UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Region • 2023

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1. Joe and Arlene took their three children to Fuzzy's on Tuesday. Joe ordered a burrito for $6.95, Arlene ordered a chicken fajita salad for $7.95, and each child ordered a breakfast taco. If a breakfast taco costs $2.95 and the tax rate is 8.25%, what is the total cost?

(A) $25.68 **(B) $25.71** **(C) $25.74** **(D) $25.77** **(E) $25.80**

2. The total energy of a sphere rolling down an incline without slipping is $E = .7mv^2 + mgh$, where E is the energy (J), m is the mass (kg), v is the velocity (m/s), g is the acceleration of gravity (m/s^2), and h is the height (m). Find v if E = 27.6 J, m = 1.72 kg, g = 9.807 m/s^2, and h = 1.53 m. (nearest hundredth)

(A) 1.16 m/s **(B) 1.18 m/s** **(C) 1.20 m/s** **(D) 1.22 m/s** **(E) 1.24 m/s**

3-4. Given the following points with coordinates: $A(-4, 8)$, $B(8, 2)$, $C(2, -2)$, $D(6, 6)$, and $E(r, 3)$.

3. $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{CD}$ at point $P(a, b)$. $a + b =$ _____. (nearest tenth)

(A) 7.8 **(B) 8.0** **(C) 8.2** **(D) 8.4** **(E) 8.6**

4. If point E lies on the perpendicular bisector of $\overline{AB}$, then r = _____. (nearest tenth)

(A) 1.0 **(B) 1.2** **(C) 1.4** **(D) 1.6** **(E) 1.8**

5. A good approximation for a person's maximum heart rate is given by $MHR = 220 - x$, where MHR = maximum heart rate in beats per minute and x = a person's age. Randy recently ran a 3-mile run in 30 minutes. He ran with his heart rate at 85% of his MHR and his heart beat 4080 times during the run. How old is Randy?

(A) 58 **(B) 60** **(C) 62** **(D) 64** **(E) 66**

6. Salem left Canadian and traveled north on State Highway 60 on his new bicycle at 24 mph for 10 minutes. Then he turned east and headed toward Lake Marvin on FM 2266. He traveled at 28 mph, arriving at the lake 22 minutes after turning east. What is the straight-line distance from Canadian to Lake Marvin? (nearest tenth)

(A) 11.0 mi **(B) 11.2 mi** **(C) 11.4 mi** **(D) 11.6 mi** **(E) 11.8 mi**

7-8. Given: $h(x) = \dfrac{\sqrt{x}}{\sqrt{16 - x^2}}$.

7. Find the domain of $h(x)$.

(A) $[0, \infty)$ **(B) $[0, 4]$** **(C) $(0, 4)$** **(D) $[0, 4)$** **(E) $(0, \infty)$**

8. Find the range of $h(x)$.

(A) $[0, \infty)$ **(B) $[0, 4]$** **(C) $(0, 4)$** **(D) $[0, 4)$** **(E) $(0, \infty)$**

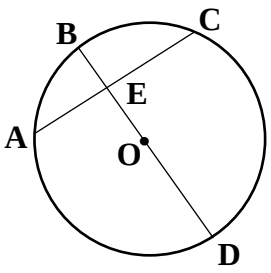
9. Mary trimmed a large square picture so that it would fit into a frame. She trimmed 10 inches from the length and 6 inches from the width. The area of the trimmed picture is 780 square inches. What was the perimeter of the original picture before she trimmed it? (nearest inch)

(A) 132 in **(B) 136 in** **(C) 140 in** **(D) 144 in** **(E) 148 in**

10. In Southeast Idaho, the number of cougars in a protected area varies directly as the number of acres the area covers and inversely as the square of the number of grizzly bears in the area. The protected area in Teton County covers 600 acres, has 88 cougars and has 12 grizzly bears. If the protected area in Fremont County covers 964 acres and has 22 grizzly bears, how many cougars live in this area?

