



UNIVERSITY INTERSCHOLASTIC LEAGUE

# Mathematics

District • 2024



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1. Tommy took Telena to the Trail Dust for her birthday. He ordered the 8-oz Center Cut Filet for \$23.99 and iced tea for \$2.15. She ordered the 16-oz Ribeye for \$26.99 and iced tea for \$2.15. They shared a slice of chocolate cake which cost \$6.99. The tax rate was 8.125%. Tommy paid with four \$20 bills and he told the waitress to keep the change as a tip. How much was her tip?
- (A) \$12.34      (B) \$12.45      (C) \$12.56      (D) \$12.67      (E) \$12.78
2. Coach Hood persuaded the Canadian Education Foundation to purchase some new calculators for the High School and the Middle School UIL teams. A Swiss DM 32 calculator cost \$249.00 and an HP Prime calculator cost \$144.99. The foundation agreed to purchase a total of 40 calculators at a total cost of \$8087.82. How many Swiss DM 32 calculators did they purchase?
- (A) 20      (B) 21      (C) 22      (D) 23      (E) 24
3. A round trip ticket on American Airlines from Dallas to Idaho Falls cost \$840. Avis will rent a RAV4 for \$57 per day. Arlene estimates that food and other expenses will cost \$55 per day. She has budgeted \$2500 and she wants to stay as long as possible. According to her projected costs, what is the maximum length of her stay in Idaho if she does not exceed her budget?
- (A) 10 days      (B) 12 days      (C) 14 days      (D) 16 days      (E) 18 days
4. Last summer, Anthony took a 4-day trip from Seminole to Astoria, Oregon. The total distance was 1785 miles and the total drive time was 27 hr 37 min. On day one, he traveled 525 miles at an average speed of 63 mph. On day two, he traveled 475 miles at an average speed of 67 mph. On day three, he traveled 425 miles at an average speed of 71 mph. What was his average speed on day four? (nearest whole number)
- (A) 58 mph      (B) 60 mph      (C) 62 mph      (D) 64 mph      (E) 66 mph
5. Melania left her home in Afton and cycled due north for 30 minutes at an average speed of 26 mph. Then she turned east and cycled for 45 minutes at an average speed of 24 mph. Next, she turned south and cycled for 75 minutes at an average speed of 22 mph. She stopped for lunch and calculated that she was \_\_\_\_\_ miles from home. (nearest tenth)
- (A) 23.1      (B) 23.4      (C) 23.7      (D) 24.0      (E) 24.3
6. Diezel can milk 8 cows in 50 minutes. Zeven can milk 12 cows in 70 minutes. Monday morning, Diezel began milking cows at The Afton Dairy at 5:00 AM. Zeven arrived at 6:45 AM and they worked together until all 97 cows at the dairy had been milked. What time was it when they finished? (nearest minute)
- (A) 10:35 AM      (B) 10:41 AM      (C) 10:47 AM      (D) 10:53 AM      (E) 10:59 AM
7. Consider the function  $f(x) = \frac{7-3x}{9-4x}$ . If  $g(x)$  is the inverse function of  $f(x)$ , then  $g(1) =$  \_\_\_\_\_.
- (nearest tenth)
- (A) 1.4      (B) 1.6      (C) 1.8      (D) 2.0      (E) 2.2

8. Walt made a 176 on Test A, a 182 on Test B, a 184 on Test C, a 186 on Test D and a 198 on Test E. What score will he need to make on Test F to have an overall average of 190?

- (A) 212                      (B) 214                      (C) 216                      (D) 218                      (E) 220

9. Dylan has a small farm west of Brock where he raises frogs, hogs and dogs. The number of frogs is 7 more than 5 times the number of hogs. The number of hogs is 3 times the number of dogs. If there are a total of 121 creatures to take care of on the farm, how many frogs does Dylan have?

