# Problem Sequencing UIL High School Number Sense Test

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#### Problem 1 - 20 \*

- 1) Addition, subtraction, multiplication, & division of Integers, Mixed Numbers, Fractions, and Decimals
- 2) Order of Operations
- 3) Use of the Distributive Property
- 4) Comparison of Fractions and Decimals
- 5) Multiplication Short-Cuts
- 6) Squaring Numbers
- 7) Conversion Problems (either way):

Percent/Fractions, English/Metric, Roman Numerals/Arabic Numerals, Measurement units

(length, weight, capacity, time)

- 8) Greatest Common Divisor (GCD) and Least Common Multiple (LCM)
- 9) Percent Problems
- 10) Mean, Median, & Mode
- 11) Sums of Integers
- 12) Remainder Problems
- 13) Consumer Type Problems
- 14) Number Theory Problems Involving: Prime Numbers, Divisors, Sums of Divisors, etc.

#### **Problems 21 - 40 \***

- 1) Powers of Numbers
- 2) Substitution
- 3) Word Problems
- 4) Inverses
- 5) Absolute Value
- 6) Ratio/Proportion
- 7) Square Roots/Cube Roots
- 8) Sets
- 9) Base System Conversion Problems
- 10) Solving Simple Equations
- 11) Systems of Equations
- 12) Repeating Decimals to Fractions
- 13) More Remainder Type Problems
- 14) Perimeter & Area of Polygons and Circles
- 15) Sequences
- 16) Quadratic & Cubic Equation Problems

#### **Problems 41 - 60 \***

- 1) Laws of Exponents
- 2) Right Triangle Problems
- 3) Coordinate Geometry Problems
- 4) Regular Polygon Problems
- 5) Inequalities
- 6) Applications of Theorems from Geometry
- 7) Direct and Inverse Variation
- 8) Sequences & Series (Finite & Infinite)
- 9) Complex Numbers
- 10) Logarithms & Logarithmic Equations
- 11) Factorials, Permutations, & Combinations
- 12) Probability/Odds
- 13) Conics
- 14) Binomial Theorem (Expansion)
- 15) Base System Problems Using Operations
- 16) Roots of equations
- 17) Polygonal numbers

#### **Problems 61 - 70 \***

- 1) Volume & Surface Area
- 2) Greatest Integer
- 3) Application of Remainder Theorem
- 4) Trigonometry
- 5) Determinants
- 6) Matrices
- 7) Vectors
- 8) Composite Functions
- 9) Bases Involving Decimals or Fractions
- 10) Polar/Rectangular Coordinates

#### **Problems 71 -80 \***

- 1) Function domains and ranges
- 2) Modular Arithmetic
- 3) Limits
- 4) Derivatives
- 5) Slopes of Tangent Lines
- 6) Horizontal & Vertical Asymptotes
- 7) Determining Critical Values
- 8) Maximum & Minimum Problems
- 9) Definite Integration
- 10) Inverse functions
- \* A type of problem from a particular section could appear later in the test.

Example: A base problem could appear as problem #55, but should not appear earlier than problem #21.

# UIL Number Sense Contest

# Basic Ideas, Shortcuts and Problems #1-20 from the Sequence Chart

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# What Pops into Your Mind?

- 1. 1728
- 2. 512
- 3. 1331
- 4. 289
- **5. 6.25%**
- 6. 2.828...
- 7. 3.141...
- 8. 2.718...
- 9. 1.618...
- 10. 0.142857142857142857...

Mental Math -- How fast can you work these?

5. 
$$40 \div 0.625 =$$

6. 
$$33\frac{1}{3}\%$$
 of  $60 =$ 

7. 
$$31^2 =$$

# Math Magic (Number Sense Strategies (Tricks) )

- A. Memorize the first 35 squares, the first 15 cubes, and the square roots of 2, 3, 5, 6, 7, 8, & 10.
- B. Know the "One-sies" equivalents. (Fractions-Decimals-Percents)  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{6}$ ,  $\frac{1}{7}$ ,  $\frac{1}{8}$ ,  $\frac{1}{9}$ ,  $\frac{1}{10}$ ,  $\frac{1}{11}$ ,  $\frac{1}{12}$ ,  $\frac{1}{16}$
- C.  $\frac{3}{5} + \frac{5}{3} = ?$  (Is it a trick? Is it magic? See proof on page 4)
- D. Find the average of 25, 36, and 47 using a focus number.
- E. LCM (24, 42) is?
- F. Write 0.1222... as a fraction.
- G.  $(37 \times 13 + 19) \div 8$  has a remainder of?

