UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational A • 2023

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1. On his birthday, Darius received a $250 gift card from Academy Sports in Groom. He purchased Pegasus running shoes for $128.00, a Nike running shirt for $24.95, Nike running shorts for $33.85, and Nike socks for $6.95. If the tax rate is 8.25%, how much is left on his gift card?

   (A) $40.15   (B) $40.19   (C) $40.23   (D) $40.27   (E) $40.31

2. Consider line $\overrightarrow{AB}$ with points $A(8, 6)$ and $B(-6, -10)$. If the point $(1, b)$ lies on $\overrightarrow{AB}$, then b = ___.

   (A) -2 (B) $-\dfrac{12}{7}$ (C) $-\dfrac{10}{7}$ (D) $-\dfrac{8}{7}$ (E) $-\dfrac{6}{7}$

3. Chay has a jar full of nickels, dimes and quarters. The jar has a total of 264 coins with a value of $37.20. There are 20 more quarters than nickels in the jar. How many dimes are in the jar?

   (A) 83   (B) 88   (C) 93   (D) 98   (E) 103

4. Which of the following values is not in the solution to $3 \le x + 7 < 12$?

   (A) -4 (B) -1 (C) 1   (D) 4   (E) 5

5. Samuel is solving the quadratic equation $x^2 + 10x - 6 = 0$ by completing the square. His second step is $x^2 + 10x + c = 6 + c$. The value of c is _____.

   (A) 5   (B) 6   (C) 10   (D) 12   (E) 25

6. The perimeter of $\triangle ABC$ shown on the right is ____. (nearest tenth)

   (A) 42.6   (B) 42.9   (C) 43.2   (D) 43.5   (E) 43.8

7. The coordinates of the midpoint of $\overline{BC}$ are (a, b).
   $a + b =$ _____. (nearest tenth)

   (A) 2.0   (B) 2.2   (C) 2.4   (D) 2.6   (E) 2.8

8. The area of $\triangle ABC$ is _____. (nearest tenth)

   (A) 78.8   (B) 79.2   (C) 79.6   (D) 80.0   (E) 80.4

9. Point D (not shown) lies on $\overline{AC}$ such that $\overrightarrow{BD}$ bisects $\angle ABC$. DC = _____. (nearest tenth)

   (A) 6.6   (B) 6.8   (C) 7.0   (D) 7.2   (E) 7.4
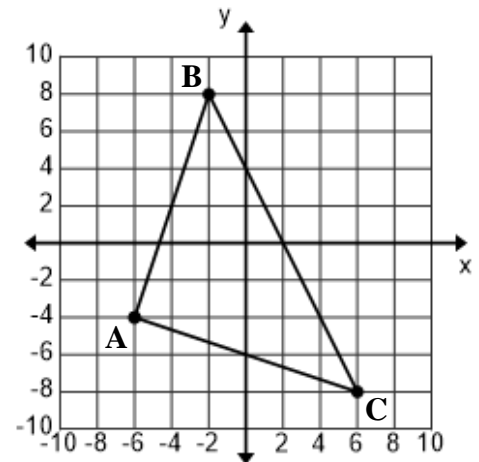
Problems 6, 7, 8, 9

10. Four times the complement of $\angle A$ is $36°$ greater than the supplement of $\angle A$. $m\angle A =$ _____.

   (A) $44°$ (B) $46°$ (C) $48°$ (D) $50°$ (E) $52°$

Problems 11-12. The base of a pyramid is a square with each side equal to 14 cm. The height is 10 cm.

11. The volume of the pyramid is _____. (nearest whole number)

   (A) 650 cm^3 (B) 653 cm^3 (C) 656 cm^3 (D) 659 cm^3 (E) 662 cm^3

12. The total surface area of the pyramid is _____. (nearest whole number)

   (A) 522 cm^2 (B) 526 cm^2 (C) 530 cm^2 (D) 534 cm^2 (E) 538 cm^2

13. The volume of a cone is $768\pi \text{ cm}^3$ and the height of the cone is 12 cm. What is the diameter of the circular base? (nearest whole number)

