UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

District • 2023

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1. After math practice on Thursday, the Holliday math team drove to the Whataburger in Wichita Falls for supper. The principal gave them $50 to spend. They ordered 5 cheeseburger combos. A combo cost $7.85 plus tax. If the tax rate is 8.25% and if an apple pie cost $1.25 plus tax, how many apple pies could they order?

(A) 2 **(B) 3** **(C) 4** **(D) 5** **(E) 6**

2. To pay for some new HP Prime G2 calculators for Mr. C's math team, Crosby, Stills, and Nash agreed to perform at SHS with all proceeds going to the math department. Student tickets cost $15.00 and adult tickets cost $25.00. If they raised $7,700 by selling 372 tickets, how many adult tickets were sold?

(A) 208 **(B) 210** **(C) 212** **(D) 214** **(E) 216**

3. Line L_1 contains the points $(-8, 6)$ and $(4, -10)$. Line L_2 is parallel to L_1 and contains the point $(-6, -12)$. The y-intercept of L_2 is $(0, b)$. The value of b is _____.

(A) –22 **(B) –20** **(C) –18** **(D) –16** **(E) –14**

4. Tal rented a car at the airport when his plane landed in Boise. The city of Boise charges an upfront fee of $20 to rent a car at the airport. He was also charged $25 per day and $0.55 per mile. If Tal used his car for five days and his final bill was $241.25, how many miles did he drive during his stay?

(A) 175 **(B) 177** **(C) 179** **(D) 181** **(E) 183**

5. Anthony can wash and wax a car in 45 minutes while Jacob needs one hour to wash and wax a car. If Anthony works by himself for two hours before being joined by Jacob, how much time will it take for them to wash and wax 16 cars? (nearest minute)

(A) 7 hr 31 min **(B) 7 hr 34 min** **(C) 7 hr 37 min** **(D) 7 hr 40 min** **(E) 7 hr 43 min**

6. The value of Warith's house is increasing by 9.35 percent each year. If his house is worth $378,000 on January 1st, 2023, what should it be worth on January 1st, 2035? (nearest dollar)

(A) $1,100,441 **(B) $1,101,552** **(C) $1,102,663** **(D) $1,103,774** **(E) $1,104,885**

7. If the values of the roots of the function $f(x) = 7x^2 + 14x - 105$ are a and b, then $\frac{a+b}{ab}$ = _____.

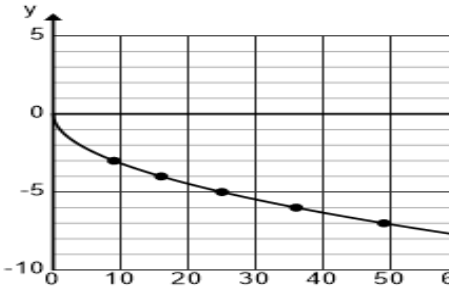
(A) 0.125 **(B) $0.1\overline{3}$** **(C) $0.14\overline{2}$** **(D) 0.15** **(E) $0.1\overline{6}$**

8. The graph of y = h(x) begins at the point $(0, 0)$. h(121) = ___.

(A) –9 **(B) –10** **(C) –11** **(D) –12** **(E) –13**

9. If g(x) is the inverse function of h(x), find the domain of g(x).

(A) $[0, 121]$ **(B) $[-121, \infty)$** **(C) $[0, \infty)$**
(D) $(-\infty, \infty)$ **(E) $(-\infty, 0]$**

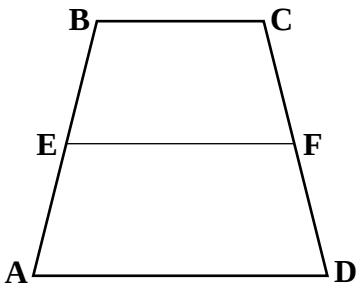


Problems 8, 9

10. The number of 2-liter cokes sold at Walmart each month varies inversely as the price. In a month when the price was $1.80, they sold 3448 2-liter cokes. If the price is reduced to $1.20 the next month, what is the expected number of cokes that will be sold?

(A) 5166 **(B) 5168** **(C) 5170** **(D) 5172** **(E) 5174**

11-12. Consider isosceles trapezoid ABCD. $\overline{EF}$ is the median.
BC = BE = 12. $m\angle BAD = 80°$.