H. 
$$35 \times 35 = ?$$
  $35 \times 45 = ?$   $35 \times 55 = ?$   $35 \times 65 = ?$ 

I. 
$$\frac{13}{16} \times 13 = ?$$

J. 
$$53 \times 47 = ?$$

K. Change 234 base 5 to base 10.

L. 
$$36^2 + 57^2 = ?$$

# Math Magic (solutions and tricks)

C. 
$$\frac{3}{5} + \frac{5}{3} = 2 \frac{4}{15}$$
 (Is it magic?)

$$\frac{a}{b} + \frac{b}{a}$$
 Proof

Let 
$$x = \frac{a}{b} + \frac{b}{a}$$

$$\mathbf{x} = \frac{(\mathbf{a}^2 + \mathbf{b}^2)}{\mathbf{a}\mathbf{b}}$$

(common denominator)

$$x-2=\frac{(a^2+b^2)}{ab}-2$$

(subtract 2 from both sides)

$$x-2=\frac{(a^2+b^2-2ab)}{ab}$$

(common denominator)

$$x-2=\frac{(a-b)^2}{ab}$$

(binomial square)

$$x=2+\frac{(a-b)^2}{ab}$$

(solve for x)

D. The average of 25, 36, and 47 is 36.

Using 35 as a focus number,

add 10 to 25; subtract 1 from 36; subtract 12 from 47

$$-10+1+12=3$$
.

Since 3 divided by three numbers is 1, then 35 + 1 = 36.

E. LCM(24, 42) = 168

$$24 \div 6 = 4$$
 and  $4 \times 42 = 168$ 

F. 
$$0.1222... = 11/90$$

12 - 1 = 11 and there is 1 repeater, the 2, hence one 9 and 1 non-repeater, the 1, hence one 0

G. 
$$(37 \times 13 + 19) \div 8$$
 has a remainder of 4

 $37 \div 8$  has remainder of 5,  $13 \div 8$  has remainder of 5, and  $19 \div 8$  has remainder of 3 So,  $5 \times 5 + 3 = 28$  and  $28 \div 8$  has remainder of 4

H. 
$$35 \times 35 = 1225$$
  $35 \times 45 = 1575$   $35 \times 55 = 1925$   $35 \times 65 = 2275$   $a5 \times b5 = a \times b +$ the integer portion of  $(a + b) \div 2$  then put either 25 or 75 on the end depending on whether  $(a + b)$  is even or odd

I. 
$$\frac{13}{16} \times 13 = 10\frac{9}{16}$$

numerator ---> 16 - 13 = 3, and  $3^2 = 9$  whole number ---> 13 - 3 = 10

J. 
$$53 \times 47 = 2491$$
  
difference of squares  $(50 + 3)(50 - 3) = 50^2 - 3^2 = 2491$ 

K. 234 base 5 to base 10 = 69

$$2 \times 25 + 3 \times 5 + 4 \times 1 = 69$$

$$L. \quad 36^2 + 57^2 = 4545$$

note 
$$3 + 7 = 10$$
 and  $6 - 5 = 1$   
so  $(3^2 + 6^2) \times 101 = 4545$ 

#### **SHORTCUTS**

### I. Multiplying numbers ending in 5

- A. First digits are equal: 1) always ends in 25
  - 2) multiply first digit by first digit plus 1

Ex: 
$$35 \times 35 = 3 \times (3 + 1)$$
 and ends in  $25 = 1225$   
65 x 65 = 6 x (6 + 1) and ends in  $25 = 4225$ 

- B. First digits differ by 1: 1) always ends in 75
  - 2) multiply smallest first digit by largest first digit plus 1

Ex: 
$$45 \times 35 = 3 \times (4 + 1)$$
 and ends in  $75 = 1575$   
 $65 \times 75 = 6 \times (7 + 1)$  and ends in  $75 = 4875$ 

- C. First digits differ by an even number: 1) always ends in 25
  - 2) add first digits and divide by 2
  - 3) multiply first digits and add quotient from step 2