- (A) 91                      (B) 93                      (C) 95                      (D) 97                      (E) 99

10. The area of  $\triangle ABC$  is \_\_\_\_\_. (nearest tenth)

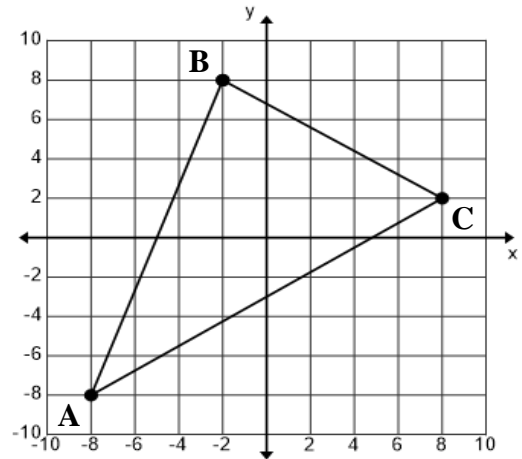
- (A) 97.2                      (B) 97.4                      (C) 97.6  
(D) 97.8                      (E) 98.0

11. The graph of  $y = f(x)$  is the perpendicular bisector of  $\overline{AB}$ .  $f(-21) =$  \_\_\_\_\_. (nearest tenth)

- (A) 5.8                      (B) 6.0                      (C) 6.2  
(D) 6.4                      (E) 6.6

12. The length of the longest median of  $\triangle ABC$  is \_\_\_\_\_. (nearest tenth)

- (A) 17.0                      (B) 17.2                      (C) 17.4  
(D) 17.6                      (E) 17.8



Problems 10, 11, 12

13-14. Consider  $\triangle DEF$  with  $DE = 12$ ,  $EF = 9$ , and  $m\angle DEF = 90^\circ$ . Point  $G$  lies on  $\overline{DF}$  such that  $m\angle EGF = 90^\circ$ .

13.  $DG =$  \_\_\_\_\_. (nearest tenth)

- (A) 7.2                      (B) 7.8                      (C) 8.4                      (D) 9.0                      (E) 9.6

14. The area of  $\triangle EGF =$  \_\_\_\_\_. (nearest hundredth)

- (A) 19.22                      (B) 19.33                      (C) 19.44                      (D) 19.55                      (E) 19.66

15. Consider  $\triangle HIJ$  with  $HI = 16$ ,  $IJ = 25$ , and  $m\angle HIJ = 56^\circ$ . Point  $K$  lies on  $\overline{HJ}$  such that ray  $\overrightarrow{IK}$  bisects  $\angle HIJ$ . If  $HK = 8.1265$ , then  $KJ =$  \_\_\_\_\_. (nearest tenth)

- (A) 12.1                      (B) 12.3                      (C) 12.5                      (D) 12.7                      (E) 12.9

16. If the area of regular hexagon  $ABCDEF$  is 62.3798, then  $AE =$  \_\_\_\_\_. (nearest tenth)

- (A) 7.7                      (B) 7.9                      (C) 8.1                      (D) 8.3                      (E) 8.5

17. The total area of a pyramid with a square base is 464.49. If the diagonal of the base is 16.55, what is the volume of the pyramid? (nearest whole number)
- (A) 577            (B) 580            (C) 583            (D) 586            (E) 589
18. Consider an arithmetic sequence in which 5 times the third term equals the 13<sup>th</sup> term. If the 8<sup>th</sup> term is 60, what is the sum of the first 18 terms?
- (A) 1280            (B) 1284            (C) 1288            (D) 1292            (E) 1296
19. The frequency of a vibrating string, fixed at both ends, varies directly with the square root of the tension the string is under and inversely with the effective length of the string. A string that was initially under a tension of 60 N, with an effective length of 42 cm, produced sound with a frequency of 512 Hz. What will the frequency be if the tension is increased to 120 N and the effective length is reduced to 36 cm? (nearest whole number)
- (A) 833 Hz            (B) 836 Hz            (C) 839 Hz            (D) 842 Hz            (E) 845 Hz
20. A ball is dropped from a height of 18 feet. On each bounce, it rebounds two-thirds of the distance it fell. How far does the ball fall on its 10<sup>th</sup> fall? (nearest tenth)
- (A) 4.0 in            (B) 4.4 in            (C) 4.8 in            (D) 5.2 in            (E) 5.6 in
21. Two different samples of radioactive materials are decaying. Sample A has an initial mass of 3.22 g and has a half-life of 96 hours. Sample B has an initial mass of 6.86 g and has a half-life of 64 hours. Find the time required for the remaining amount in sample A to equal the remaining amount in sample B. (nearest tenth)
- (A) 207.3 hr            (B) 208.4 hr            (C) 209.5 hr            (D) 210.6 hr            (E) 211.7 hr
22. Jacob left port at 1:00 PM and sailed due east at 18 mph. Benjamin left port at 2:00 PM and sailed at 30° south of west at 15 mph. How far apart are they at 6:00 PM? (nearest whole number)
- (A) 143 mi            (B) 145 mi            (C) 147 mi            (D) 149 mi            (E) 151 mi
- 23-24. The swimming pool at Teresa's house has a pipe that can fill the pool in 72 hours. The drain can empty the pool in 90 hours. The pool is shaped like a regular pentagon with each side 24 feet long and it has a constant depth of 4 feet. The pool was empty at  $t = 0$ .
23. The fill pipe was turned on at  $t = 0$ . The drain was accidentally opened 12 hours later. The pool was finally full at  $t =$  \_\_\_\_\_ hours. (nearest whole number)
- (A) 296            (B) 300            (C) 304            (D) 308            (E) 312
24. When the pool is completely full, how many gallons of water does it hold? (nearest whole number)
- (A) 29,653            (B) 29,657            (C) 29,661            (D) 29,665            (E) 29,669