   (A) 26.9 cm   (B) 27.3 cm   (C) 27.7 cm   (D) 28.1 cm   (E) 28.5 cm

14. The hypotenuse of an isosceles right triangle is 24.0416. The area of the triangle is _____. (nearest tenth)

   (A) 144.5   (B) 145.6   (C) 146.7   (D) 147.8   (E) 148.9

15. Jen can mow 3 large yards in 8 hr. Tom can mow 5 large yards in 16 hr. If they work together, how long would it take them to mow 12 large yards? (nearest minute)

   (A) 16 hr 35 min   (B) 16 hr 48 min   (C) 17 hr 1 min   (D) 17 hr 14 min   (E) 17 hr 27 min

16. Find the number that is $\dfrac{5}{12}$ of the way from $-3\dfrac{1}{3}$ to $8\dfrac{1}{2}$.

   (A) $\dfrac{37}{24}$ (B) $\dfrac{113}{72}$ (C) $\dfrac{115}{72}$ (D) $\dfrac{13}{8}$ (E) $\dfrac{119}{72}$

17. If $f(x) = x^2$, then $\dfrac{f(x+h)-f(x)}{h} =$ _____.

   (A) $2x$ (B) $2x+h$ (C) $\dfrac{2x}{h}$ (D) x^2+h (E) $2x+2h$

18. Dad's age is two more than three times Abe's age and Dad's age is eight more than twice Connie's age. Connie is six years older than Abe. What is the sum of their ages?

   (A) 90   (B) 92   (C) 94   (D) 96   (E) 98

19. Find the total number of distinct diagonals that can be drawn from the vertices of a regular decagon?

(A) 24   (B) 32   (C) 35   (D) 36   (E) 42

20. Consider $\triangle ABC$ with point D on $\overline{AC}$ such that $\overline{BD} \perp \overline{AC}$. If $m\angle ABC = 90°$, $AD = 10.8$ and $DC = 19.2$, then $BD =$ _____. (nearest tenth)

(A) 14.0   (B) 14.2   (C) 14.4   (D) 14.6   (E) 14.8

21. Find the domain of the function $f(x) = \dfrac{\sqrt{x^2 - 25}}{|x - 6|}$.

(A) $(-\infty, -5] \cup [5, \infty)$ (B) $(-\infty, -5) \cup (5, 6) \cup (6, \infty)$ (C) $(-\infty, -5) \cup (5, \infty)$

(D) $(-\infty, \infty)$ (E) $(-\infty, -5] \cup [5, 6) \cup (6, \infty)$

22. How many distinguishable permutations can be formed using the letters from the word Massachusetts?

(A) 64,864,800   (B) 129,729,600   (C) 259,459,200   (D) 518,918,400   (E) 1,037,836,800

23. Consider the circle $x^2 + y^2 - 8x + 10y - 8 = 0$. The area of the circle is _____. (nearest whole number)

(A) 154   (B) 156   (C) 158   (D) 160   (E) 162

24. $\sin(x) \tan\left(\dfrac{\pi}{2} - x\right) =$ _____.

(A) $\cos(x)$ (B) $\sin(2x)$ (C) $\tan(x)$ (D) $-\tan(x)$ (E) $-\cos(x)$

25. Consider a geometric sequence in which the first term is $22\dfrac{1}{2}$ and the fifth term is $4\dfrac{4}{9}$. What is the seventh term of the sequence?

(A) $\dfrac{52}{27}$ (B) $\dfrac{158}{81}$ (C) $\dfrac{160}{81}$ (D) 2   (E) $\dfrac{164}{81}$

26. Find the eccentricity of the ellipse. $16x^2 + 25y^2 - 128x + 150y + 81 = 0$. (nearest hundredth)

(A) 0.60   (B) 0.67   (C) 0.75   (D) 0.80   (E) 0.83

27. The pressure of an ideal gas varies directly with the temperature and inversely with the volume. The values of the initial state were 2.0 atm, 6.0 L, and 300 K. What will the pressure be in the final state if the volume is reduced to 2.0 L and the temperature is increased to 500 K? (nearest tenth)

 (A) 9.8 atm (B) 10.0 atm (C) 10.2 atm (D) 10.4 atm (E) 10.6 atm

28. The value of my RAV4 Prime (plug-in hybrid) depreciates exponentially. I originally paid $55,200 on July 8, 2022. On July 8, 2025, the value had fallen to $51,500. If the value continues to depreciate exponentially, what is the expected value of my RAV4 Prime on July 8, 2030?