(A) 42 **(B) 62** **(C) 82** **(D) 102** **(E) 122**

11-12. Consider the circle on the right with center O.
Chord $\overline{AC}$ intersects diameter $\overline{BD}$ at point E.
$\overline{AC} \perp \overline{BD}$, $BD = 20$, and $AC = 16$.



Problems 11, 12

11. BE = _____. (nearest tenth)

(A) 3.9 **(B) 4.0** **(C) 4.1**
(D) 4.2 **(E) 4.3**

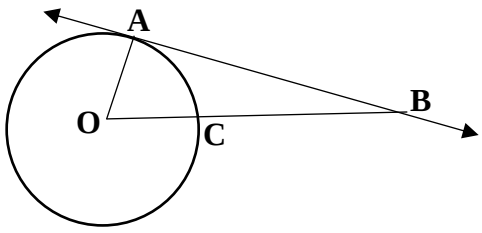
12. Find the area of sector AOD. (nearest tenth)

(A) 107.4 **(B) 108.5** **(C) 109.6** **(D) 110.7** **(E) 111.8**

13. Triangle ABC is similar to triangle DEF. AB = 24, BC = 18, AC = 20, and EF = 15. Find the perimeter of triangle DEF. (nearest tenth)

(A) 51.7 **(B) 51.9** **(C) 52.1** **(D) 52.3** **(E) 52.5**

14-15. Line $\overleftrightarrow{AB}$ is tangent to the circle at point A.
Point O is the center of the circle.
$AO = x$, $AB = 5x - 5$, and $BC = 4x - 4$.



Problems 14, 15

14. Find the perimeter of triangle AOB. (nearest tenth)

(A) 86.4 **(B) 87.6** **(C) 88.8**
(D) 90.0 **(E) 91.2**

15. Find the area of the region inside triangle AOB but outside sector AOC. (nearest whole number)

(A) 122 **(B) 125** **(C) 128** **(D) 131** **(E) 134**

16. The following points are the vertices of a right triangle. $A(-6, -2)$, $B(4, b)$, and $C(6, -2)$.
If $m\angle ABC = 90°$ and $b > 0$, then b = _____. (nearest hundredth)

(A) 2.47 **(B) 2.59** **(C) 2.71** **(D) 2.83** **(E) 2.94**

17. Consider triangle DEF with DE = 13, EF = 15, and DF = 17. Find the length of the longest median of the triangle. (nearest tenth)

(A) 11.5 (B) 12.3 (C) 13.1 (D) 13.9 (E) 14.7

18. Consider triangle GHI with a point J that lies on side $\overline{GI}$. GH = 24, HI = 36, IJ = 24 and ray $\overrightarrow{HJ}$ bisects $\angle GHI$. GJ = _____. (nearest tenth)

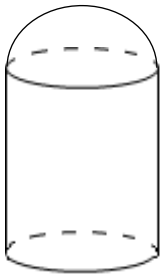
(A) 15.6 (B) 15.8 (C) 16.0 (D) 16.2 (E) 16.4

19. Consider triangle KLM with vertices $K(-8,-3)$, $L(2,7)$, and $M(5,-6)$. Triangle KLM is a/an _____ triangle.

(A) scalene (B) isosceles (C) equilateral (D) right (E) obtuse

20. A hemisphere sits on top of a cylinder as shown on the right. The total surface area of the figure shown is 1520 cm^2. The height of the cylinder is 3 times the radius. Find the volume of the figure shown on the right. (nearest whole number)

(A) 4536 cm^3 (B) 4540 cm^3 (C) 4544 cm^3
(D) 4548 cm^3 (E) 4552 cm^3



Problem 20

21. Dwayne "the Rock" Johnson weighs 21 stones. There are 8 stones in a hundredweight, 20 hundredweight in a ton, and 9.81 newtons in a kilogram. He weighs _____ newtons. (nearest whole number)

(A) 1160 (B) 1164 (C) 1168 (D) 1172 (E) 1176

22. The amount of a radioactive substance present is decreasing exponentially. At t = 0, there was 100 mg of the substance present. At t = 27 years, there was 59 mg of the substance present. Predict the amount present at t =144 years. (nearest hundredth)

