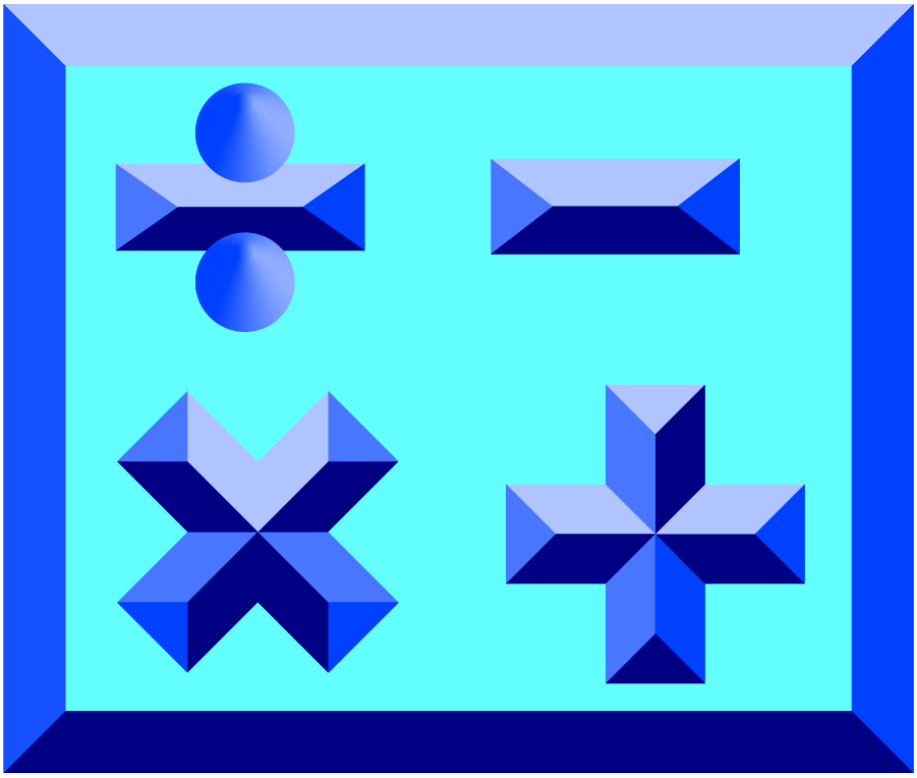
UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

State • 2018



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YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $2 + 3 \times (5 - 7) \div (1 - 4) \times 6 + 8$

(A) 28 (B) 22 (C) $10\frac{1}{3}$ (D) -2 (E) -6

2. One million forty thousand six hundred eighty is subtracted from two million three hundred thousand five hundred seven. The difference is multiplied by nine. The digits in the product are added together. What is the sum of the digits?

(A) 9 (B) 16 (C) 18 (D) 22 (E) 27

3. Find the arithmetic mean of the median, the mode and the range of these numbers: 5, 5, 20, 18, 4, 3, 60, 81, 2, & 55. (nearest whole number)

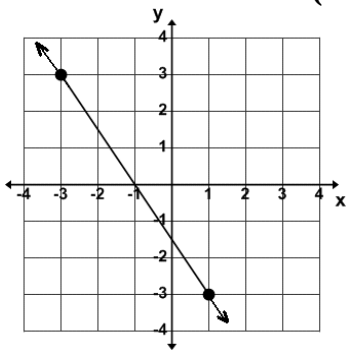
(A) 10 (B) 12 (C) 26 (D) 32 (E) 34

4.

The distances between the hash marks (|) are equal. Find $P + Q + R + S$.

(A) 0.75 (B) .5 (C) 0.25 (D) — 0.75 (E) — 1.5

5. A line perpendicular to the line shown at (— 1, 0) contains which of the following points.



(A) (— 13, — 8) (B) (— 15, — 11) (C) (— 14, — 7) (D) (13, 11) (E) (11, 7)

6. Which of the following set(s) of numbers is/are closed under multiplication but not subtraction?
I. Integers II. Naturals III. Primes IV. Rationals

(A) I only (B) II only (C) I, II & IV (D) II & IV (E) None of them

7. If $6x^2 - 7x + c = (ax - 5)(bx + 1)$, where a and b are integers then $a + b + c =$ _____.

(A) — 5 (B) — 1 (C) 0 (D) 5 (E) 10

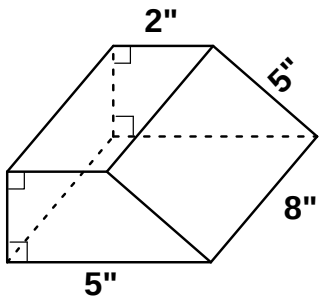
8. $\angle A$ and $\angle B$ are complementary angles with m $\angle A = 2x - 1$ and m $\angle B = 3x + 4$. $\angle C$ is supplementary to $\angle B$. Find m$\angle C$.