 (A) $45,416.16 (B) $45,531.17 (C) $45,646.18 (D) $45,761.19 (E) $45,876.20

29. Consider the baseball diamond at SHS. Home plate and the three bases are located at the vertices of a square with the length of each side of the square being 90 ft. The pitcher's mound lies on a straight line from home plate to second base and it is located 60 ft 6 in from home plate. How far is it from the pitcher's mound to first base? (nearest tenth)

 (A) 63.7 ft (B) 63.9 ft (C) 64.1 ft (D) 64.3 ft (E) 64.5 ft

30. At the Moulton Fall Festival, cash prizes were awarded for the top twenty places in the peach pie contest. First place received $500, second place received $475, third place received $450, fourth placed received $425 and so on. What was the total amount of prize money awarded?

 (A) $5200 (B) $5225 (C) $5250 (D) $5275 (E) $5300

31. Six couples plan to attend a concert and sit in the same row. Each row has 12 seats. If the two members of each couple are to sit together, how many different seating arrangements are possible?

 (A) 24,060 (B) 36,060 (C) 42,060 (D) 46,080 (E) 48,040

32. The point $(-7, b)$ lies of the curve defined by the parametric equations $\begin{array}{l} x = t - 5 \\ y = t^2 \end{array}$. $b =$ _____.

 (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

33. Convert the rectangular equation $2x - 3y - 5 = 0$ to polar form.

 (A) $r = \dfrac{5}{\cos^2 \theta - \sin^3 \theta}$ (B) $r = \dfrac{5}{2\cos \theta - 3\sin \theta}$ (C) $r = \dfrac{5}{2\sin \theta - 3\cos \theta}$

 (D) $r = 5\sec(\theta)$ (E) $r = 5\csc(\theta)$

34. Find the perimeter of the triangle with vertices $(4, -5, 6)$, $(3, -2, -1)$ and $(-5, 7, -9)$. (nearest tenth)

 (A) 43.4 (B) 43.6 (C) 43.8 (D) 44.0 (E) 44.2

35. Three of the zeros of $f(x) = x^4 + bx^3 + cx^2 + dx + 12 = 0$ are $-2, 3, 1+\sqrt{3}$. $f(2) = $ _____.

(A) –8   (B) –4   (C) 0   (D) 4   (E) 8

36. Consider the graph of $f(x) = 3 - 2\cot\left(\dfrac{4\pi x}{3} + \dfrac{\pi}{6}\right)$. The period of the graph is _____.

(A) $\dfrac{3}{4}$ (B) $\dfrac{4}{3}$ (C) $\dfrac{3}{2}$ (D) 2   (E) 3

37. Assume that the earth rotates about its axis once every 23 hr 56 min 3.72 sec. Find the linear speed of a person sitting in a chair on the equator. The radius of the earth is 3960 miles. (nearest tenth)

(A) 1037.4 mph   (B) 1038.5 mph   (C) 1039.6 mph   (D) 1040.7 mph   (E) 1041.8 mph

38. The graph of $r = 12\cos(8\theta)$ has _____ rose petals.

(A) 6   (B) 8   (C) 12   (D) 16   (E) 24

39. On January 1, 2020, Lily placed \$15,000 into account A where it earns 7% annual interest compounded semiannually. On the same day, she also placed \$14,500 into account B that earns 7.5% annual interest compounded monthly. Of the following choices, which is the earliest day in which the balance in account B exceeds the balance in account A?

(A) July 10, 2025   (B) Sept 10, 2025   (C) Nov 10, 2025   (D) Jan 10, 2026   (E) March 10, 2026

