UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational B • 2023

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1. **The equation for kinetic energy is $K = \frac{1}{2}mv^2$, where K is kinetic energy in joules, m is mass in kilograms, and v is velocity in meters/second. Find the mass of an object with a kinetic energy of 184 joules and a velocity of 18.8 meters/second. (nearest hundredth)**

   (A) 1.04 kg **(B) 1.25 kg** **(C) 1.46 kg** **(D) 1.67 kg** **(E) 1.88 kg**

2. **Consider a line, L_1, that contains the points $A(3, 8)$ and $B(-4, -6)$. If the point $C(a, 14)$ lies on L_1, what is the value of a?**

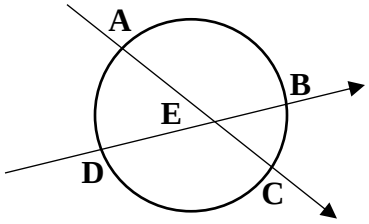
   (A) 4 **(B) 5** **(C) 6** **(D) 7** **(E) 8**

3. **Consider three consecutive prime numbers such that the sum of the numbers is 143. Find the product of the three numbers.**

   (A) 103,071 **(B) 107,113** **(C) 111,155** **(D) 115,197** **(E) 119,239**

4. **The measure of minor arc AB is $128°$ and the measure of minor arc CD is $112°$. Find the measure of $\angle BEC$.**

   (A) 54° **(B) 56°** **(C) 58°** **(D) 60°** **(E) 62°**



Problem 4

5. **At Pizza Heaven in Rankin, a large pizza cost $12.00 plus $1.25 per topping. A coke cost $2.25. The Tubbs family ordered a large pizza with 3 toppings and 4 cokes. The Wyatt family ordered a large Pizza with 4 toppings and 5 cokes. Mr. Wyatt also ordered a salad. If the total cost for both families was $62.37 after an 8.00% tax was added, what was the cost of the salad?**

   (A) $3.75 **(B) $4.00** **(C) $4.25** **(D) $4.50** **(E) $4.75**

6. **Mr. Clanton plans to hire Alexis, Skylee and Arthur to paint the 40 classrooms at the new elementary school during the summer. Alexis can paint a classroom in 8 hours, Skylee can paint a classroom in 9 hours and Arthur can paint a classroom in 10 hours. If they work together, how many hours are required to paint the 40 classrooms? (nearest whole number)**

   (A) 115 hr **(B) 117 hr** **(C) 119 hr** **(D) 121 hr** **(E) 123 hr**

7. **My final grade is determined by four tests and the final exam, which counts twice. I scored 86, 82, 89 and 95 on my tests. If I need to have an average of 89.5 or higher to make an A, what is the minimum score that I need to make on the final exam in order to make an A?**

   (A) 91 **(B) 92** **(C) 93** **(D) 94** **(E) 95**

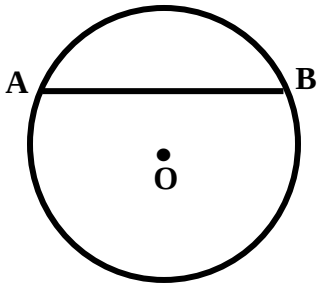
8. **Siep is driving his 2022 Dodge Ram 3500 Pickup on I-10 at a speed of 75 mph. If each tire has a radius of 16 inches, what is the rotational speed of the tires? (nearest whole number)**

   (A) 764 rpm **(B) 770 rpm** **(C) 776 rpm** **(D) 782 rpm** **(E) 788 rpm**

9. An equilateral triangle has a side length of 14. Find the circumference of a circle that has the same area as the triangle. (nearest tenth)

(A) 32.4 **(B) 32.7** **(C) 33.0** **(D) 33.3** **(E) 33.6**

10-11. Consider the circle with center O shown on the right. The length of chord $\overline{AB}$ is 24. The area of the circle is 530.929.