25.  $\frac{36(\cos 150^\circ + i \sin 150^\circ)}{9(\cos 30^\circ + i \sin 30^\circ)} = \underline{\hspace{2cm}}$ .

- (A)  $-2 - 2\sqrt{3}i$     (B)  $-2\sqrt{3} + 2i$     (C)  $-2 + 2\sqrt{3}i$     (D)  $-2\sqrt{3} - 2i$     (E)  $2 + 2\sqrt{3}i$

26. Grandpa told Michael that he would pay for his first year of graduate school at A&M. The current estimated cost for one year is \$29,266. Michael anticipates entering graduate school in 4 years. How much will Grandpa need to place in an account that earns 6% annual interest compounded monthly to pay for Michael's first year if the estimated cost does not increase? (nearest dollar)

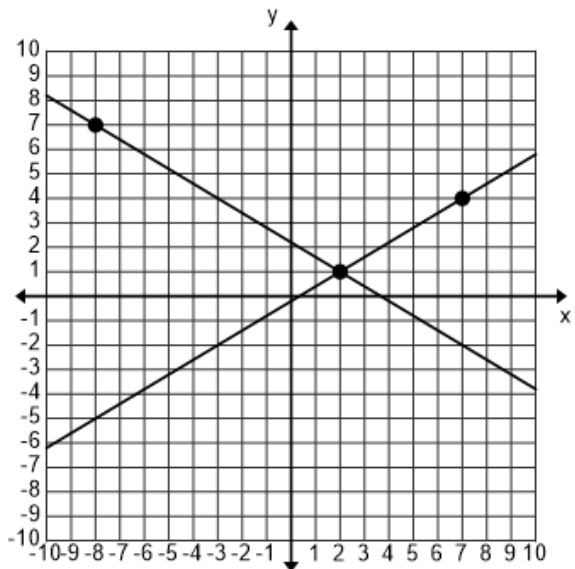
- (A) \$23,002    (B) \$23,013    (C) \$23,024    (D) \$23,035    (E) \$23,046

27. Find the acute angle formed by the two intersecting lines shown on the right. (nearest tenth)

- (A)  $61.3^\circ$     (B)  $61.6^\circ$     (C)  $61.9^\circ$   
 (D)  $62.2^\circ$     (E)  $62.5^\circ$

28. The two lines are the asymptotes of a hyperbola. The equation of the hyperbola is of the form  $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ . Given: a and b are integers with  $a < b < 9$ . One of the foci is the point  $(h, k + c)$ .  $a + b + c = \underline{\hspace{2cm}}$ . (nearest tenth)

- (A) 13.0    (B) 13.2    (C) 13.4  
 (D) 13.6    (E) 13.8



Problems 27, 28

29. If the area of the circle  $x^2 + y^2 - 12x + 4y + f = 0$  is  $49\pi$ , then  $f = \underline{\hspace{2cm}}$ .

- (A) -15    (B) -12    (C) -9    (D) -6    (E) -3

30. Three of the zeros of  $f(x) = x^4 + bx^3 + cx^2 + dx + h$  are  $-4, 3,$  and  $1 + \sqrt{7}$ . If b, c, d, and h are integers, then  $f(4) = \underline{\hspace{2cm}}$ .