40. One of the foci of the hyperbola shown on the right has coordinates $(a, 0)$.
 a = _____. (nearest tenth)

(A) 6.5
(B) 6.7
(C) 6.9
(D) 7.1
(E) 7.3

41. Find the area bounded by the graph of the right branch of the hyperbola and the line x = 10. (nearest tenth)

(A) 20.2
(B) 20.4
(C) 20.6
(D) 20.8
(E) 21.0

Problems 40, 41
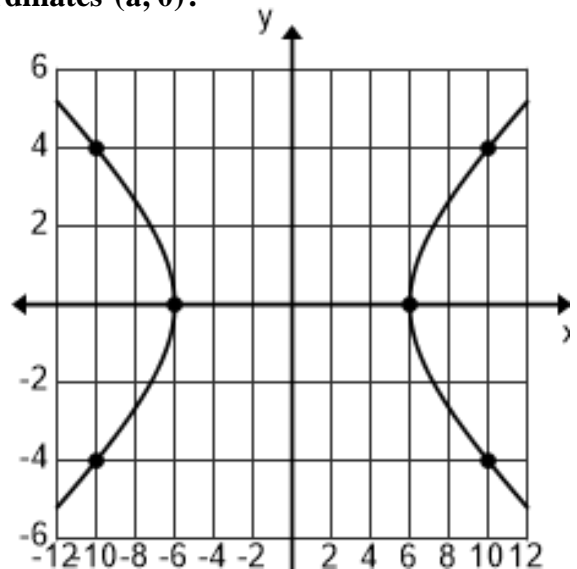
42. A 50-foot ladder is leaning against a building. If the base of the ladder is being pulled away from the building at a rate of 4 feet per second, then the top of the ladder will move down the wall at a rate of _____ feet per second at the moment the base of the ladder is 14 feet from the base of the building.

(A) $\dfrac{7}{6}$ (B) $\dfrac{5}{4}$ (C) $\dfrac{4}{3}$ (D) $\dfrac{3}{2}$ (E) $\dfrac{5}{3}$

43. Consider the graph of $f(x) = 3\sin(x) + 2\cos(2x)$. How many values of x exist in the interval $(6, 10)$ such that there is a horizontal tangent at x?

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5

44. Find the average value of $f(x) = 2\sin(x) + 5\cos(x)$ on the interval $[6, 8]$. (nearest tenth)

(A) 4.3   (B) 4.5   (C) 4.7   (D) 4.9   (E) 5.1

45-46. Let $y = f(x)$ be the solution to the differential equation $\dfrac{dy}{dx} = \dfrac{x}{2y}$ with the initial condition $f(4) = 3$.

45. Use Euler's method to approximate $y(5)$ using two steps of equal size starting at $x = 4$. (nearest ten-thousandth)

(A) 3.6648   (B) 3.6668   (C) 3.6688   (D) 3.6708   (E) 3.6728

46. Find the exact value of y(5). (nearest thousandth)

(A) 3.6702   (B) 3.6722   (C) 3.6742   (D) 3.6762   (E) 3.6782

47-48. Consider the region bounded by the graphs of $y_1 = .5x^2$ and $y_2 = 4 - x$.

47. Find the area of the specified region. (nearest tenth)

(A) 18.0   (B) 18.4   (C) 18.8   (D) 19.2   (E) 19.6

48. Find the volume of the solid generated by revolving the specified region about the x-axis. (nearest whole number)

(A) 350   (B) 353   (C) 356   (D) 359   (E) 362

49. If $P_3(x)$ is the third degree Maclaurin polynomial for $f(x) = e^x$, then $f(.5) - P_3(.5) =$ _____. (nearest ten-thousandth)

(A) 0.0021   (B) 0.0023   (C) 0.0025   (D) 0.0027   (E) 0.0029

50. The position of a particle is given by the parametric equations $x(t) = 3e^{4t}$ and $y(t) = \ln(t^3 + 3t - 2)$. Find the velocity vector when $t = 3$.

(A) $\left\langle 12e^{12}, \dfrac{30}{39} \right\rangle$ (B) $\left\langle 12e^{6}, \dfrac{15}{17} \right\rangle$ (C) $\left\langle 12e^{12}, \dfrac{30}{37} \right\rangle$ (D) $\left\langle 12e^{4}, \dfrac{15}{17} \right\rangle$ (E) $\left\langle 12e^{12}, \dfrac{15}{17} \right\rangle$
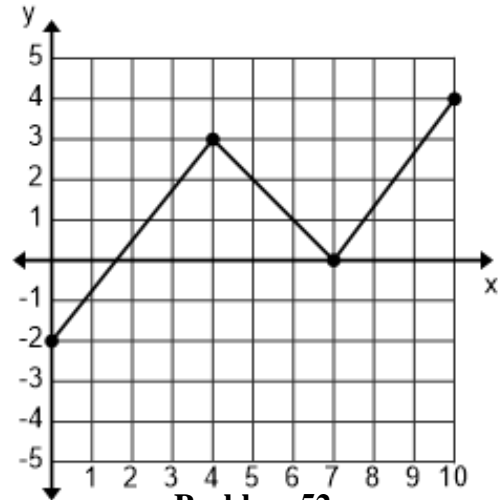
51. The second derivative of a function h(x) is given by $h''(x) = x^2(x-2)^3(x-8)^5$. Of the following values of x, which of these does the graph of h(x) have a point of inflection? $\{0, 2, 8\}$