Problems 11, 12

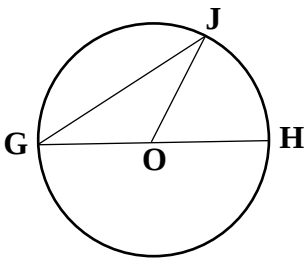
11. Draw auxiliary line segment $\overline{EC}$. Find the area of triangle EBC. (nearest tenth)

(A) 70.3 **(B) 70.6** **(C) 70.9** **(D) 71.2** **(E) 71.5**

12. Find the area of trapezoid ABCD. (nearest whole number)

(A) 382 **(B) 385** **(C) 388** **(D) 391** **(E) 394**

13-14. Consider the circle with center O and diameter $\overline{GH}$. The measure of minor arc GJ = 110° and GH = 18.



Problems 13, 14

13. The area of sector JOH is _____. (nearest tenth)

(A) 48.7 **(B) 49.1** **(C) 49.5** **(D) 49.9** **(E) 50.3**

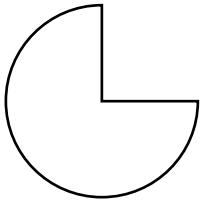
14. The perimeter of triangle GOJ is _____. (nearest tenth)

(A) 32.5 **(B) 32.7** **(C) 32.9** **(D) 33.1** **(E) 33.3**

15. A regular hexagon is inscribed in a circle. If the area of the circle is 452, then the perimeter of the hexagon is _____. (nearest whole number)

(A) 68 **(B) 70** **(C) 72** **(D) 74** **(E) 76**

16. The area of the three-quarter circle is 530. Find the perimeter of the three-quarter circle. (nearest whole number)


Problem 16

(A) 97 **(B) 99** **(C) 101** **(D) 103** **(E) 105**

17. A right circular cone has a radius of 7.75 and a total surface area of 462. Find the volume of the cone. (nearest whole number)

(A) 495 **(B) 499** **(C) 503** **(D) 507** **(E) 511**

18. The center of a circle is in quadrant IV and the circumference of the circle is 16π. The equation of the circle is $x^2 + y^2 + ax + 8y - 39 = 0$. The value of a is _____.

(A) –8 **(B) –6** **(C) –4** **(D) –2** **(E) –1**

19. Consider right triangle ABC with $m\angle C = 90°$. Point D lies on $\overline{AB}$, $\overline{CD} \perp \overline{AB}$, AC = 6 and AB = 10. Find the area of triangle ACD. (nearest hundredth)

(A) 8.53 **(B) 8.64** **(C) 8.75** **(D) 8.86** **(E) 8.97**

20. Consider $f(x) = 2x^3 + bx^2 + 4x - 8$. If $f(3) = 31$, then b = _____.

(A) –3 **(B) –2** **(C) –1** **(D) 1** **(E) 2**

21. Consider four consecutive even negative integers (in increasing order) such that the product of the first and third is 122 greater than the product of -25 and the fourth. Find the sum of the four integers.

(A) –108 **(B) –104** **(C) –100** **(D) –96** **(E) –92**

22. How many even four-digit counting numbers are less than 7000?

(A) 1500 **(B) 2000** **(C) 2500** **(D) 3000** **(E) 3500**

23. If the distance from the point $(e, -12)$ to the line $y = \frac{3}{5}x + 6$ is $\frac{66}{\sqrt{34}}$, then e = ____. $(e > -15)$

(A) –11 **(B) –10** **(C) –9** **(D) –8** **(E) –7**

24. Consider the conic with equation $9x^2 - 4y^2 - 36x - 24y - 36 = 0$. If the coordinates of the foci are (a, b) and (c, b) then $a + b + c =$ _____. (nearest tenth)

(A) 1.0 **(B) 1.2** **(C) 1.4** **(D) 1.6** **(E) 1.8**

25. Two cables are attached to a vertical tower from a point on the ground. The angle between the cables is 20°. The longer cable is 270 feet long and is attached to the top of the tower. The shorter cable is attached to the tower 105 feet below the top of the tower. Find the length of the shorter cable. (nearest whole number)

(A) 195 ft **(B) 198 ft** **(C) 201 ft** **(D) 204 ft** **(E) 207 ft**

26. Consider an arithmetic sequence in which the sixth term is 47 and the twelfth term is 95. What is the product of the eighteenth and nineteenth terms?

(A) 21,579 **(B) 21,586** **(C) 21,593** **(D) 21,600** **(E) 21,607**

27. If $f(x) = \frac{3x-4}{5x-6}$ and $h(x) = \frac{-2x+5}{-3x-8}$, then $\left(h^{-1} \circ f^{-1}\right)(1) =$

(A) –13 **(B) –10** **(C) –7** **(D) –4** **(E) –1**

28. Three of the roots of the fourth-degree polynomial $x^4+bx^3+cx^2+dx+e$ are $-2,\ 3,$ and $1-\sqrt{5}$. If b, c, d, and e are rational numbers, then $b+c+d+e=$ _____.

