UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

State • 2023

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1. The absolute pressure P, in pascals (Pa), on an object at a given depth h, in meters (m), is given by $P = P_0 + \rho gh$, where P_0 is the atmospheric pressure at the surface of the water, ρ is the density of the water, and g is the acceleration of gravity. Find the absolute pressure on an object 60 m below the surface of the ocean. $\left(P_0 = 1.013 \times 10^5 \text{ Pa}, \rho = 1023 \text{ kg/m}^3, g = 9.807 \text{ m/s}^2\right)$ (nearest whole number)

(A) 703,238 Pa (B) 703,242 Pa (C) 703,246 Pa (D) 703,250 Pa (E) 703,254 Pa

2. Ted met Terry at The Sportsman for breakfast. Terry ordered the deluxe omelet for $8.95 and orange juice for $2.35. Ted ordered eggs and pancakes for $9.75 and coffee for $1.95. The tax rate is 8.25%. If they added a $5.00 tip, what was the total cost?

(A) $29.75 (B) $29.80 (C) $29.85 (D) $29.90 (E) $29.95

3. Given: points $A(-6,-3)$, $B(6,-9)$, and $C(12,9)$. Line L_1 contains point C and is parallel to $\overleftrightarrow{AB}$. If point $D(-9,b)$ lies on line L_1, then $b =$ _____. (nearest tenth)

(A) 18.6 (B) 18.9 (C) 19.2 (D) 19.5 (E) 19.8

4. Pam's Pizza offers small, medium and large pizzas. On Friday night, they sold 138 pizzas. They sold 5 more small pizzas than large pizzas and they sold 11 more medium pizzas than small pizzas. How many medium pizzas did they sell?

(A) 54 (B) 55 (C) 56 (D) 57 (E) 58

5. Diane took a trip from Plano to Billings, a distance of 1424 miles. Her total drive time was 20 hr 30 min. She drove 512 miles in 7 hr 45 min on day 1. She drove 488 miles in 6 hr 36 min on day 2. What was her average speed on day 3? (nearest tenth)

(A) 68.6 mph (B) 68.9 mph (C) 69.2 mph (D) 69.5 mph (E) 69.8 mph

6. Mom is 28 years older than Cathy. Grandma's age is 3 more than 6 times Cathy's age. Twice Mom's age is 5 more than Grandma's age. How old is Cathy?

(A) 10 (B) 11 (C) 12 (D) 13 (E) 14

7. Lance left Lubbock and cycled 10 miles west. Then he turned south and cycled 15 miles. Next, he turned east and cycled 25 miles. Finally, he turned south and cycled 10 miles. How far was Lance from where he started? (nearest tenth)

(A) 28.6 mi (B) 28.8 mi (C) 29.0 mi (D) 29.2 mi (E) 29.4 mi

8. Given: points $A(-6,-8)$, $B(4,10)$, and $C(-6,-2)$. Find the distance from point C to the midpoint of $\overline{AB}$. (nearest tenth)

(A) 5.8 (B) 6.0 (C) 6.2 (D) 6.4 (E) 6.6

9. Consider four consecutive positive even numbers in ascending order. Four times the sum of the first and second is 42 greater than three times the sum of the third and fourth. What is the largest number?

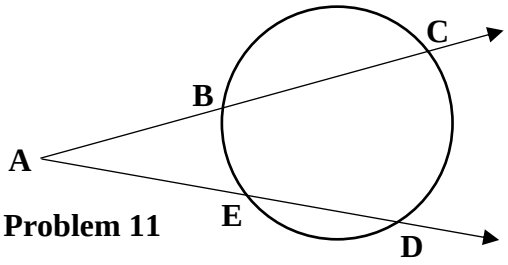
(A) 36 **(B) 38** **(C) 40** **(D) 42** **(E) 44**

10. The resistance of a length of copper wire is directly proportional to the length of the wire and inversely proportional to the cross-sectional area of the wire. The resistance of a 20.0-m length of copper wire with a cross sectional area of 3.46×10^{-6} m^2 is $0.100\ \Omega$. If the resistance of a 32.0-m length of copper wire is $0.626\ \Omega$, what is the diameter of the 32.0-m length of wire? (nearest hundredth)

(A) 1.02 mm **(B) 1.04 mm** **(C) 1.06 mm** **(D) 1.08 mm** **(E) 1.10 mm**

11. Consider the circle on the right. The measure of minor arc CD is 114° and the measure of minor arc EB is 48°. The measure of $\angle CAD$ is _____.