(A) 146.2° (B) 120.2° (C) 143.8° (D) 123.8° (E) 162.6°

9. **The distance from Austin, TX to Ft. Worth, TX by way of I-35 is 190 miles and the distance from Austin, TX to Laredo, TX by way of I-35 is 235 miles. Willie Kann leaves Ft. Worth on I-35 at 7:00 a.m. driving toward Laredo at an average speed of 55 mph due to construction and I-35 traffic. Betty Wheel leaves Laredo on I-35 at 8:00 a.m. driving toward Ft. Worth at an average speed of 70 mph. How far will Betty have driven when they meet? (nearest mile)**

(A) 277 miles (B) 218 miles (C) 163 miles (D) 248 miles (E) 207 miles

10. **Find the total surface area of the trapezoidal prism shown. (nearest sq. in)**



(A) 160 sq. in (B) 156 sq. in (C) 142 sq. in (D) 141 sq. in (E) 134 sq. in

11. **Rose Gardner built a square wooden deck with a side length of 18 ft. She cut out a circle in the center of the deck with a diameter of 10 ft. to put in a pool. What is the area of the deck that she will have to stain? (nearest sq. ft)**

(A) 292 sq. ft (B) 64 sq. ft (C) 10 sq. ft (D) 224 sq. ft (E) 245 sq. ft

12. **The graph of $x^2 + y^2 + 10x - 2y + 10 = 0$ is a circle with a center (h, k) and a radius r. Find $h + k - r$.**

(A) 0 (B) -5 (C) -8 (D) -4 (E) 10

13. **The set of Lucas numbers is {2,1,3,4,7,11, ...}, where $L_0 = 2$ and $L_1 = 1$. The set of Fibonacci numbers is {0,1,1,2,3,5, ...}, where $F_0 = 0$ and $F_1 = 1 = F_2$. $L_{15} = F_x + F_y$. $x + y$ = ?**

(A) 32 (B) 31 (C) 30 (D) 28 (E) 26

14. **An "*emirp*" number is a prime number that becomes a new prime number when the digits are reversed. Single digit primes and palindromic primes cannot be *emirp* numbers. The sum of the prime numbers less than 40 which are considered to be *emirp* numbers is ?**

(A) 80 (B) 117 (C) 98 (D) 61 (E) 85

15. **Speedy randomly selected a letter from the words NASCAR RACE. What is the probability he selected a consonant?**

(A) $33\frac{1}{3}\%$ (B) 40% (C) 50% (D) 60% (E) $66\frac{2}{3}\%$

16. If two dice are tossed, what are the odds that the sum of the top faces is 2, 3, 7, or 12? (nearest whole %)

(A) 28% **(B) 20%** **(C) 38%** **(D) 13%** **(E) 67%**

17. Polynomial equations with integer coefficients and only integer solutions are known as _____ equations.

(A) Archimedian **(B) Boolean** **(C) Mersenne** **(D) Mandelbrot** **(E) Diophantine**

18. Mr. White's 'bath tub mat' pattern table consists of 19 columns and 12 rows. Only 7 rows are shown. Determine the number in column 14 row 12.

1				1				2				3				5		
			2				3				5				8			
		3				5				8				13				21
	5				8				13				21				34	
8				13				21				34				55		
			21				34				55				89			
		34				55				89				144				233

(A) 1,595 **(B) 1,974** **(C) 2,584** **(D) 2,843** **(E) 3,571**

19. Find the sum of the x-values in $\left\{ x \mid \sin(2x) - \cos(x) = 0, x \in [\frac{\pi}{2}, 2\pi) \right\}$? (nearest tenth)

(A) 9.4 **(B) 4.1** **(C) 5.2** **(D) 4.8** **(E) 8.9**

20. Larry and Moe live on the river bank on one side of the river, and Curly lives on the river bank on the other side. The distance across the river is 80 yards. Curly rows his canoe to Larry's house on a bearing of 125°, then walks due north to Moe's house, then rows Moe's boat on a bearing of 210° back to his house. How far did Curly travel? (nearest yard)