Ex: 
$$65 \times 25 = 6 \times 2 + ((6+2)/2)$$
 and ends in  $25 = 6 \times 2 + 4$  and ends in  $25 = 1625$   
  $35 \times 95 = 3 \times 9 + ((3+9)/2)$  and ends in  $25 = 3 \times 9 + 6$  and ends in  $25 = 3325$ 

- D. First digits differ by an odd number: 1) always ends in 75
  - 2) add first digits and divide by 2
  - 3) multiply first digits and add integer part of quotient

Ex: 
$$85 \times 55 = 8 \times 5 + (int((8+5)/2))$$
 and ends in  $75 = 8 \times 5 + 6$  and ends in  $75 = 4675$   
  $35 \times 65 = 3 \times 6 + (int((3+6)/2))$  and ends in  $75 = 3 \times 6 + 4$  and ends in  $75 = 2275$ 

# II. Multiplying by 11 or Teens

- A. Multiply by 11: 1) bring down units digit
  - 2) add two digits at a time
  - 3) bring down first digit plus any carry

- B. Multiply by teens: 1) multiply units digit of the teen times units digit
  - 2) multiply units digit of the teen times other digits and add back plus carry
  - 3) bring down first digit plus any carry

Ex: 
$$72 \times 13 = (7 + C) & (3 \times 7 + 2) & (3 \times 2) = 7 & 23 & 6 = (7 + 2) & 3 & 6 = 936$$
  
 $164 \times 12 = (1 + C) & (2 \times 1 + 6 + C) & (2 \times 6 + 4 + C) & (2 \times 4) = 1968$ 

#### III. Multiplying by 25 or 75

- A. Multiply by 25: 1) divide by 4
  - 2) last two digits 00, 25, 50, or 75 depends on the remainder

Ex: 
$$64 \times 25 = 64 \div 4 = 16 \times 0$$
 & and remainder digits = 1600  $57 \times 25 = 57 \div 4 = 14 \times 1$  & add remainder digits = 1425

- B. Multiply by 75: 1) divide by 4
  - 2) last two digits 00, 25, 50, or 75 depends on the remainder
  - 3) multiply results by 3

Ex: 
$$64 \times 75 = 64 \div 4 = 16 \text{ R } 0 \text{ \& add remainder digits} = 1600 \times 3 = 4800$$
  
 $57 \times 75 = 57 \div 4 = 14 \text{ R } 1 \text{ \& add remainder digits} = 1425 \times 3 = 4275$ 

#### IV. Dividing by 25

- A. Divide by 25: 1) multiply by 4
  - 2) place decimal so the answer has 2 decimal places

Ex: 
$$64 \div 25 = 64 \times 4 = 256 \& place decimal = 2.56$$
  
 $57 \div 25 = 57 \times 4 = 228 \& place decimal = 2.28$ 

## V. Multiplying by numbers when first or last digits total 10

- A. Multiply when units digits total 10 and first digits are equal:
  - 1) multiply first digit times first digit plus 1
  - 2) multiply units digits

- B. Multiply when first digits total 10 and units digits are equal:
  - 1) multiply first digits and add the units digit
  - 2) square the units digit

Ex: 
$$27 \times 87 = 2 \times 8 + 7 & 7 \times 7 = 16 + 7 & 49 = 2349$$
  
 $43 \times 63 = 4 \times 6 + 3 & 3 \times 3 = 24 + 3 & 9 = 2709$ 

#### VI. Multiplying by difference of squares

A. Algebra:  $a^2 - b^2 = (a + b)(a - b)$ :

1) easiest to see shortcut by examples

Ex: 
$$53 \times 47 = (50 + 3) \times (50 - 3) = 50^2 - 3^2 = 2500 - 9 = 2491$$
  
 $28 \times 32 = (30 - 2) \times (30 + 2) = 30^2 - 2^2 = 900 - 4 = 896$ 

#### **VII. Least Common Multiple**

- A.  $LCM(a,b) = a \div GCF \times b$ : 1) find the greatest common factor (GCF)
  - 2) divide one number by the GCF
  - 3) multiply quotient times the other number

Ex: LCM( 8,14) --- GCF = 2 --- 
$$8 \div 2 = 4$$
 --->  $4 \times 14 = 56$  ---> LCM( 8,14) =  $56$  LCM(24,99) --- GCF =  $3$  ---  $24 \div 3 = 8$  --->  $8 \times 99 = 792$  ---> LCM(24,99) =  $792$ 