Problems 10, 11

10. Find the measure of minor arc AB. (nearest tenth)

(A) 131.5
(B) 132.6
(C) 133.7
(D) 134.8
(E) 135.9

11. Find the area bounded by minor arc AB and chord $\overline{AB}$. (nearest tenth)

(A) 138.7 **(B) 139.1** **(C) 139.5** **(D) 139.9** **(E) 140.3**

12. If $\Delta ABC \approx \Delta DEF$, $AB = 10$, $AC = 8$ and $DE = 7$, then DF = _____. (nearest tenth)

(A) 5.6 **(B) 5.8** **(C) 6.0** **(D) 6.2** **(E) 6.4**

13. Consider the Fibonacci type series $12+9+21+30+51+...+558+903$. The sum of the series is ____.

(A) 2339 **(B) 2343** **(C) 2347** **(D) 2351** **(E) 2355**

14. Consider an arithmetic sequence whose 4^{th} term is 33 and 9^{th} term is 73. Find the sum of the first 13 terms of the sequence.

(A) 738 **(B) 741** **(C) 744** **(D) 747** **(E) 750**

15-16. Consider the geometric sequence $48, a, b, c, 23\frac{4}{27}, d, ...$ with $a > 0$.

15. b = _____.

(A) $32.\overline{3}$ **(B) $32.\overline{6}$** **(C) 33** **(D) $33.\overline{3}$** **(E) $33.\overline{6}$**

16. The sum of the first 8 terms of the sequence is _____. (nearest whole number)

(A) 209 **(B) 213** **(C) 217** **(D) 221** **(E) 225**

17. A right circular cone has a diameter of 12 and a slant height of 10. Find the volume of the cone. (nearest tenth)

(A) 300.8 **(B) 301.2** **(C) 301.6** **(D) 302.0** **(E) 302.4**

18. Madison plans to take 4 days to drive from Rankin, TX to Bellingham, WA, a distance of 1964 miles. Madison drove 485 miles at an average speed of 59 mph on Monday, 611 miles at an average speed of 67 mph on Tuesday, and 447 miles at an average speed of 62 mph on Wednesday. What average speed must Madison travel at on Thursday in order to have an overall average speed of 63 mph for the entire trip? (nearest tenth)

(A) 62.9 mph (B) 63.2 mph (C) 63.5 mph (D) 63.8 mph (E) 64.1 mph

19. Coach Perkins has 3 posts, 8 wings and 4 guards on his team. His starting lineup must consist of one post, two wings and two guards. How many starting lineups are possible?

(A) 96 (B) 232 (C) 368 (D) 504 (E) 640

20. If $f(x) = 17x^{\left(\frac{3}{2}\right)}$ and $h(x) = \sqrt[3]{x-22}$, then $f\left(h(86)\right) =$ _____. (nearest whole number)

(A) 136 (B) 138 (C) 140 (D) 142 (E) 144

21. Find the remainder when $x^4 - 3$ is divided by $x^2 - 1$.

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

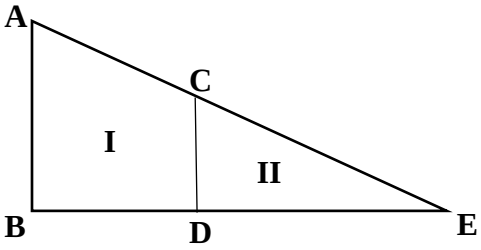
22. When the vector $v = -18i + 5j$ is converted to polar coordinates, one correct answer, rounded to the nearest tenth is $v = \langle 18.7, \theta° \rangle$. θ could be _____°

(A) -254.5 (B) -195.5 (C) 74.5 (D) 166.5 (E) 195.5

23-24. $\overline{AB}$ is parallel to $\overline{CD}$ and $\overline{CD} \perp \overline{BE}$.
$AB = 8.00$, $CD = 4.75$, $CE = 10.50$

23. AC = _____. (nearest hundredth)

(A) 6.94 (B) 7.06 (C) 7.18
(D) 7.30 (E) 7.42



Problems 23, 24

24. Find the area of region I. (nearest hundredth)

(A) 36.40 (B) 37.51 (C) 38.62 (D) 39.73 (E) 40.84

25. On Rihot's farm, the number of cows varies inversely as the number of pigs and directly as the number of chickens squared. When there was 400 cows, he had 20 chickens and 40 pigs. How many cows will he have when there are only 4 pigs and 16 chickens?