(A) 313 yds **(B) 370 yds** **(C) 415 yds** **(D) 452 yds** **(E) 532 yds**

21. Which of the following functions are even functions? f(x) =
I. $x^2 - 1$ II. $x^5 - x^3 - x$ III. $x^2 - 2x - 1$

(A) I only **(B) II only** **(C) I & II but not III** **(D) all of them** **(E) none of them**

22. The 3rd term of a geometric sequence is $\frac{2}{27}$. The 4th term is $\frac{4}{81}$. Find the sum of the first 5 terms of this geometric sequence.

(A) $\frac{211}{486}$ **(B) $\frac{200}{243}$** **(C) $\frac{64}{81}$** **(D) $\frac{422}{2187}$** **(E) $\frac{227}{243}$**

23. Let f(x) = 3x — 1, g(x) = 2x + 4, and h(x) = 4x — 1. Find $g\left(f\left(h(-x)+1\right)-1\right)$

(A) — 24x **(B) — 24x — 8** **(C) — 24x — 4** **(D) — 24x + 4** **(E) — 24x + 8**

24. Find the digit in the ten-thousandth place of the series $\frac{3^1}{1!} - \frac{3^3}{3!} + \frac{3^5}{5!} - \frac{3^7}{7!} + \frac{3^9}{9!} - \ldots$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

25. How many non-negative proper fractions in lowest terms have a denominator of 42?

(A) 17 (B) 14 (C) 12 (D) 10 (E) 7

26. Find the digit in the units place of the integer 8^{2018}.

(A) 8 (B) 6 (C) 4 (D) 2 (E) cannot be determined

27. How many ordered pairs (A, B) exist such that 1,468,AB9 ÷ 9 has a remainder of 7?

(A) 4 (B) 6 (C) 7 (D) 11 (E) 15

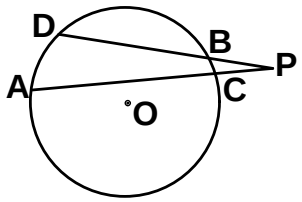
28. If the roots of $x^3 + bx^2 + cx + d = 0$ are -6, -3, and -1, then $b + c + d$ equals:

(A) -10 (B) 55 (C) 19 (D) 45 (E) -1

29. How many integers are in the solution set for $|3x + 4| - 7 \leq 11$?

(A) 7 (B) 10 (C) 11 (D) 12 (E) 18

30. Given the circle with center O shown with DB = 5 cm, BP = 2 cm, and CP = 1.5 cm. Find AP.



(A) 9.333... cm (B) 5.5 cm (C) 8.5 cm (D) 7.8333... (E) 13.333... cm

31. Let $A + B = 18$ and $A \times B = 32$. Find $A - B$, where $A < B$.

(A) -14 (B) -25 (C) 7 (D) 14 (E) 25

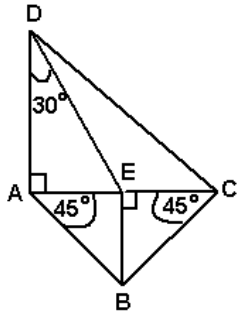
32. The roots of $x^3 + 4x^2 + x - 1 = 0$ are d, e, and f. Find $(d + e)(e + f)(f + d)$.

(A) -5 (B) -3 (C) -2 (D) -1 (E) 4

33. Let $f(x) = |2 - 3x| - |2x + 3|$. Find the minimum value of f(x).

(A) -4.5 (B) $-4.333...$ (C) -4.25 (D) -4.2 (E) $-4.111...$

34. Find the perimeter of the quadrilateral BCDE if BE = 3". (nearest inch)



(A) 21 in (B) 28 in (C) 18 in (D) 22 in (E) 24 in

35. Find P + Q if $\begin{bmatrix} 1 & P \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} Q \\ 5 \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \end{bmatrix}$

(A) — 4 (B) — 3 (C) 3 (D) 8 (E) 9

36. The polar equation $r = 2\csc(\theta)\cot(\theta)$ written in rectangular form is:

(A) $x^2 + y^2 = \sqrt{2}$ **(B)** $x + y = 2$ **(C)** $x^2 = \sqrt{2y}$ **(D)** $y^2 = 2x$ **(E)** $y = \sqrt{2x}$