(A) 32° **(B) 33°** **(C) 34°**
(D) 35° **(E) 36°**



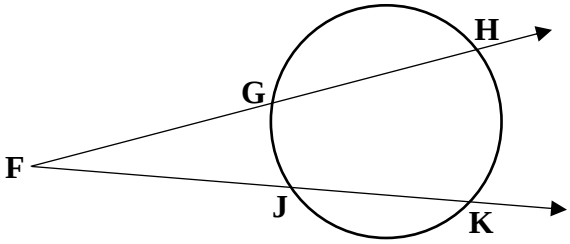
Problem 11

12. Consider equilateral triangle PQR with an inscribed circle. If the area of triangle PQR is 77.8, the area of the inscribed circle is _____. (nearest tenth)

(A) 45.8 **(B) 46.1** **(C) 46.4** **(D) 46.7** **(E) 47.0**

13. Consider the circle on the right. If FG = 18, GH = 16 and JK = 8, then FJ = _____. (nearest tenth)

(A) 20.7
(B) 20.9
(C) 21.1
(D) 21.3
(E) 21.5



Problem 13

14. Point C(4, 6) is rotated 90° counterclockwise about the origin to point E. Point E is reflected across the y-axis to point F. Point F is translated horizontally 15 units to the right to point G. If the coordinates of point G are (e, f), then $e+f =$ _____.

(A) 22 **(B) 23** **(C) 24** **(D) 25** **(E) 26**

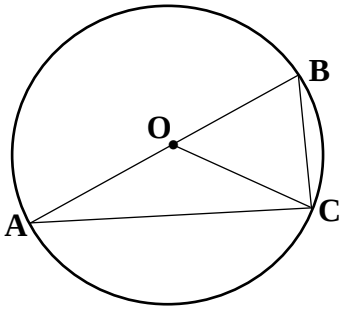
15. Consider kite ABCD. AB = BC = 12. CD = DA = 20. Diagonal AC = 12. Find the area of kite ABCD. (nearest whole number)

(A) 165 **(B) 168** **(C) 171** **(D) 174** **(E) 177**

16. The base of a pyramid is a regular hexagon with a perimeter of 72. The height of the pyramid is 18. Find the volume of the pyramid. (nearest whole number)

(A) 2236 **(B) 2239** **(C) 2242** **(D) 2245** **(E) 2248**

17-18. The circle shown on the right has an area of 452.
The measure of $\angle BAC$ is 30°.
Point O is the center of the circle.



Problems 17, 18

17. Find the area of triangle ABC. (nearest whole number)

(A) 122 **(B) 125** **(C) 128**
(D) 131 **(E) 134**

18. Find the area of sector BOC. (nearest tenth)

(A) 74.1 **(B) 74.4** **(C) 74.7** **(D) 75.0** **(E) 75.3**

19-20. The vertices of triangle DEF are $D(0, 8)$, $E(16, -4)$, and $F(14, 10)$.