(A) 5.88 mg (B) 5.92 mg (C) 5.96 mg (D) 6.00 mg (E) 6.04 mg

23. The graph of a cubic function has a local minimum at $A(7,-4)$ and a point of symmetry at $B(0,6)$. If a local maximum occurs at point $C(a,b)$, then $a+b=$ _____. (nearest whole number)

(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

24. Consider a regular pentagon with a perimeter of 23. Find the area of a circle inscribed in the pentagon. (nearest tenth)

(A) 30.3 (B) 30.6 (C) 30.9 (D) 31.2 (E) 31.5

25. Given: $f(x)=\frac{8-3x}{9-2x}$. If $g(x)$ is the inverse function of $f(x)$, then $g(7)=$ _____. (nearest tenth)

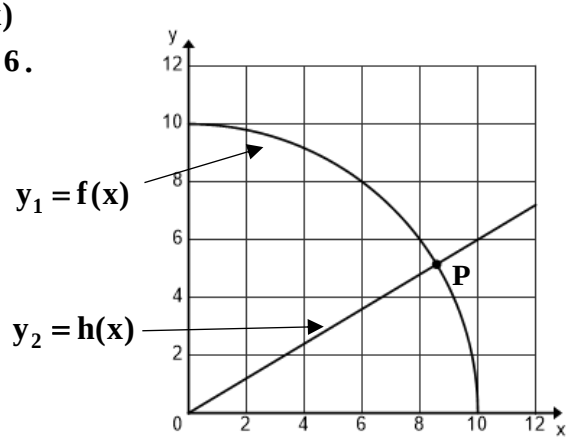
(A) 4.6 (B) 4.8 (C) 5.0 (D) 5.2 (E) 5.4

26. The graphs of quarter circle $y_1 = f(x)$ and line $y_2 = h(x)$ intersect in the first quadrant at point $P(a, b)$. $h(10) = 6$. $a + b =$ _____. (nearest hundredth)

(A) 13.60 (B) 13.63 (C) 13.66
(D) 13.69 (E) 13.72

27. Find the area of the first quadrant region bounded by the curves $y_1 = f(x)$, $y_2 = h(x)$, and $x = 0$. (nearest tenth)

(A) 51.5 (B) 51.7 (C) 51.9
(D) 52.1 (E) 52.3



Problems 26, 27

28. Erica left Muenster High School at 12:00 PM and cycled at 22 mph on a bearing of 15°. At 1:30 PM, Halle left Muenster High School and cycled at 25 mph on a bearing of 215°. How far apart were they at 4:30 PM? (nearest whole number)

(A) 163 mi (B) 165 mi (C) 167 mi (D) 169 mi (E) 171 mi

29. Consider the circle $x^2 + y^2 = 25$ and the point $A(10, 0)$. The length of a tangent line segment from point A to the circle is _____. (nearest hundredth)

(A) 8.44 (B) 8.55 (C) 8.66 (D) 8.77 (E) 8.88

30. Consider the ellipse $4x^2 + 3y^2 - 40x - 30y + 127 = 0$. The coordinates of the foci are (a, b) and (a, c). $b + c =$ _____. (nearest tenth)

(A) 9.6 (B) 9.8 (C) 10.0 (D) 10.2 (E) 10.4

31. Consider a hyperbola centered at the origin with a vertex at $(0, -2)$ and an asymptote with equation $y = x$. The coordinates of the foci are (a, b) and (a, c). $|b - c| =$ _____. (nearest tenth)

(A) 5.1 (B) 5.3 (C) 5.5 (D) 5.7 (E) 5.9

32. Which of the following is one of the four fourth roots of $-8 - 8\sqrt{3}\,i$?

(A) $1 - \sqrt{3}\,i$ (B) $-\sqrt{3} - i$ (C) $-1 - \sqrt{3}\,i$ (D) $-1 + \sqrt{3}\,i$ (E) $\sqrt{3} + i$

33. There are 830 seniors at Pokie High. Two hundred are taking BC Calculus, 196 are taking AB Calculus, and 192 are taking Statistics. Forty-four are taking all three courses, 80 are only taking BC Calculus, 94 are only taking AB Calculus, and 66 are only taking Statistics. How many seniors are not taking any of these three courses?