- (A) 16    (B) 18    (C) 20    (D) 22    (E) 24

31. Russell shoots free throws every day after practice. On Tuesday, after 15 minutes, he had made only 60% of his free throws. At this point, Becci came into the gym and Russell began to focus. He got on a hot streak and made 40 free throws in a row. If this increased his free throw percentage for the day to 70%, how many free throws did he attempt on Tuesday?

- (A) 154    (B) 156    (C) 158    (D) 160    (E) 162

32. If  $f(x) = 2x^2 - 3$  and  $h(x) = x^2 \div 25$ , then  $(h \circ f)(-3) = \underline{\hspace{2cm}}$ .

- (A) 3    (B) 6    (C) 9    (D) 12    (E) 15

33. The angle of elevation from a mouse on the ground to a hawk on the edge of the roof of the Canadian State Bank was  $36^\circ$ . The mouse cautiously moved toward the bank and 80 seconds later, the angle of elevation had changed to  $48^\circ$ . If the height of the bank building is 126 feet, at what rate did the mouse move toward the bank? (nearest tenth)

- (A) 8.8 in/s      (B) 9.0 in/s      (C) 9.4 in/s      (D) 9.6 in/s      (E) 9.8 in/s

34. A parabola is concave down with the point  $(-3, -2)$  as the vertex. The point  $(3, -11)$  lies on the parabola. If the point  $(-6, b)$  also lies on the parabola, then  $b =$  \_\_\_\_\_. (nearest hundredth)

- (A)  $-5.00$       (B)  $-4.75$       (C)  $-4.50$       (D)  $-4.25$       (E)  $-4.00$

35-36. Consider  $\triangle ABC$  with vertices  $A(-4, 3, 5)$ ,  $B(2, -5, 3)$ , and  $C(7, -6, 8)$ .

35.  $m\angle BAC =$  \_\_\_\_\_. (nearest tenth)

- (A)  $26.8^\circ$       (B)  $27.0^\circ$       (C)  $27.2^\circ$       (D)  $27.4^\circ$       (E)  $27.6^\circ$

36. Find the area of  $\triangle ABC$ . (nearest tenth)

- (A) 33.6      (B) 33.9      (C) 34.2      (D) 34.5      (E) 34.8

37. The point in rectangular coordinates,  $(a, 5)$ , is on the polar graph  $r^2 \sin(2\theta) = 24$ .  $a =$  \_\_\_\_\_. (nearest tenth)

- (A) 2.0      (B) 2.1      (C) 2.2      (D) 2.3      (E) 2.4

38. Consider the unit circle with an angle  $\theta$  in quadrant III such that  $\cos\theta = -0.5$  and with an angle  $\alpha$  also in quadrant III such that  $\sin\alpha = -0.5$ .  $\cos(\theta - \alpha) =$  \_\_\_\_\_.

- (A)  $-\frac{\sqrt{3}}{2}$       (B)  $-\frac{1}{2}$       (C) 0      (D)  $\frac{1}{2}$       (E)  $\frac{\sqrt{3}}{2}$

39-40. Consider the conic  $5x^2 + 4xy + 3y^2 - 32 = 0$ .

39. The graph of the conic is \_\_\_\_\_.

- (A) an ellipse      (B) a parabola      (C) a hyperbola      (D) a line      (E) 2 parallel lines

40. The angle of rotation of the graph of the conic is \_\_\_\_\_. (nearest tenth)  
(nearest tenth)

- (A)  $30.9^\circ$       (B)  $31.1^\circ$       (C)  $31.3^\circ$       (D)  $31.5^\circ$       (E)  $31.7^\circ$

41. The slope of the line tangent to the graph of  $y = f(x)$  at  $x = -2$  is \_\_\_\_\_.  
(nearest tenth)

- (A) 2.0                      (B) 2.1                      (C) 2.2  
(D) 2.3                      (E) 2.4

42. Find the area bounded by the graphs of  $y = f(x)$  and  $y = g(x)$ . (nearest tenth)

- (A) 28.0                      (B) 28.2                      (C) 28.4  
(D) 28.6                      (E) 28.8

43. Find the volume of the solid generated when the region bounded by the graphs of  $y = f(x)$  and  $y = g(x)$  is revolved about the line  $y = -8$ . (nearest whole number)