19. Triangle DEF is a/an _____ triangle. Choose two of the given choices.

I. scalene II. isosceles III. equilateral IV. acute V. obtuse VI. right

(A) I, V **(B) II, IV** **(C) III, IV** **(D) I, VI** **(E) II, VI**

20. The area of triangle DEF is _____. (nearest whole number)

(A) 96 **(B) 98** **(C) 100** **(D) 102** **(E) 104**

21. A circle with center O has a diameter of 30. Chord $\overline{AB}$ is perpendicular to diameter $\overline{CD}$. If AB = 18, find the area bounded by chord $\overline{AB}$ and minor arc AB. (nearest tenth)

(A) 35.6 **(B) 35.9** **(C) 36.2** **(D) 36.5** **(E) 36.8**

22. The average price for a loaf of bread was \$0.22 on January 1, 1962. On January 1, 2022, the average price was up to \$2.50. If the price of bread is used to calculate the rate of inflation, what was the average annual rate of inflation from 1962 to 2022? (nearest hundredth)

(A) 4.09% **(B) 4.13%** **(C) 4.17%** **(D) 4.21%** **(E) 4.25%**

23. Consider an arithmetic sequence in which the fourth term is 31 and the twelfth term is 103. Find the sum of the first twenty-seven terms.

(A) 3261 **(B) 3263** **(C) 3265** **(D) 3267** **(E) 3269**

24. A math consultant earned \$80,000 his first year with Raytheon. Each year his salary increased by 8%. What was the total amount earned over the first 12 years? (nearest dollar)

(A) \$1,518,134 **(B) \$1,518,143** **(C) \$1,518,152** **(D) \$1,518,161** **(E) \$1,518,170**

25. A hawk is positioned on top of a tall building looking down at a mouse on the ground that is heading straight toward the base of the building at a constant speed of 6 inches per second. At t = 0, the angle of elevation from the mouse to the hawk is 15°. At t = 240 sec, the angle of elevation from the mouse to the hawk is 25°. How tall is the building? (nearest tenth)

(A) 75.2 ft (B) 75.4 ft (C) 75.6 ft (D) 75.8 ft (E) 76.0 ft

26. Consider a location on the earth at sea level and at a latitude of $36° 20' 10''$ north. If the radius of Earth is 3960 miles, then the linear speed of a point on the earth's surface at this location is _____. (nearest tenth)

(A) 832.9 mph (B) 834.0 mph (C) 835.1 mph (D) 836.2 mph (E) 837.3 mph

27. On Monday, there was heavy traffic at 7:30 AM and Kay only averaged 56 mph on her commute to work. She arrived 16 minutes late. On Tuesday, Kay left for work at 6:30 AM. Traffic was better and she averaged 70 mph and arrived on time. How far is her commute to work? (nearest tenth)

(A) 73.8 mi (B) 74.1 mi (C) 74.4 mi (D) 74.7 mi (E) 75.0 mi

28. Three of the roots of $f(x)$, a fourth-degree polynomial, are $-4, 5,$ and $1-\sqrt{3}$. If $f(x) = x^4 + bx^3 + cx^2 + dx + e$ and b, c, d and e are rational numbers, then $b+c+d+e =$ _____.

(A) 55 (B) 57 (C) 59 (D) 61 (E) 63

29. The range of the function $f(x) = \text{Cos}^{-1}(x)$ is _____.

(A) $[-1, 1]$ (B) $(-1, 1)$ (C) $[-\pi, \pi]$ (D) $(0, \pi)$ (E) $[0, \pi]$

30. If $\text{Sin}^{-1}(x) + \text{Sin}^{-1}(y) = \frac{\pi}{2}$, then $x^2 + y^2 =$ _____.

(A) 0.50 (B) 0.75 (C) 1.00 (D) 1.25 (E) 1.50

31. The coordinates of the foci of the hyperbola $4y^2 - x^2 + 8y - 4x - 4 = 0$ are (a, b) and (a, c). $|c - b| + a =$ _____. (nearest hundredth)

(A) 2.41 (B) 2.44 (C) 2.47 (D) 2.50 (E) 2.53

32. The perihelion of Earth's orbit about the sun is 147.095×10^6 km and the aphelion is 152.100×10^6 km. Find the eccentricity of Earth's orbit. (nearest ten-thousandth)