## VIII. Division by 9

- A. xyz divided by 9: 1) add x plus y plus z and put sum over 9 (be sure to reduce)
  - 2) add x plus y plus carry
  - 3) bring down x plus carry

Ex. 
$$201 \div 9 = (2 + C) & (2 + 0 + C) & (2 + 0 + 1)/9 = 22 3/9 = 22 1/3$$
  
  $1240 \div 9 = (1 + C) & (1 + 2 + C) & (1 + 2 + 4 + C) & (1 + 2 + 4 + 0)/9 = 137 7/9$ 

## IX. Multiplying numbers close to 100

- A. Numbers close to and below 100:
  - 1) A = 100 minus first number and B = 100 minus second number
  - 2) subtract A from the second number (or vice versa)
  - 3) multiply A and B

Ex. 
$$96 \times 99 --> A = 4 & B = 1 --> 99 - 4 \text{ (or } 96 - 1) = 95 --> 4 \times 1 = 4 --> 96 \times 99 = 9504$$
  
 $92 \times 97 --> A = 8 & B = 3 --> 97 - 8 \text{ (or } 92 - 3) = 89 --> 8 \times 3 = 24 --> 92 \times 97 = 8924$ 

- B. Numbers close to and above 100:
  - 1) A = first number minus 100 and B = second number minus 100
  - 2) add A to the second number (or vice versa)
  - 3) multiply A and B

Ex. 
$$106 \times 103 \longrightarrow A = 6 & B = 3 \longrightarrow 6 + 103 \text{ (or } 3 + 106) = 109 \longrightarrow 6 \times 3 = 18 \longrightarrow 10918$$
  
 $112 \times 105 \longrightarrow A = 12 & B = 5 \longrightarrow 12 + 105 \text{ (or } 5 + 112) = 117 \longrightarrow 12 \times 5 = 60 \longrightarrow 11760$ 

#### X. Repeating decimals converted to fractions

- A. All digits repeat:
  - 1) the number of digits that repeat is the number of 9's in the denominator
  - 2) one set of the repeating digits is the numerator (be careful to reduce)

Ex: 0.13 --> two repeaters means two 9's --> 13/99 0.341341341... --> three repeaters means three 9's --> 341/999

- B. Some digits repeat and some don't:
  - 1) the number of digits that repeat is the number of 9's in the denominator
  - 2) the number of non-repeating digits is the number of 0's in the denominator
  - 3) subtract the non-repeating digits from the number before repetition starts for the umerator

Ex: 0.12424... --> two repeaters and one non-repeater means two 9's and one 0 --> 124 - 1 = 123 --> 0.12424... = 123/990

 $0.12\overline{35}$  --> two repeaters and two non-repeaters means two 9's and two 0's --> 1235 - 12 = 1223 --> 0.12353535... = 1223/9900

#### **Problem 1 - 20 \***

- 1) Addition, subtraction, multiplication, & division of Integers, Mixed Numbers, Fractions, and Decimals
- 2) Order of Operations
- 3) Use of the Distributive Property
- 4) Comparison of Fractions, Decimals and Percents
- 5) Multiplication Short-Cuts
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Percent/Fractions, English/Metric, Roman Numerals/Arabic Numerals

- 8) Greatest Common Divisor (GCD) and Least Common Multiple (LCM)
- 9) Percent Problems
- 10) Mean, Median, & Mode
- 11) Sums of Integers
- 12) Remainder Problems
- 13) Consumer Type Problems
- 14) Number Theory Problems Involving:
  Prime Numbers, Divisors, Sums of Divisors, etc.

\*\*\* A type of problem from this section can appear later in the test, as well.

# Any questions on any of these?

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$$(1) \ 322 + 327 + 2024 = \underline{\hspace{1cm}}$$

$$(2) \ \ 3\frac{2}{7} - 2\frac{2}{3} = \underline{\hspace{1cm}}$$

$$(3) (5.6)(7+8) = \underline{\hspace{1cm}}$$

(4) 
$$\frac{5}{8} \div \frac{4}{5} =$$

(5) 
$$31^2 =$$

(11) If CDs cost \$4.75 each or a 3-pack for \$12.95, then how much is saved by buying a 3-pack? \$\_\_\_\_\_

$$(12) \sqrt[3]{2744} = \underline{\hspace{1cm}}$$

(13) 8% tax on \$322.00 is \$\_\_\_\_\_

$$(14) \ \ 3 \div (2-7) \times 2 + 2 - 3 = \underline{\hspace{1cm}}$$

(16) 
$$\frac{5}{8}$$
 of 96 is \_\_\_\_\_

(19) 
$$33\frac{1}{3}\%$$
 of 20% of 15 is \_\_\_\_\_\_

\*(20) 
$$300(\sqrt{2} + \sqrt{7}) =$$
\_\_\_\_\_

#### **District Answers**

- (1) 2,673 (2)  $\frac{13}{21}$ (3) 84 (4) .78125,  $\frac{25}{32}$ (5) 961 (6)  $\frac{43}{12}$ (7) 392