(A) 432 (B) 434 (C) 436 (D) 438 (E) 440

34. The parametric equations $x(t) = (v_0 \cos\theta)t$ and $y(t) = y_0 + (v_0 \sin\theta)t - 16.087t^2$ can be used to model the path of a projectile. If $v_0 = 110$ ft/s, $\theta = 36°$, and $y_0 = 120$ ft, how far will the projectile travel horizontally from the moment it is launched until the instant it hits the ground? (nearest whole number)

(A) 469 ft (B) 472 ft (C) 475 ft (D) 478 ft (E) 481 ft

35. Find the distance from the point $(6, 2, 10)$ to the plane $3x + 3y + z = 12$. (nearest hundredth)

(A) 5.05 (B) 5.16 (C) 5.27 (D) 5.38 (E) 5.49

36. Rick's backyard pool is shaped like a regular pentagon with each side equal to 20 feet. The pool has a constant depth of 4 feet. If the pipe filling the pool delivers 40 gallons of water per minute, how long will it take to completely fill the pool? (nearest tenth)

(A) 7.8 hr (B) 8.0 hr (C) 8.2 hr (D) 8.4 hr (E) 8.6 hr

37. Consider the sequence $3, 7, 12, 18, 25, 33, \ldots$ Find the sum of the first 20 terms of the sequence.

(A) 1958 (B) 1960 (C) 1962 (D) 1964 (E) 1966

38. A circle with center P has a radius of 6, and it is externally tangent to a circle with center O, which has a radius of 9. Points A and B lie on circle O. From P, tangents $\overrightarrow{PA}$ and $\overrightarrow{PB}$ are drawn to the circle with center O. Find the area that is inside kite PAOB, but outside both circles. (nearest tenth)

(A) 9.7 (B) 9.9 (C) 10.1 (D) 10.3 (E) 10.5

39. The graph of $r = \dfrac{4\sin\theta + \sqrt{16\cos^2\theta + 16\sin^2\theta}}{16\cos^2\theta}$ is a parabola of the form $y = ax^2 + bx + c$.

$a + b + c =$ _____.

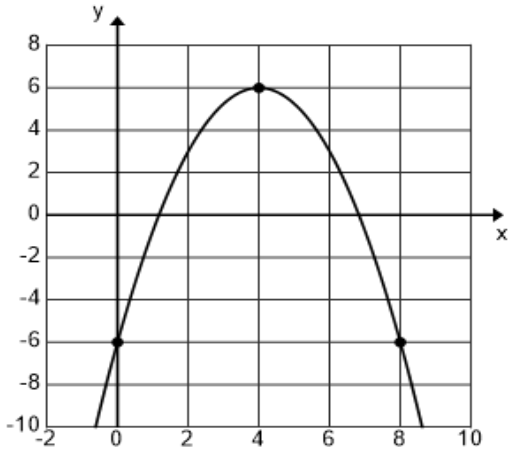
(A) $1\frac{1}{2}$ (B) $1\frac{3}{4}$ (C) $1\frac{7}{8}$ (D) 2 (E) $2\frac{1}{8}$

40. The focus of the graph shown on the right is the point $F(a, b)$. $b =$ _____. (nearest hundredth)

(A) 5.25 (B) 5.33 (C) 5.50
(D) 5.67 (E) 5.75

41. Consider the line tangent to the graph of the parabola at x = 6. The x-intercept of this tangent line is the point $H(c, 0)$. $c =$ _____. (nearest tenth)

(A) 6.8 (B) 7.0 (C) 7.2
(D) 7.4 (E) 7.6



Problems 40, 41

42-44. Consider the functions $f(x) = -.2(x+1)^2 + 10$ and $g(x) = 0.4(x-1)^2 - 4$.