(A) 0.0167 (B) 0.0191 (C) 0.0215 (D) 0.0239 (E) 0.0263

33. Consider the sequence 5, 7, 11, 19, 35, 67,… The ninth term of the sequence is _____.

(A) 511 (B) 512 (C) 513 (D) 514 (E) 515

34. The graph of $x^2 + 4xy + 4y^2 - 1 = 0$ is _____.

(A) an ellipse (B) a hyperbola (C) a parabola (D) a line (E) two parallel lines

35. Planet X has 36-hour days and nine 40-day months. In month 5, the average temperature of the city of Xanadu varies sinusoidally each day with an average low temperature of 10° at t = 0 and an average high temperature of 90° at t = 18 hours. On average, the temperature is 60° or higher for _____ hours each day in month 5. (nearest tenth)

(A) 14.7 (B) 14.9 (C) 15.1 (D) 15.3 (E) 15.5

36. Consider the curve defined by the parametric equations $x(t) = e^{t+1}$ and $y(t) = e^{3t}$. The rectangular equation that represents the curve is $y = f(x)$. $f(10) =$ _____. (nearest tenth)

(A) 49.4 (B) 49.6 (C) 49.8 (D) 50.0 (E) 50.2

37. Consider the curve defined by the polar equation $r = \cot^2\theta\csc\theta$. The rectangular equation that represents the curve is $y = h(x)$. $h(8) =$ _____. (nearest tenth)

(A) 4.0 (B) 4.3 (C) 4.6 (D) 4.9 (E) 5.2

38. A 1.00-meter-long string is cut into two pieces. One piece is formed into a circle and the other piece is formed into a square. If the area of the circle is equal to the area of the square, what is the diameter of the circle? (nearest tenth)

(A) 15.0 cm (B) 15.2 cm (C) 15.4 cm (D) 15.6 cm (E) 15.8 cm

39. The general form of the equation of the plane passing through the point $P(2, 3, 5)$ and perpendicular to the vector $n = 7i - 4j + 6k$ is $ax + by + cz + d = 0$. $a + b + c + d =$ _____.

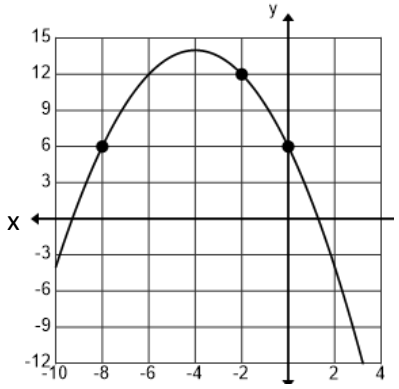
(A) –25 (B) –23 (C) –21 (D) –19 (E) –17

40. The equation of the directrix of the graph shown on the right is y = k. k = _____.

(A) 14.25 (B) 14.5 (C) 14.75
(D) 15 (E) 15.25

41. The shortest distance from the origin to the graph of the parabola is _____. (nearest hundredth)

(A) 1.21 (B) 1.23 (C) 1.25
(D) 1.27 (E) 1.29



Problems 40, 41

x	0	2	4	6	8	10
f(x)	6	11	20	21	2	-49

42. The table above gives selected values for the differentiable function f. In which of the following intervals must there be a number c such that $f'(c) = -9.5$?

(A) (0, 2) **(B) (2, 4)** **(C) (4, 6)** **(D) (6, 8)** **(E) (8, 10)**

43. The line normal to the curve $y = .25x^3 - x^2 + .75x - 1$ at x = 3 intersects the x-axis at the point (a, 0). a = _____. (nearest hundredth)

(A) 1.17 **(B) 1.28** **(C) 1.39** **(D) 1.50** **(E) 1.61**

44. Let f be a continuous function such that $\int_0^{15} f(x)dx = 12$, $\int_{15}^{20} f(x)dx = -4$, and $\int_{10}^{20} f(x)dx = 6$.

What is the value of $\int_0^{10} f(x)dx$?