- (8) 80
- (9) 1
- \*(10) 35,468 39,200
- (11) 1.30
- (12) 14
- (13) 25.76
- (14)  $-2.2, -\frac{11}{5}, -2\frac{1}{5}$ (15) 12
- (16) 60
- (17) 72
- **(18)** 9
- **(19)** 1
- \*(20) 1,158 1,278

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(2) 
$$2024 + 3 \times 2024 =$$

(4) 
$$27^2 =$$

(5) 
$$\frac{5}{16} =$$
\_\_\_\_\_\_% (decimal)

(6) 
$$333 \times \frac{1}{37} =$$

(7) 
$$33 \times 24 =$$

(13) 
$$33 \times \frac{31}{34} =$$
\_\_\_\_\_\_ (mixed number)

$$(14) \ \ 20 \div (2-4) \times 3 + 30 = \underline{\hspace{1cm}}$$

$$(15) \ \ 30 \div \frac{2}{5} = \underline{\hspace{1cm}}$$

(17) 
$$30 \div 1\frac{1}{5} =$$
(18)  $33^2 - 29^2 = 31 \times$ 

$$(18) \ \ 33^2 - 29^2 = 31 \times \underline{\hspace{1cm}}$$

$$(19) \ \frac{1}{64} - \frac{1}{16} - \frac{1}{4} = \underline{\hspace{1cm}}$$

\*(20) 
$$33 \times \left(\sqrt{20} + \sqrt{24}\right) =$$
\_\_\_\_\_

# **Regional Answers**

- (1) 1,694
- (2) 8,096
- (3) 10.08
- (4) 729
- (5) 31.25
- **(6)** 9
- (7) 792
- (8) 2
- (9) 1,710
- \*(10) 4,456 4,924
- (11) 223
- (12) 43
- $(13) \ \ 30\frac{3}{34}$
- **(14)** 0
- (15) 75 (16) 37.5,  $\frac{75}{2}$ ,  $37\frac{1}{2}$ (17) 25

- $\begin{array}{c}
  (17) \ 26 \\
  (18) \ 8 \\
  (19) \ -\frac{19}{64} \\
  *(20) \ 294 324
  \end{array}$

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(1) 
$$5 \times 15 + 2024 =$$

(2) 
$$24 \div \frac{2}{3} - 15.5 =$$

(3) 
$$\frac{7}{9} \div \frac{2}{3} =$$

(4) 
$$\frac{7}{8} =$$
 \_\_\_\_\_\_(decimal)

$$(5) \ 5^3 - 15^2 + 24 = \underline{\hspace{1cm}}$$

(7) 
$$666 \times \frac{2}{37} =$$

(8) 
$$MMXXIV + XV \times V =$$
 (Arabic Numeral)

(9) Which is larger, 
$$\frac{7}{12}$$
 or 0.58?

(11) If hankies cost \$1.50 each or a dozen for \$15.75, then how much is saved by buying a dozen? \$\_\_\_\_\_

$$(12) \ \ 24 \div (20 - 16) + 12 - 8 \times 4 = \underline{\hspace{1cm}}$$

$$(15) \ \frac{1}{27} - \frac{1}{9} - \frac{1}{3} = \underline{\hspace{1cm}}$$

$$(16) \ \ 3+5+7+9+...+19+21=\underline{\hspace{1.5cm}}$$

$$(18) \ \ 26.25 \times 64 = \underline{\hspace{1cm}}$$

$$(19) \ \ 102\frac{1}{2} \times 64 = \underline{\hspace{1cm}}$$

\*(20) 
$$2024 \times \left(\sqrt{15} + \sqrt{5}\right) =$$
\_\_\_\_\_

#### **State Answers**

- (1) 2,099 (2) 20.5,  $\frac{41}{2}$ ,  $20\frac{1}{2}$ (3)  $\frac{7}{6}$ ,  $1\frac{1}{6}$ (4) .875