(A) 17 **(B) 20** **(C) 23** **(D) 26** **(E) 29**

29. The sound level in decibels, β, is given by $\beta=10\log\left(\frac{I}{10^{-12}}\right)$, where I is the intensity of sound in W/m^2. Andrew is playing his trumpet, producing a sound level of 88 dB. If twelve other musicians join him and they all play their trumpets at the same intensity as Andrew, what is the sound level of all of the trumpets playing together? (nearest whole number)

(A) 88 dB **(B) 99 dB** **(C) 110 dB** **(D) 121 dB** **(E) 132 dB**

30. Assume the temperature on a typical day in January in Idaho Falls varies sinusoidally with a low of $12°\,F$ at 5:00 AM and a high of $29°\,F$ at 5:00 PM. What is the expected temperature at midnight? (nearest tenth)

(A) $17.7°\,F$ **(B) $18.0°\,F$** **(C) $18.3°\,F$** **(D) $18.6°\,F$** **(E) $18.9°\,F$**

31. Angle A is in quadrant II and angle B is in quadrant III. If $\sin A=\frac{3}{5}$ and $\cos B=-\frac{5}{13}$, then $\tan(A+B)=$ _____. (nearest hundredth)

(A) 0.56 **(B) 0.59** **(C) 0.62** **(D) 0.65** **(E) 0.68**

32. The partial fraction decomposition of $\frac{x+8}{x^2+x-6}$ is $\frac{A}{x-2}+\frac{B}{x+3}$. $A+B=$ _____.

(A) 1 **(B) 2** **(C) 3** **(D) 4** **(E) 5**

33. Find the angle between the vectors $u=\langle 4,-6\rangle$ and $v=\langle 12,8\rangle$ is _____ rad. (nearest hundredth)

(A) 1.13 **(B) 1.24** **(C) 1.35** **(D) 1.46** **(E) 1.57**

34. Consider the sequence $1, 5, 12\frac{1}{2}, 20\frac{5}{6}, 26\frac{1}{24}, \ldots$. Find the eighth term in the sequence. (nearest hundredth)

(A) 15.02 **(B) 15.14** **(C) 15.26** **(D) 15.38** **(E) 15.50**

35. A ball is dropped from a height of six feet and begins bouncing. Each bounce is three-fourths of the height of the previous bounce. Which bounce is the first bounce in which the height of the bounce is less than one foot?

(A) 5 **(B) 6** **(C) 7** **(D) 8** **(E) 9**

36. For the final exam in calculus, Mrs. Wilcox gave her class a list of 18 study problems. Of these, 10 will be on the exam. If Emmy knows how to correctly solve 16 of these, find the probability that she will correctly solve all 10 problems on the final exam. (nearest thousandth)

(A) 0.141 **(B) 0.162** **(C) 0.183** **(D) 0.204** **(E) 0.225**

37. Consider an ellipse in which the vertices are $(0, 4)$ and $(10, 4)$ and the endpoints of the minor axis are $(5, 2)$ and $(5, 6)$. What is the eccentricity of the ellipse? (nearest hundredth)

(A) 0.84 **(B) 0.86** **(C) 0.88** **(D) 0.90** **(E) 0.92**

38. Find the area of a triangle with vertices $A(6,4,2)$, $B(8,6,10)$, and $C(6,2,8)$. (nearest tenth)

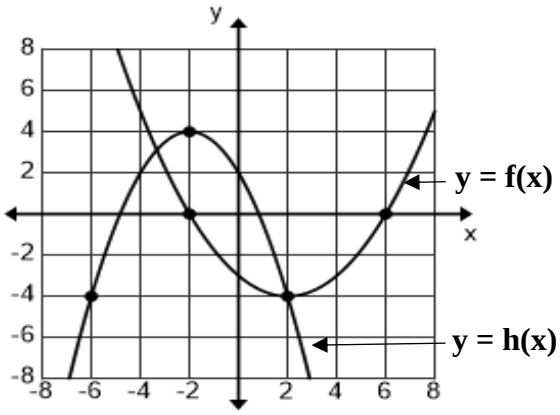
(A) 14.5 **(B) 14.8** **(C) 15.1** **(D) 15.4** **(E) 15.7**