37. Let $f(x) = (3x + 1)^2$. The tangent to f(x) at (a, b) is perpendicular to y = 2 + x. Find a + b.

(A) $-\frac{7}{18}$ **(B)** $-\frac{13}{36}$ **(C)** $-\frac{5}{12}$ **(D)** $-\frac{2}{9}$ **(E)** $-\frac{4}{9}$

38. Let $f(x) = x^3 - 5x^2 + 8x - 4$ and $g(x) = (2x + 1)^2$. Find $f'(g'(-1))$

(A) 0 (B) 172 (C) 96 (D) 180 (E) 140

39. If the probability of scoring 150 or more on this test is 55%, what are the odds of scoring less than 150 on this test?

(A) $\frac{9}{20}$ **(B)** $\frac{3}{10}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{11}{30}$ **(E)** $\frac{9}{11}$

40. Which of the sets of numbers does 55 belong to?

I. Unhappy II. Unlucky III. Evil

(A) II only (B) I & II only (C) I & III only (D) all of these (E) none of these

41. Let $f(x) = \sqrt{3 - \sqrt{5x + 7}}$ **. The domain of f(x) is** $\{x \mid p \le x \le q\}$**. Find** $\frac{P + Q}{2}$**.**

(A) 0.4 (B) 2 (C) — 1 (D) — 0.5 (E) — 1.4

42. Find the sum of the measure of an interior angle of a regular heptagon, the measure of a central angle of a regular octagon, and the measure of an exterior angle of a regular nonagon. (nearest degree)

(A) 236° (B) 226° (C) 214° (D) 202° (E) 136°

43. Point P (— 2, 5) lies on the x-y plane. P is reflected across the line y = — x to point Q. Point Q is rotated 90° counter clockwise about the origin to point R. Point R is translated vertically — 3 units to point S(x, y). Find $x + y$

(A) — 10 (B) — 7 (C) — 3 (D) 4 (E) 7

44. Phil Witwater has a tank that uses two pipes to input water and fill the tank. It has another pipe that is used to output water and drain the tank. One of the input pipes can fill the empty tank in 8 hours by itself with the output pipe shut off. The tank can be fully drained in 18 hours with the input pipes shut off. The tank can be filled with both input pipes open and the output pipe open in 6 hours. How long would the other input pipe take to fill the tank by itself with the output pipe shut off? (nearest min)

(A) 14 hrs 24 min (B) 4 hrs 30 min (C) 9 hrs (D) 5 hrs 32 min (E) 10 hrs 17 min

45. The attendance rate last year at the Millersview's *4-Act Play Theater* is modeled by the function $A(m) = 100 + 40\cos((\frac{\pi}{4})(m + 4))$, $0 \leq m \leq 12$, where m is in months and A(m) is in people per month. If the theater opened on Jan. 1, when did the theater's attendance rate first reach a maximum?

(A) 12 months (B) 8 months (C) 6 months (D) 4 months (E) 2 months

46. Lotta Koins is putting the coins from her bank in stacks of 5 coins each. She has pennies, nickels, dimes, quarters, and half dollars. How many different stacks of 5 coins can she make if each stack has to have at least one half dollar?

(A) 126 (B) 35 (C) 124 (D) 54 (E) 70

47. Ucandoette High School has three administrators, five teachers, seven female students and nine male students. A special council consisting of one administrator, two teachers, three female students and four male students is formed. How many different ways can the council be formed?

(A) 174 (B) 728 (C) 4,440 (D) 132,300 (E) 1,961,256

48. Willie When put two blue balls, three white balls, and five red balls in a bucket. He draws out one ball. If the ball is blue, he gets $3.00. If it is white he gets nothing. If it is red he has to pay $1.00. What is the mathematical expectation value if he does this many times? (nearest cent)

(A) 10¢ (B) 30¢ (C) 40¢ (D) $1.00 (E) $1.10

49. Find $f(-1) - f(2) + f(-3)$ if $f(x) = \begin{cases} 3x - 2 & \text{if } x < -2 \\ -2x + 1 & \text{if } -2 \le x \le 1 \\ -1 - 3x & \text{if } x > 1 \end{cases}$

(A) -17 **(B)** -1 **(C)** 1 **(D)** 7 **(E)** 12

50. How many distinct triangles can be made using three sticks at a time from six sticks measuring 12 inches, 10 inches, 8 inches, 6 inches, 6 inches, and 5 inches?