- (A) 1744                      (B) 1748                      (C) 1752                      (D) 1756                      (E) 1760

44. What is the arc length of  $y = f(x)$  between  $x = -3$  and  $x = 1$ ? (nearest hundredth)

- (A) 12.73                      (B) 12.76                      (C) 12.79                      (D) 12.82                      (E) 12.85

45. Given:  $\frac{dy}{dx} = 4 - e^{-x} - y$  and  $y(0) = 1$ . Use Euler's method with a step size of  $h = 0.1$  to approximate the value of  $y(0.2)$ . (nearest thousandth)

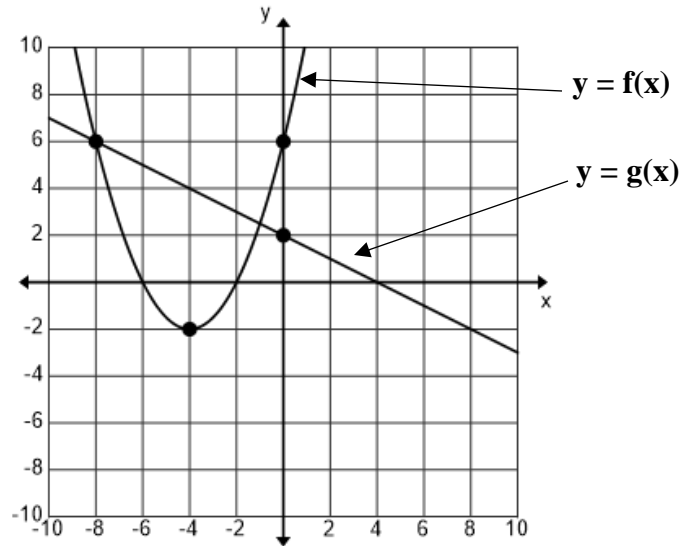
- (A) 1.386                      (B) 1.388                      (C) 1.390                      (D) 1.392                      (E) 1.394

46. Given:  $F(x) = \int_0^{3x} \cos(2t)dt$ .  $F'(2) =$  \_\_\_\_\_. (nearest hundredth)

- (A) 2.53                      (B) 2.55                      (C) 2.57                      (D) 2.59                      (E) 2.61

47. Find the average value of  $f(x) = 2\sin(3x) - 0.1e^{0.2x}$  over the interval  $[0, 12]$ . (nearest thousandth)

- (A) -0.366                      (B) -0.355                      (C) -0.344                      (D) -0.333                      (E) -0.322



Problems 41, 42, 43, 44

48-49. Suppose that the quail population in Hemphill County was essentially zero in 2019. Suppose also that the Panhandle Conservation Club released 100 quail into Hemphill County on March 1, 2020. On March 1, 2023, the population reached 180 quail. Professors from TAMU in Canyon estimate that Hemphill County can sustain no more than 500 quail.

48. Find a logistic differential equation that models the rate of change of the quail population. The greatest rate of increase of the quail population according to the model is \_\_\_\_\_ quail per year. (nearest whole number)

- (A) 28                      (B) 30                      (C) 32                      (D) 34                      (E) 36

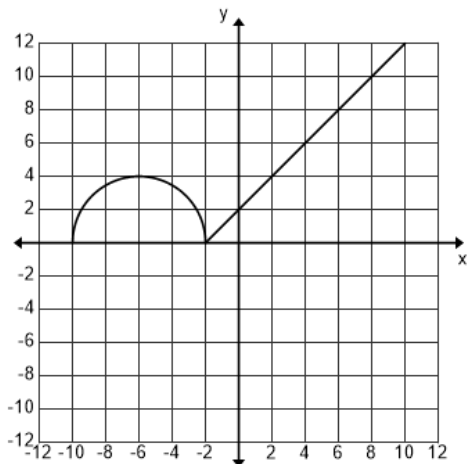
49. Find a general solution to this logistic differential equation that models the quail population at any time  $t$ . This model predicts that the quail population on March 1, 2040 will be \_\_\_\_\_ quail. (nearest whole number)