39. The directrix of the graph of y = h(x) is the line y = c. The value of c is _____.

(A) 4.125 **(B) 4.25** **(C) 4.375**
(D) 4.5 **(E) 4.625**

40. If the parabolas intersect at points A and B, then AB = _____. (nearest tenth)

(A) 8.9 **(B) 9.0** **(C) 9.1**
(D) 9.2 **(E) 9.3**



Problems 39, 40, 41, 42

41. The area of the region bounded by the graphs of the parabolas is _____. (nearest tenth)

(A) 18.4 **(B) 18.6** **(C) 18.8** **(D) 19.0** **(E) 19.2**

42. Find the arc length of the graph of y = f(x) on the interval [0, 8]. (nearest tenth)

(A) 13.2 **(B) 13.4** **(C) 13.6** **(D) 13.8** **(E) 14.0**

43. A rectangle is to be inscribed between the graph of $y = 16 - x^2$ and the x-axis with its base on the x-axis. What is the maximum area of such a rectangle? (nearest tenth)

(A) 44.9 **(B) 46.0** **(C) 47.1** **(D) 48.2** **(E) 49.3**

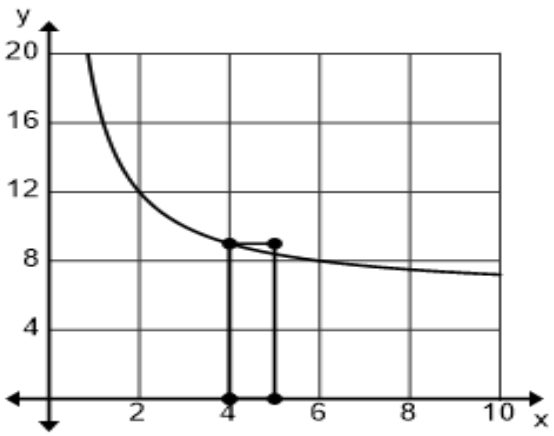
44. Find the sum of the series. $2 - \frac{4}{3} + \frac{4}{15} - \frac{8}{315} + \ldots$

(A) sin(2) **(B) cos(2)** **(C) e^2** **(D) ln(2)** **(E) 2sin(1)**

45. Find the area in the second quadrant bounded by the x-axis, the y-axis, and the graph of $r(\theta) = 2\theta + 3\sin(\theta),\ 0 \le \theta \le 2\pi$. (nearest tenth)

(A) 33.9 **(B) 34.2** **(C) 34.5** **(D) 34.8** **(E) 35.1**

46. Consider the graph of $y_1 = \frac{12}{x} + 6$. Use the Left Rectangle Approximation Method with six rectangles of equal width to approximate the area bounded by the curves $y_1 = \frac{12}{x} + 6,\ y_2 = 0$, $x_1 = 2$, and $x_2 = 8$. One of the rectangles is shown on the right. (nearest hundredth)



Problems 46, 47

(A) 55.11 **(B) 55.22** **(C) 55.33**
(D) 55.44 **(E) 55.55**

47. Find the volume of the solid generated when the region bounded by the curves $y_1 = \frac{12}{x} + 6,\ y_2 = 0,\ x_1 = 2,\ x_2 = 8$ is revolved around the line x = 12. (nearest whole number)

(A) 2369 **(B) 2373** **(C) 2377** **(D) 2381** **(E) 2385**

48. The Panhandle Coffee Shop keeps their dining area at a constant 72°. They serve their famous coffee at exactly 175° and the temperature of the coffee changes at the rate $r(t) = -5.89e^{-.0766t}$ degrees per minute. Darius received a phone call at the moment his coffee was served and his coffee cooled for exactly five minutes before he was able to take his first sip. What was the temperature of the coffee when he took his first sip? (nearest whole number)

(A) 139° **(B) 143°** **(C) 147°** **(D) 151°** **(E) 155°**

49. (rad) The derivative of the function f is given by $f'(x) = 2x^3 - 6\sin\left(x^2\right) + 2$. On the interval $(-2, 2)$, at which of the following values does f have a relative minimum? (nearest thousandth)

I. –0.535 **II. 0.669** **III. 1.260**

(A) I only **(B) I, III only** **(C) II only** **(D) III only** **(E) I, II, III**

50-51. (rad) The position of an object moving in the xy-plane is given by $(x(t), y(t))$, $0 \le t \le \frac{5\pi}{12}$, with $\frac{dx}{dt} = 4t\sin(t)$ cm/s and $\frac{dy}{dt} = 4t\cos(t)$ cm/s. At t = 0, the position of the object is $(4, 8)$.