42. Find the value c that satisfies the conclusion of the Mean Value Theorem for the function f(x) on $[-12, -2]$? (nearest hundredth)

(A) –7.24 **(B) –7.16** **(C) –7.08** **(D) –7.00** **(E) –6.92**

43. Find the area bounded by the graphs of $y_1 = f(x)$ and $y_2 = g(x)$. (nearest whole number)

(A) 85 **(B) 87** **(C) 89** **(D) 91** **(E) 93**

44. Find the volume of the solid of revolution generated when the region bounded by the graphs of $y_1 = f(x)$ and $y_2 = g(x)$ is revolved around the line $x = -6$. (nearest whole number)

(A) 3379 **(B) 3382** **(C) 3385** **(D) 3388** **(E) 3391**

x	f	f′	g	g′
1	–3.75	1.5	3	–4
2	–2	2	–2	–6
3	0.25	2.5	–9	–8
4	3	3	–18	–10

45. The table above shows values of differentiable functions f and g. If $h(x) = f\big(g(x)\big)$, then $h'(1) =$ _____.

(A) –16 **(B) –14** **(C) –12** **(D) –10** **(E) –8**

46. Consider the curve defined by the parametric equations $x(t) = 5t^{(1.5)} + 4$ and $y(t) = 3t^2 - 2$. The length of the curve from t = 1 to t = 3 is given by the expression _____.

(A) $\int_1^3 \sqrt{1 + \frac{144t}{225}}\, dt$ **(B)** $\int_1^3 \sqrt{\left(5t^{1.5} + 4\right)^2 + \left(3t^2 - 2\right)^2}\, dt$ **(C)** $\int_1^3 \sqrt{\frac{225t^2}{4} + 36t^2}\, dt$

(D) $\int_1^3 \sqrt{\frac{225t}{4} + 36t^2}\, dt$ **(E)** $\int_1^3 \sqrt{1 + \frac{144t^2}{225}}\, dt$

47. Consider the line tangent to the curve $3x^2 - y^3 = 2$ when x = 1. The y-intercept of this line is the point $(0, b)$. b = _____. (nearest tenth)

(A) –1.2 **(B) –1.0** **(C) –0.8** **(D) –0.6** **(E) –0.4**

48. Let x and y be functions of time t that are related by the equation $3x^2 + 2y^2 - 6xy = 99$. At time t = 2, x = 3, $y < 0$, and $\frac{dx}{dt} = 6$. Find the value of $\frac{dy}{dt}$ at t = 2. (nearest tenth)

(A) 7.0 **(B) 7.2** **(C) 7.4** **(D) 7.6** **(E) 7.8**

t (hr)	0	4	8	12	16	20	24
T (deg)	2°	10°	20°	24°	18°	7°	2°

49-51. The table above shows the temperature, T, at 4-hour intervals, on January 4th at a cabin in the mountains south of Pocatello, Idaho. Assume that $T'(t)$ is a differentiable function of time.

49. Use the table to find the approximate value of $T'(8)$ in _____ degrees per hour. (nearest hundredth)

(A) 1.69 **(B) 1.72** **(C) 1.75** **(D) 1.78** **(E) 1.81**

50. Using the table above, estimate the average temperature over the 24-hour time interval by using a trapezoidal approximation. The length of each subinterval is 4 hours. (nearest tenth)

(A) 12.9° **(B) 13.1°** **(C) 13.3°** **(D) 13.5°** **(E) 13.7°**

51. A professor at ISU modeled the temperature with the function $T(t) = 13 - 11\cos\left(\frac{t+1}{4}\right)$. Use his model to find the average temperature over the time interval $0 \le t \le 24$ hours. (nearest thousandth)

(A) 13.403° **(B) 13.514°** **(C) 13.625°** **(D) 13.736°** **(E) 13.847°**

52-53. The distribution of IQ scores is approximately normal with a mean of 100 and a standard deviation of 15.