(A) 2 **(B) 4** **(C) 6** **(D) 8** **(E) 10**

45. A particle moves along the x-axis so that at time $t \geq 0$ its acceleration is given by $a(t) = 6t$. At t = 0, the velocity of the particle is -6 and the position of the particle is 3. How far did the particle travel from t = 0 to t = 4? (nearest whole number)

(A) 45 **(B) 46** **(C) 47** **(D) 49** **(E) 51**

46. Consider the first quadrant region bounded above by the curve $y = 6 + (0.01)e^{3x}$, below by the x-axis, on the left by the y-axis and on the right by the line $x = 2$. Find the periphery of this region. (nearest tenth)

(A) 23.1 **(B) 23.4** **(C) 23.7** **(D) 24.0** **(E) 24.3**

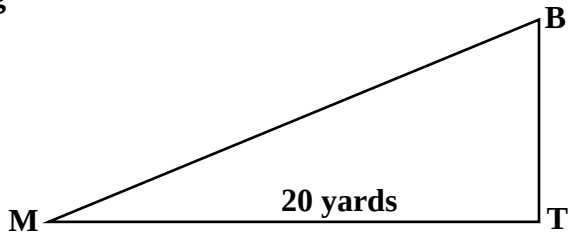
47. A north-south jogging trail intersects an east-west road at point D. At t = 0, a runner leaves point D traveling south at 7.5 mph. At the same time, a car is one mile east of point D traveling west at 30 mph. The distance between the runner and the car is a minimum at t = _____. (nearest tenth)

(A) 111.8 sec **(B) 112.9 sec** **(C) 114.0 sec** **(D) 115.1 sec** **(E) 116.2 sec**

48. Given: the parametric equations $x(t) = 2\sqrt{t}$ and $y(t) = \frac{t^2 - 1}{2}$. Find the value of $\frac{d^2y}{dx^2}$ when $t = 4$.

(A) –4 **(B) 2** **(C) 4** **(D) 6** **(E) 8**

49. Teresa (T) was 20 yards from a mouse (M) when she released a balloon (B) from the ground that began rising vertically at a constant rate of 10 feet per second. The rate of change of $\angle BMT$ at the instant the balloon is 80 feet above the ground is _____ rad/s. (nearest thousandth)



Problem 49

(A) 0.056 **(B) 0.060** **(C) 0.064**
(D) 0.068 **(E) 0.072**

50. The length of the curve $y = 3\cos\left(\frac{x}{4}\right)$ from $x = \pi$ to $x = 2\pi$ is given by

(A) $\int_{\pi}^{2\pi} \sqrt{1+\frac{9}{16}\cos^2\left(\frac{x}{4}\right)}\,dx$ **(B)** $\int_{\pi}^{2\pi} \sqrt{1+3\cos\left(\frac{x}{4}\right)}\,dx$ **(C)** $\int_{\pi}^{2\pi} \sqrt{1+12\sin^2\left(\frac{x}{4}\right)}\,dx$

(D) $\int_{\pi}^{2\pi} \sqrt{\left(1-\frac{3}{4}\sin\left(\frac{x}{4}\right)\right)}\,dx$ **(E)** $\int_{\pi}^{2\pi} \sqrt{1+\frac{9}{16}\sin^2\left(\frac{x}{4}\right)}\,dx$

51. The function $y = f(t)$ models the amount of a substance present at time t, in years. On March 1, 2022, (t = 0), there was 100 g present. The function satisfies the differential equation $\frac{dy}{dt} = -0.005y^2$. Find $f(t)$ and determine the amount present on March 1, 2040. (nearest gram)

(A) 10 g **(B) 20 g** **(C) 25 g** **(D) 50 g** **(E) 75 g**

52. The following list gives the five-number summary for the maximum bench press for a large group of Texas high school football players. 184, 220, 240, 320, 480
About what percent of the players had a maximum bench press between 220 and 480 pounds?