50. Find the speed of the object at t = $\frac{\pi}{6}$. (nearest hundredth)

(A) 1.87 cm/s (B) 1.98 cm/s (C) 2.09 cm/s (D) 2.20 cm/s (E) 2.31 cm/s

51. The position of the object at t = $\frac{\pi}{3}$ is (a, b). b = _____. (nearest hundredth)

(A) 5.19 cm (B) 6.30 cm (C) 7.41 cm (D) 8.52 cm (E) 9.63 cm

52. Consider the first quadrant region bounded by the y-axis, the line x = 4, the line y = 10, and the curve $y = 2\ln(5 - x)$. This region is the base of a solid by cross sections in which each cross section is a square perpendicular to the x-axis. What is the volume of the solid? (nearest whole number)

(A) 250 (B) 254 (C) 258 (D) 262 (E) 266

Year	1911	1931	1951	1971	1995
Distance	50 ft 11 in	51 ft 1.25 in	52 ft 6.25 in	57 ft 1in	60 ft

The progression of the world record in the men's triple jump is shown in the table above. Use this table for problems 53 and 54.

53. Professor Stat instructed his students to find the LSRL for the data. The linear regression model overestimates the true value of the 1951 distance by _____. (nearest hundredth)

(A) 1.35 ft (B) 1.44 ft (C) 1.53 ft (D) 1.62 ft (E) 1.71 ft

54. Use the LSRL for the data and predict what the world record should be in 2022. (nearest inch)

(A) 62 ft 3 in (B) 62 ft 6 in (C) 62 ft 9 in (D) 63 ft (E) 63 ft 3 in

55. Assume the mean hang time of a punt for all NFL punters over the 2022 season was 4.40 seconds with a standard deviation of 0.25 seconds. If Jordan Stout had a mean hang time of 4.82 seconds for the 2022 season, what percentile did that place him at?

(A) 89th (B) 91st (C) 93rd (D) 95th (E) 97th

56. Consider a random variable X that is normally distributed with a mean of 75 and a standard deviation of 16. The approximate interquartile range for this distribution is _____. (nearest tenth)

(A) 20.5 (B) 21.6 (C) 22.7 (D) 23.8 (E) 24.9

57. A random sample of 500 Texas high school students is used to estimate the proportion of Texas high school students who participate in UIL academics. What is the maximum margin of error if a 96 percent confidence interval is to be constructed? (nearest thousandth)

(A) 0.028 **(B) 0.034** **(C) 0.040** **(D) 0.046** **(E) 0.052**

University	Texas	A&M	Tech	TCU
Students	96	82	112	70

58. A random sample of 360 high school seniors in the Texas Panhandle were asked which university they hoped to attend. Students were asked to choose between Texas, A&M, Tech, and TCU. The results are in the table above. Researchers had expected a ratio of 3:3:4:2 for their choices. An appropriate test at the $\alpha = 0.05$ level was performed to see if the observed values differ from what was expected. Based on a P-value of _____, researchers concluded that there was insufficient evidence to show that student choices differ from what was expected.

(A) 0.262 **(B) 0.283** **(C) 0.304** **(D) 0.325** **(E) 0.346**

59. Ninety-five percent of the Olympic athletes who have been using steroids will test positive using a new test just developed. Ninety-eight percent of Olympic athletes who have not been using steroids will test negative using the new test. If ten percent of Olympic athletes have been using steroids, what percent of Olympic athletes will test positive using the new test? (nearest tenth)

(A) 9.1% **(B) 10.2%** **(C) 11.3%** **(D) 12.4%** **(E) 13.5%**

60. In the Fort Bend school district, 16 out of 88 randomly selected high school seniors plan to study computer science in college, while 21 out of 72 juniors plan to study computer science in college. A 96% confidence interval for the difference between the proportion of high school seniors who plan to study computer science in college and the proportion of high school juniors who plan to study computer science is to be calculated. What is the standard error of the difference? (nearest ten-thousandth)

(A) 0.0591 **(B) 0.0612** **(C) 0.0633** **(D) 0.0654** **(E) 0.0675**

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University Interscholastic League
MATHEMATICS CONTEST
HS ● District ● 2023
Answer Key

1. D	**21. C**	**41. D**
2. C	**22. D**	**42. C**
3. B	**23. D**	**43. E**
4. A	**24. A**	**44. A**
5. E	**25. D**	**45. C**
6. E	**26. C**	**46. A**
7. B	**27. A**	**47. E**
8. C	**28. E**	**48. D**
9. E	**29. B**	**49. B**
10. D	**30. C**	**50. C**
11. C	**31. B**	**51. E**
12. A	**32. A**	**52. C**
13. C	**33. E**	**53. E**
14. B	**34. E**	**54. B**
15. C	**35. C**	**55. D**
16. C	**36. C**	**56. B**
17. E	**37. E**	**57. D**
18. B	**38. D**	**58. E**
19. B	**39. D**	**59. C**
20. A	**40. A**	**60. E**