52. It has been reported that President Biden has an IQ of 115. What percentile does that place him at?

(A) 80th **(B) 82nd** **(C) 84th** **(D) 86th** **(E) 88th**

53. Find the interquartile range for the distribution of IQ scores. (nearest tenth)

(A) 19.8 **(B) 20.2** **(C) 20.6** **(D) 21.0** **(E) 21.4**

Event	Probability
Obese	.33
Obese and has diabetes	.12
Is not obese and does not have diabetes	.59

54. A study of a large population of adult men in Montana was done in which researchers were interested in the relationship between obesity and diabetes. The results of the study are in the table above. If four adult men are selected at random, find the probability that at least two of them have diabetes. (nearest hundredth)

(A) 0.14 **(B) 0.16** **(C) 0.18** **(D) 0.20** **(E) 0.22**

55. The office manager at Aire Texas conducts follow-up phone calls with customers one week after work was done on their AC units. She reviews her notes on the phone calls to evaluate customer satisfaction. She wants to estimate the proportion of customers who are satisfied with the work done by her workers. Of the following, which is the smallest sample size needed to estimate the proportion of satisfied customers within five percent at a 96% confidence level.

(A) 320 **(B) 354** **(C) 388** **(D) 422** **(E) 456**

Runner	Mean 440 yd time in seconds	Standard Deviation in seconds
Bruce	48.8	0.4
Keenan	50.2	1.1
Kelly	49.7	0.6
Joe	48.2	0.8

56. Coronado High School has had the same four runners on the mile relay the last two years. The times, in seconds, for all four runners over the last two years are approximately normal with means and standard deviations shown in the table above. Assuming each runner's individual times are independent, find the probability that the total team time at a randomly selected track meet is less than 195 seconds. (nearest hundredth)

(A) 0.11 (B) 0.13 (C) 0.15 (D) 0.17 (E) 0.19

57. Events A and B are independent, P(A) = 0.5, and P(A and B) = 0.15. P(A or B) = _____. (nearest hundredth)

(A) 0.55 (B) 0.60 (C) 0.65 (D) 0.70 (E) 0.75

58. In Idaho, all high school junior boys are required to see how far they can run in 12 minutes. A distance of 1.632 miles is at the 30^{th} percentile for the distribution of distances for all junior boys. The distribution is approximately normal with a standard deviation of 0.225 miles. Find the mean distance run in 12 minutes for Idaho junior boys. (nearest hundredth)

(A) 1.75 mi (B) 1.78 mi (C) 1.81 mi (D) 1.84 mi (E) 1.87 mi

Days	0-2	3-5	6-7	Total
Heart Disease	566	288	146	1000
No Heart Disease	162	314	438	914
Total	728	602	584	1914

59-60. Researchers wanted to know if there is an association between exercise and heart disease. A large random sample of men in their 70s were studied. The average number of days each man exercised each week and his heart condition were recorded. Results of the study are in the table above. A Chi-Square Test for Association/Independence was performed.

59. Find the expected count for the 6-7 days/Heart Disease cell. (nearest whole number)

(A) 269 (B) 278 (C) 287 (D) 296 (E) 305

60. The contribution of the 0-2 days/No Heart Disease cell to the Chi-Square statistic is _____. (nearest whole number)

(A) 75 (B) 81 (C) 87 (D) 93 (E) 99

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University Interscholastic League
MATHEMATICS CONTEST
HS ● Region ● 2023
Answer Key

1. B	**21. C**	**41. B**
2. D	**22. D**	**42. D**
3. D	**23. A**	**43. A**
4. A	**24. E**	**44. C**
5. B	**25. C**	**45. D**
6. A	**26. E**	**46. D**
7. D	**27. A**	**47. B**
8. A	**28. E**	**48. B**
9. D	**29. C**	**49. C**
10. A	**30. C**	**50. D**
11. B	**31. D**	**51. B**
12. D	**32. C**	**52. C**
13. A	**33. D**	**53. B**
14. D	**34. E**	**54. C**
15. B	**35. A**	**55. D**
16. A	**36. E**	**56. A**
17. E	**37. B**	**57. C**
18. C	**38. A**	**58. A**
19. B	**39. C**	**59. E**
20. B	**40. D**	**60. E**