(A) 2 only   (B) 2, 8 only   (C) 0, 8 only   (D) 0, 2 only   (E) 0, 2, 8

52. The graph of the piecewise function f(x) is shown on the right. Find the value of $\int_{0}^{10} f'(x)dx$.

(nearest hundredth)

(A) 6.00
(B) 7.75
(C) 9.50
(D) 11.25
(E) 13.00

Problem 52

Use the table below and the information below the table for problems 53 and 54.

	1	2	3	4	5	6	7	8	9
Score	166	172	154	205	221	198	144	172	188

Phoenix entered the Irion County Bowling Marathon last week. Each contestant is required to bowl nine games in six hours. The table above show the results.

53. What is the difference in the mean score and the median score, a positive number?

(A) 2   (B) 4   (C) 6   (D) 8   (E) 10

54. Find the interquartile range of the scores.

(A) 6   (B) 23.75   (C) 41.5   (D) 59.25   (E) 77

55. Alessandra flipped a fair coin 6 times and it came up heads all 6 times. She decided to flip the coin one more time. What is the probability that the seventh flip will produce a tails? (nearest ten-thousandth)

(A) 0.0078   (B) 0.1309   (C) 0.2500   (D) 0.3772   (E) 0.5000

Use the table below and the information below the table for problems 56-58.

M.S. Time	12.25	11.88	11.36	10.75	10.33	9.91
H.S. Score	139	148	153	164	171	178

Christopher decided that he would warm up for his number sense competitions in 2023 by taking a middle school number sense test 30 minutes before the high school competition started. The table above shows the time, in minutes, it took him to complete the middle school warm up tests and his scores at the six high school meets he has attended so far this year.

56. Christopher's coach plotted the data and observed a strong, negative, linear relationship. His coach analyzed the data and his statistical software generated a LSRL with a correlation of $r =$ _____. (nearest ten-thousandth)

    (A) -0.9967   (B) -0.9922   (C) -0.9885   (D) -0.9843   (E) -0.9812

57. Use the data in the table above to predict his score at the next competition if he took 9.66 minutes to complete a middle school warm up test. (nearest whole number)

    (A) 180   (B) 182   (C) 184   (D) 186   (E) 188

58. Find the value of the residual for the data point $(11.88, 148)$. (nearest tenth)

    (A) 1.5   (B) 1.7   (C) 1.9   (D) 2.1   (E) 2.3

59-60. A researcher is testing the claim that 62% of Americans believe that the economy is the most important issue in America today. In a survey involving a simple random sample of 1200 Americans, 782 responded by placing the economy as their top concern. The researcher performed an appropriate test with $H_0 : p = 0.62$ and, $H_a : p \neq 0.62$. He used an $\alpha = 0.05$ level as his standard.

59. What is the value of the test statistic? (nearest hundredth)

    (A) 2.02   (B) 2.14   (C) 2.26   (D) 2.38   (E) 2.50

60. What was the conclusion of the researcher after studying the test results?

    (A) Based on a p-value of 0.06, he failed to reject H_0.
    (B) Based on a p-value of 0.06, he rejected H_0.
    (C) Based on a p-value of 0.02, he failed to reject H_0.
    (D) Based on a p-value of 0.02, he rejected H_0.
    (E) Based on a p-value of 0.01, he failed to reject H_0.

University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitational A ● 2023
Answer Key

1. D	21. E	41. A
2. A	22. A	42. A
3. B	23. A	43. C
4. E	24. A	44. A
5. E	25. C	45. D
6. C	26. A	46. C
7. A	27. B	47. A
8. D	28. E	48. E
9. E	29. A	49. E
10. C	30. C	50. E
11. B	31. D	51. B
12. E	32. C	52. A
13. C	33. B	53. D
14. A	34. A	54. C
15. E	35. E	55. E
16. C	36. A	56. A
17. B	37. C	57. B
18. E	38. D	58. D
19. C	39. B	59. C
20. C	40. B	60. D