(A) 256 (B) 832 (C) 1408 (D) 1984 (E) 2560

26. If $f(x) = 6x - 1$ and $h(x) = x^2 + 8$, then $(h \circ f)(2) =$ _____.

(A) 127 (B) 129 (C) 131 (D) 133 (E) 135

27. The roots of the equation $12x^2 + bx + c = 0$ are $-\frac{1}{3}$ and $\frac{3}{4}$. $b + c =$ _____.

(A) –8 **(B)** –6 **(C)** –4 **(D)** –2 **(E)** 2

28. The six members of the Elkins math team were seated in a circle around a large circular table along with their coach in yesterday's math practice. If there were seven seats in the room, how many seating arrangements were possible?

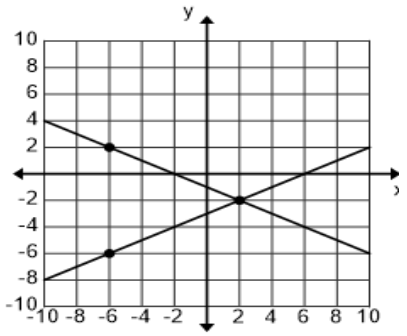
(A) 24 **(B)** 120 **(C)** 720 **(D)** 5040 **(E)** 40320

29. $x^2 + y^2 - 8x - 12y - 36 = 0$ is the equation of a circle with an area of _____. (nearest hundredth)

(A) 276.46 **(B)** 278.56 **(C)** 280.66 **(D)** 282.76 **(E)** 284.86

30. Find the acute angle formed by the two intersecting lines shown on the right. (nearest hundredth)

(A) 53.13° **(B)** 54.24° **(C)** 55.35° **(D)** 56.46° **(E)** 57.57°



Problems 30, 31

31. The asymptotes of a hyperbola are graphed on the right. If the equation of the hyperbola centered at (h, k) is $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{4} = 1$, and one of the vertices is the point (m, n) then $m + n =$ _____. (nearest tenth)

(A) –4.4 **(B)** –4.2 **(C)** –4.0 **(D)** –3.8 **(E)** –3.6

32. The perimeter of a regular octagon is 32. What is the area of the octagon? (nearest whole number)

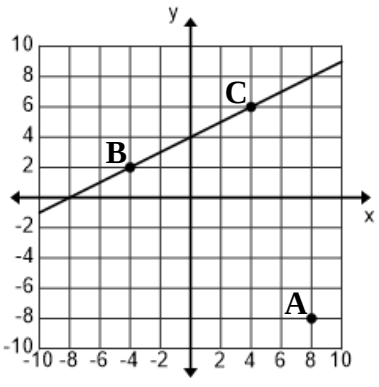
(A) 75 **(B)** 77 **(C)** 79 **(D)** 81 **(E)** 83

33. Find the distance from point A to the line $\overleftrightarrow{BC}$ shown on the right. (nearest tenth)

(A) 13.5 **(B)** 13.7 **(C)** 13.9 **(D)** 14.1 **(E)** 14.3

34. Draw auxiliary lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$. Find the perimeter of ΔABC. (nearest tenth)

(A) 38.8 **(B)** 39.1 **(C)** 39.4 **(D)** 39.7 **(E)** 40.0



Problems 33, 34, 35

35. Find the area of ΔABC. (nearest whole number)

(A) 64 **(B)** 66 **(C)** 68 **(D)** 70 **(E)** 71

36. At 6:00 AM, the hour hand and the minute hand of my clock point in opposite directions. How long will it be until the hands point in the same direction? (nearest tenth)

(A) 32.1 min (B) 32.3 min (C) 32.5 min (D) 32.7 min (E) 32.9 min

37. Allison Engineering has three open positions. Twelve A&M grads apply and ten UT grads apply. If the first position must be filled by an A&M grad, then how many different ways can these positions be filled?

(A) 840 (B) 2940 (C) 5040 (D) 7140 (E) 9240

38. Ivan wants to find a set of parametric equations to represent the graph of $y = -4x^2 + 6$.

If $x = -t + 3$, and $y = at^2 + bt + c$, then $a + b + c =$ _____.