- (A) 483                      (B) 485                      (C) 487                      (D) 489                      (E) 491

50. Find the interval of convergence of  $\sum_{n=0}^{\infty} \frac{(-1)^n (x+2)^n}{3^n}$ .

- (A)  $(-2, 2)$                       (B)  $[-5, 1]$                       (C)  $(-1, 5)$                       (D)  $(-5, 1)$                       (E)  $(-\infty, \infty)$

51-52. The continuous function  $f$  shown on the right is defined for  $-10 \leq x \leq 10$ . Let  $h$  be the function defined by  $h(x) = \int_{-10}^x f(t) dt$ .



51. Find  $h(10) =$  \_\_\_\_\_. (nearest whole number)

- (A) 89                      (B) 91                      (C) 93  
(D) 95                      (E) 97

52. The graph of  $h$  is concave down over the interval  $(a, b)$ .  $a + b =$  \_\_\_\_\_.

- (A) -14                      (B) -12                      (C) -10                      (D) -8                      (E) -6

Problems 51, 52

53. The OEA reported in January that the mean SAT score for the 2024 seniors at Mac High was 980 with a standard deviation of 120. In March, the OEA said an error had been found in the scoring and the corrected scores could be found by adding 20 points to your original score and then multiplying by 1.1. If Pistol's corrected score was 1280, what percentile does that put her in?

- (A) 91st                      (B) 93rd                      (C) 95th                      (D) 97th                      (E) 99th



54-55. Assume that the mean height of the trees in the California Redwoods National Park is 380 feet with a standard deviation of 20 feet.

54. What proportion of the trees are over 400 feet tall? (nearest hundredth)

- (A) 0.16            (B) 0.18            (C) 0.20            (D) 0.22            (E) 0.24

55. If a group of 12 trees is randomly selected, what is the probability that at least 4 of the trees will be taller than 400 feet tall? (nearest hundredth)

- (A) 0.11            (B) 0.13            (C) 0.15            (D) 0.17            (E) 0.19

56. Are students at A&M more fit than the students at UT? A random sample of 180 students at A&M found that 122 of them exercise regularly. A random sample of 200 students at UT found that 114 of them exercise regularly. Calculate a 96% confidence interval for the difference between the proportions of students at the two universities who exercise regularly.

- (A) {.0046, .2074} (B) {.0052, .2080} (C) {.0058, .2086} (D) {.0064, .2092} (E) {.0070, .2098}

57. Suppose 62% of the adult men over 40 are overweight. Of these, 36% are on an exercise program. Of the adult men over 40 who are not overweight, 44% are on an exercise program. Given that an adult man over 40 is on an exercise program, what is the probability that he is overweight? (nearest hundredth)

- (A) 0.51            (B) 0.54            (C) 0.57            (D) 0.60            (E) 0.63

58. Randy and Tommy are retired buddies who play golf together 3 days a week. The distribution of Randy's drives off the tee is approximately normal with a mean of 260 yd and a standard deviation of 8 yd. The distribution of Tommy's drives off the tee is also normal, with a mean of 280 yd and a standard deviation of 12 yd. What is the probability that Randy will drive the ball off the tee farther than Tommy on a randomly selected hole on the golf course? (nearest hundredth)

- (A) 0.08            (B) 0.11            (C) 0.14            (D) 0.17            (E) 0.20

# of tests	3	6	9	12	15	18
Meet Score	112	133	152	176	193	215

59-60. Mr. Newberry has a student who is very talented in Number Sense, but has never practiced very much. Mr. Newberry convinced him to increase the number of practice tests he takes each week by 3 after each meet. The results of the first 6 meets are in the table above. Mr. Newberry plotted the data in the table and calculated a LSRL for the data.

59. Find the value of the residual for the week the student takes 12 practice tests. (nearest tenth)

- (A) 1.4            (B) 1.6            (C) 1.8            (D) 2.0            (E) 2.2

60. Mr. Newberry convinces the student to take 30 practice tests the week of the state meet. Use the LSRL to predict the student's score at the state meet. (nearest whole number)

- (A) 291            (B) 293            (C) 295            (D) 297            (E) 299