5x & \text{if } x \leq 2 \\ mx^2 - 3x & \text{if } 2 < x \end{cases}$ be continuous and differentiable everywhere. Find m.

(A) 7.25 **(B)** 6 **(C)** 4 **(D)** 3.25 **(E)** 3.75

53. If the probability that a student in a remedial math class studies for an exam is 50%, and the probability that a student who doesn't study fails the test is 75%, then the probability that a student both passes the test and doesn't study is: (nearest whole percent)

(A) 63% **(B)** 38% **(C)** 25% **(D)** 13% **(E)** 7%

54. If $a_1 = -2$, $a_2 = -1$, $a_3 = 0$ and $a_n = [(a_{n-3}) - (a_{n-1})] \times (a_{n-2})$ for $n \geq 4$, then a_7 equals:

(A) -3 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** 3

55. If you start at $(-\frac{5\pi}{4}, 0)$ on the x-axis and travel horizontally 24 radians to the right, how many times will you cross the graph of $y = 3\cos(2x)$?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

56. Let x and y be integers such that $xy(x - y) = 96$ and $xy + x - y = 22$. Calculate $x^2 + y^2$.

(A) 68 **(B)** 69 **(C)** 71 **(D)** 72 **(E)** 74

57. The focus of the parabola $y = -x^2 + 6x - 11$ is at (x, y). Find $x + y$.

(A) 0.75 **(B)** 3 **(C)** 5.25 **(D)** -0.25 **(E)** -1.75

58. Given the function $f(x) = x^3 - x$, find the slope of the secant line between x = 1 and x = 5.

(A) 15 **(B)** 24 **(C)** 30 **(D)** 40 **(E)** no slope

59. Let $x = 4 + \cfrac{14}{4 + \cfrac{14}{4 + \cfrac{14}{4 + \cfrac{14}{4 + \ldots}}}}$. Find x. (nearest tenth)

(A) 4.5 **(B)** 6.2 **(C)** 8.5 **(D)** 5.6 **(E)** 7.5

60. The figure shown is reflected over its positive diagonal. Then it is reflected over it vertical axis. Finally, it is rotated 180° counterclockwise. Which of the following figures is the result of these three transformations?

(A)  **(B)**  **(C)**  **(D)**  **(E)** 

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University Interscholastic League
MATHEMATICS CONTEST
HS • Regional • 2018
Answer Key

1. B	**21. D**	**41. E**
2. E	**22. A**	**42. E**
3. D	**23. E**	**43. D**
4. B	**24. A**	**44. D**
5. C	**25. D**	**45. B**
6. B	**26. E**	**46. E**
7. C	**27. E**	**47. C**
8. D	**28. A**	**48. B**
9. C	**29. A**	**49. E**
10. A	**30. E**	**50. C**
11. A	**31. C**	**51. B**
12. B	**32. D**	**52. B**
13. A	**33. C**	**53. D**
14. B	**34. E**	**54. B**
15. C	**35. C**	**55. C**
16. D	**36. B**	**56. A**
17. C	**37. E**	**57. A**
18. A	**38. C**	**58. C**
19. E	**39. D**	**59. B**
20. A	**40. D**	**60. A**