(A) -14 (B) -12 (C) -10 (D) -8 (E) -6

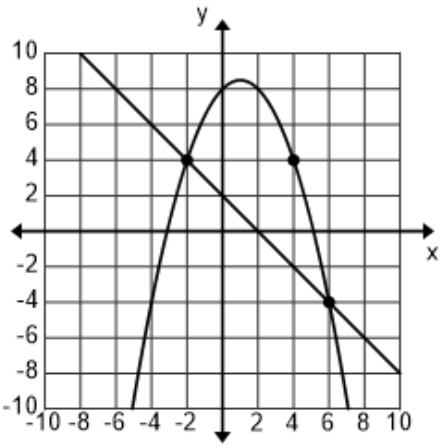
39. Find the area of the quadrilateral with vertices A(6, 4, −2), B(−4, 4, −6), C(−4, 10, −8) and D(6, 10, -4).

(nearest tenth)

(A) 61.3 (B) 63.4 (C) 65.5 (D) 67.6 (E) 69.7

40. If the parabola shown on the right intersects the line at points A and B, then AB = _____. (nearest tenth)

(A) 11.3
(B) 11.5
(C) 11.7
(D) 11.9
(E) 12.1



Problems 40, 41

41. The area bounded by the graphs of the parabola and the line shown on the right equals _____. (nearest tenth)

(A) 41.5 (B) 41.8 (C) 42.1 (D) 42.4 (E) 42.7

42. Consider the graph of $f(x) = \dfrac{x^2 + 3x - 28}{2x^2 + 9x - 35}$. Which of the following are true?

I. $f(x)$ is not continuous at $x = -7$ and $x = 4$.
II. The vertical lines $x = -7$ and $x = 2.5$ are vertical asymptotes.
III. The horizontal line $y = 0.5$ is a horizontal asymptote.

(A) I only (B) III only (C) I, II only (D) I, III only (E) I, II, III

43. If $y = \sec(x)$, then $\frac{d^2y}{dx^2} =$ _____.

(A) $\sec(x)\tan^2(x)+\sec^3(x)$ **(B)** $\sec(x)\tan(x)+\sec^2(x)$ **(C)** $\sec^2(x)\tan^2(x)+\sec^3(x)$
(D) $\sec(x)\tan(x)+\sec(x)$ **(E)** $\sec^2(x)\tan^2(x)+\sec^2(x)$

44. The first floor of the student fitness building at SIU consists of a rectangle with a semicircle on each end. A 200-meter track runs around the outside. If the designers of the building wanted to maximize the rectangular area of the first floor, how long should the radius of each semicircle be? (nearest tenth)

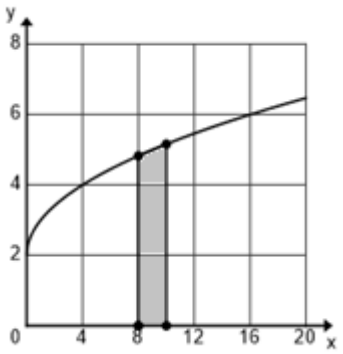
(A) 15.5 m **(B) 15.7 m** **(C) 15.9 m** **(D) 16.1 m** **(E) 16.3 m**

45. Find the area of one petal of the rose curve $r = 6\cos 3\theta$. (nearest tenth)

(A) 8.8 **(B) 9.0** **(C) 9.2** **(D) 9.4** **(E) 9.6**

46. Consider the graph shown on the right. Use the trapezoidal approximation method with six subintervals of equal width to approximate the area bounded by the curves $y_1 = \sqrt{x}+2$, $y_2 = 0$, $x_1 = 4$, $x_2 = 16$. One of the trapezoids is shown on the right. (nearest hundredth)

(A) 59.25
(B) 60.27
(C) 61.29
(D) 62.31
(E) 63.33



Problems 46, 47

47. Find the exact area of the region bounded by the curves $y_1 = \sqrt{x}+2$, $y_2 = 0$, $x_1 = 4$, $x_2 = 16$.

(A) $60\frac{5}{6}$ **(B) 61** **(C)** $61\frac{1}{6}$ **(D)** $61\frac{1}{3}$ **(E)** $61\frac{1}{2}$

48. Newton's Law of Cooling states that the rate of change in the temperature of an object is proportional to the difference between the object's temperature and the temperature of the surrounding medium. Consider an object placed in a room kept at a constant temperature of 55°. At t = 0, the temperature of the object is 120°. At t = 15 min, the temperature of the object is 105°. The temperature of the object should reach 90° at t = _____. (nearest tenth)

(A) 35.4 min **(B) 36.5 min** **(C) 37.6 min** **(D) 38.7 min** **(E) 39.8 min**

49. Naveed is evaluating $\int_0^1 \arcsin(x)dx$ using the method of integration by parts. The best choice for dv is _____.