(A) 50% **(B) 67%** **(C) 75%** **(D) 86%** **(E) 95%**

53. The Potter County Sheriff's Department reports that its response time to emergency calls is approximately normally distributed with a mean of 18 minutes and a standard deviation of 6.5 minutes. Approximately what proportion of response times are over 25 minutes? (nearest hundredth)

(A) 0.14 **(B) 0.16** **(C) 0.18** **(D) 0.20** **(E) 0.22**

	Mean	Standard Deviation
# of workouts per year	150	70
Annual medical costs	$2000	$600

54. An insurance company did a study of a large group of females aged 30 to 35. The results of the study are in the table above. The correlation for these two variables is –0.91. Find the equation of the least-squares regression line and find the predicted annual medical cost for a female in this age group who works out 300 days per year. (nearest dollar)

(A) $812 **(B) $818** **(C) $824** **(D) $830** **(E) $836**

55. Over 2000 high school seniors in Ada County took the ACT test this school year. The scores closely followed the Normal distribution with a mean of 21.6 and a standard deviation of 4.6. Two students were selected at random from the group. Find the probability that the difference in their scores was greater than 7. (nearest thousandth)

(A) 0.141 (B) 0.176 (C) 0.212 (D) 0.247 (E) 0.282

56. Cindy performed 12 independent tests of the form $H_0 : \mu = 36$ versus $H_a : \mu < 36$, each at the $\alpha = 0.05$ significance level. What is the probability of committing a Type I error with at least 3 of the 12 tests? (nearest hundredth)

(A) 0.01 (B) 0.02 (C) 0.03 (D) 0.04 (E) 0.05

Type	Apple	Cherry	Apricot	Lemon
Frequency	28	20	36	16

57-58. A researcher in Big Timber, Montana wanted to test the claim that an equal proportion of people preferred each of the four types of pies listed in the table above. He surveyed a random sample of 100 people and asked each person to identify their favorite type of pie. The results are in the table above. Assume all conditions have been met to perform a chi-square test with the null hypothesis, H_0: favorite types of pies are evenly distributed across the four types, and with $\alpha = 0.05$.

57. Based on a p-value of _____, he rejected H_0 at the $\alpha = 0.05$ level. (nearest thousandth)

(A) 0.012 (B) 0.018 (C) 0.024 (D) 0.030 (E) 0.036

58. The apricot cell contributed _____ to the chi-square statistic. (nearest hundredth)

(A) 4.40 (B) 4.51 (C) 4.62 (D) 4.73 (E) 4.84

59. Give the correct order of the following from least to greatest in a normal curve.
I. 1st quartile II. value of the 30th percentile III. value of a z-score of -1

(A) I, II, III (B) I, III, II (C) III, II, I (D) III, I, II (E) II, I, III

Return	–5.0%	5.0%	15.0%	25.0%	35.0%
Probability	0.12	0.14	0.22	0.24	0.28

60. Cantu Investments handles accounts for people agreeing to invest a minimum of two million dollars. The average annual return on investments over the last twenty years is given in the table above. How much should Audrey expect to earn in one year if she invests $3,000,000?

(A) $565,000 (B) $576,000 (C) $587,000 (D) $598,000 (E) $609,000

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University Interscholastic League
MATHEMATICS CONTEST
HS ● State ● 2023
Answer Key

1. E	**21. E**	**41. D**
2. D	**22. B**	**42. D**
3. D	**23. D**	**43. D**
4. B	**24. E**	**44. A**
5. B	**25. C**	**45. E**
6. C	**26. C**	**46. A**
7. D	**27. D**	**47. B**
8. A	**28. C**	**48. D**
9. B	**29. E**	**49. B**
10. C	**30. C**	**50. E**
11. B	**31. C**	**51. A**
12. E	**32. A**	**52. C**
13. C	**33. E**	**53. A**
14. D	**34. E**	**54. D**
15. E	**35. C**	**55. E**
16. D	**36. C**	**56. B**
17. B	**37. A**	**57. C**
18. E	**38. A**	**58. E**
19. E	**39. B**	**59. D**
20. C	**40. B**	**60. B**