(A) 20 **(B)** 16 **(C)** 14 **(D)** 12 **(E)** 10

51. Let function f = {(9, 6), (3, 9), (12, 5), (6, 7)} and function g = {(5, 6), (9, 12), (7, 4), (1,9)}. Find $(f \circ g)(9) + (g \circ f)(6)$?

(A) 5 **(B)** 9 **(C)** 11 **(D)** 15 **(E)** 16

52. Which of the following do not have an inverse function without restricting the domain?

I. $y = x^2 + 4$ **II.** $y = 2\cos(x) + 4$ **III.** $y = e^{(\sin(x))}$ **IV.** $y = 2^x$

(A) I only **(B)** III only **(C)** I, II, III but not IV **(D)** II & IV **(E)** I, II, III, & IV

53. Which of the following equations can be obtained from the graph of the function $y = 2 + \sin(x - 3)$ by applying a vertical stretch of 4 units, a vertical shift down 5 units, and a phase shift left 6 units ? y = :

(A) $\sin(4x + 6) - 5$ **(B)** $4\sin(x - 8) + 8$ **(C)** $4\cos(x - 8) - 3$
(D) $4\sin(x + 3) - 3$ **(E)** $2\sin(x + 3) - 5$

54. Which of the follow ordered pairs (P, Q) of numbers would make the graph of the polar equation $r = P + Q\cos(\theta)$ a dimpled limacon.

(A) (2, 3) **(B)** (5, 3) **(C)** (4, 4) **(D)** (1, 3) **(E)** (3, 1)

55. Integers P, Q, and R are the roots of $x^3 + Bx^2 + Cx + D = 0$. The harmonic mean of P, Q, and R is 2 and C is 27. Find B.

(A) 18 **(B)** 8 **(C)** 6 **(D)** -12 **(E)** -10

56. The function $f(x) = \begin{cases} nx^3 + 2x & \text{if } x \le 1 \\ mx^2 - x & \text{if } 1 < x \end{cases}$ is continuous and differentiable everywhere. Find m.

(A) 1.2 **(B)** 2 **(C)** 2.4 **(D)** 6 **(E)** 10

57. The area (in square units) of the region bounded by $y = x^2$, $y = x + 6$, $x = 0$ and $x = 5$ is:

(A) $23\frac{2}{3}$ **(B)** $24\frac{5}{6}$ **(C)** $24\frac{1}{3}$ **(D)** $25\frac{5}{6}$ **(E)** $26\frac{1}{6}$

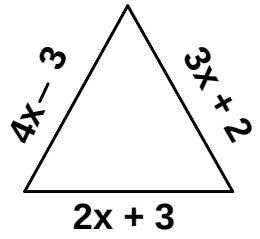
58. What is the instantaneous rate of change at x = 2 of the function f given by $f(x) = \frac{x^2 - 5}{x - 3}$

(A) — 5 **(B) — 3** **(C) 0.1.666...** **(D) 2** **(E) 3.5**

59. If $(3212_b) \div 11_b = 232_b$, then $2123_b \div 11_b = N_b$, where N_b is a 3-digit number. Find the sum of the 3 digits.

(A) 4 **(B) 5** **(C) 7** **(D) 8** **(E) 9**

60. Find the least possible perimeter of this triangle given that it is isosceles but not equilateral.



(A) 47 **(B) 43** **(C) 35** **(D) 29** **(E) 11**

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University Interscholastic League
MATHEMATICS CONTEST
HS • State • 2018
Answer Key

1. B	**21. A**	**41. D**
2. E	**22. A**	**42. C**
3. D	**23. A**	**43. A**
4. A	**24. B**	**44. E**
5. A	**25. C**	**45. D**
6. B	**26. C**	**46. E**
7. C	**27. D**	**47. D**
8. D	**28. B**	**48. A**
9. E	**29. D**	**49. B**
10. B	**30. A**	**50. D**
11. E	**31. A**	**51. B**
12. C	**32. A**	**52. C**
13. C	**33. B**	**53. D**
14. C	**34. A**	**54. B**
15. D	**35. A**	**55. E**
16. C	**36. D**	**56. D**
17. E	**37. B**	**57. E**
18. C	**38. C**	**58. B**
19. E	**39. E**	**59. C**
20. D	**40. B**	**60. E**