(A) arcsin(x) (B) x (C) sin(x) (D) cos(x) (E) dx

50. Which of the following tests will show that the series $\sum_{n=1}^{\infty}\left(\frac{n}{(n^2+1)^3}\right)$ converges?

(A) nth Term test (B) p-Series test (C) Integral test
(D) Geometric Series test (E) Telescoping Series Test

51-52. Consider the function $f(x)=\sin(x^2)$

51. For x > 0, the first local minimum occurs when x = _____. (nearest hundredth)

(A) 2.05 (B) 2.09 (C) 2.13 (D) 2.17 (E) 2.21

52. Use the first three non-zero terms of the McLaurin series for f(x) to approximate $f(.25)$. (6 decimal places)

(A) 0.062455 (B) 0.062457 (C) 0.062459 (D) 0.062461 (E) 0.062463

53-54. Eric was initially disappointed when he learned he had made 72 on his Honors Cal III test. His U.T. professor, Dr Hookem, revealed that the test scores were approximately normal with a mean of 60 with a standard deviation of 5.

53. Eric felt better after he calculated his z-score, which was _____.

(A) 1.8 (B) 2.0 (C) 2.2 (D) 2.4 (E) 2.6

54. He felt much better after his calculations placed him at the _____ percentile, based on his z-score.

(A) 91st (B) 93rd (C) 95th (D) 97th (E) 99th

Season	1981	1982	1983	1984	1985	1986	1987	1988	1988	1989
Homeruns	22	36	44	46	52	56	50	74	48	46

The table above shows the home run production for Steve Strutz of the Portland Beavers. Use this table for problems 55, 56, and 57.

55. The mode of the data is _____ homeruns.

(A) 44 (B) 46 (C) 48 (D) 50 (E) 52

56. The mean of the data is _____ homeruns. (nearest tenth)

(A) 47.0 (B) 47.2 (C) 47.4 (D) 47.6 (E) 47.8

57. Which values are considered outliers?

(A) none (B) 22 only (C) 74 only (D) 22, 74 only (E) 22, 36, 74 only

Event	Probability
Obese	0.28
Obese and has heart disease	0.16
Not obese and does not have heart disease	0.65

Use the table above for problem 58.

58. Researchers are interested in the relationship between obesity and heart disease. Suppose a person is randomly selected from a large population of males in the 50-59 age group. The table above shows the probabilities of some events related to this chance process. If two males from this population are selected at random, what is the probability that at least one of them has heart disease. (nearest thousandth)

(A) 0.387 (B) 0.407 (C) 0.427 (D) 0.447 (E) 0.467

School	Mean Math ACT Score	Standard Deviation
School A	23	4
School B	19	3

59-60. State officials wanted to know if an experimental math curriculum implemented at a school in southeast Idaho was helping students score higher on the math section of the ACT Test. Two schools which are located ten miles apart and which had similar scores using the state curriculum in 2014 were compared after one school began using an experimental math curriculum in 2015. The results of the 2020 ACT Test are given in the table above. School A was using the new curriculum while School B continued to use the standard state curriculum. One student from School A is randomly selected and one student from School B is randomly selected.

59. Find the expected value for the difference in their scores.

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

60. Find the standard deviation of the difference in their scores.

(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

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University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitational B ● 2023
Answer Key

1. A	**21. B**	**41. E**
2. C	**22. B**	**42. B**
3. B	**23. C**	**43. A**
4. D	**24. E**	**44. C**
5. E	**25. E**	**45. D**
6. C	**26. B**	**46. C**
7. C	**27. A**	**47. D**
8. E	**28. C**	**48. A**
9. B	**29. A**	**49. E**
10. D	**30. A**	**50. C**
11. A	**31. C**	**51. D**
12. A	**32. B**	**52. C**
13. E	**33. E**	**53. D**
14. B	**34. B**	**54. E**
15. D	**35. A**	**55. B**
16. D	**36. D**	**56. C**
17. C	**37. C**	**57. D**
18. C	**38. C**	**58. B**
19. D	**39. D**	**59. C**
20. A	**40. A**	**60. C**