- (5) -76(6)  $\frac{5}{6}$ (7) 36

- (8) 2,099 (9)  $\frac{7}{12}$ \*(10) 203,583 225,011
- (11) 2.25
- (12) 14
- (13) 2,170
- (14) 2,916 (15)  $-\frac{11}{27}$
- **(16) 120**
- (17) 1,600
- (18) 1,680
- (19) 6,560
- \*(20) 11,747 12,982

# UIL Number Sense Contest

# Problems #21-40 from the Sequence Chart

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# First, lets look at some ESTIMATIONS

??? Estimating - Rounding - Truncating - Reasonableness ???

## The following are from the UIL 2022 SAC test

\*(70) 
$$\sqrt[3]{9101011} =$$

\*(80) 0.1555... 
$$\times$$
 9  $\times$  10<sup>3</sup> = \_\_\_\_\_\_

#### **UIL High School Number Sense Test Problem Sequencing**

#### **Problems 21 — 40 \*\*\***

- 1) Powers of Numbers
- 2) Substitution
- 3) Word Problems
- 4) Inverses
- 5) Absolute Value
- 6) Ratio/Proportion
- 7) Square Roots/Cube Roots
- 8) Sets
- 9) Base System Problems
- **10) Solving Simple Equations**
- 11) Simultaneous Equations
- 12) Repeating Decimals to Fractions
- 13) More Remainder Type Problems
- 14) Perimeter & Area Problems of Polygons
- 15) Sequences
- 16) Quadratic & Cubic Equation Problems

\*\*\* A type of problem from this section could appear later in the test, but not in the previous section. For example: A set problem could appear in sections #21-40, #41-60, amd/or #61-80, but should not appear in section #1-20.

# Any questions on any of these?

## The University Interscholastic League Number Sense Test • HS District • 2024

(21) Find digit 
$$B > 0$$
, such that  $B32 - 32B = 405$ .

(22) If 
$$\frac{29}{33} = 0$$
.ababab..., then  $a + b =$ \_\_\_\_\_

(24) 
$$[3 + 22 \times 3 - 27] \div 4$$
 has a remainder of \_\_\_\_\_

(25) 
$$8\frac{3}{7} \times 8\frac{4}{7} =$$
 (mixed number)

(26) 
$$[\{t,r,i\} \cup \{q,u,a,d\}] \cap [\{b,i\} \cup \{p,e,n,t\}]$$
 contains how many distinct elements?

(28) If 
$$\frac{1}{7} + \frac{1}{x} = \frac{1}{3}$$
, then  $x =$ \_\_\_\_\_

\*(30) 
$$\sqrt{3222724} =$$

$$(31) \ 5993 \times 7 + 49 = \underline{\hspace{1cm}}$$

(32) If 
$$f(x) = 4x^2 + 20x + 25$$
, then  $f(15) =$ 

(39) Let 
$$\frac{x+11}{x-8} + \frac{x-8}{x+11} = 2\frac{B}{C}$$
. Find B.

\*(40) 
$$\sqrt[3]{322272024} =$$

#### **District Answers**

- (21) 7
- (22) 15
- (23) 32,402,024
- (24) 2
- $(25) \ 72\frac{12}{49}$
- **(26)** 2
- (27) 163
- $(28) \ 5.25, \frac{21}{4}, 5\frac{1}{4}$
- (29)  $\frac{8}{11}$
- \*(30) 1,706 1,884
- (31) 42,000
- (32) 1,225
- $(33) \ \frac{25}{6}, 4\frac{1}{6}$
- (34) 66
- (35) 3022
- (36) 4.80
- (37) 5.44
- (38) 2.72
- (39) 361
- \*(40) 652 719

# The University Interscholastic League Number Sense Test • HS Regional • 2024

(21)	0.58333 × 72 =
(22)	324 × 14 is
(23)	Round $\sqrt{6}$ to the nearest hundredths place
(24)	23% of 40 is% of 10
(25)	The number of positive integral factors of 30 is
(26)	$15\frac{3}{4} \times 8\frac{2}{3} = \underline{\qquad} \text{ (mixed number)}$
(27)	Divide 24 into 4 parts such that the ratio of the 4 parts is 1:2:3:4. The largest part is
(28)	$2\frac{4}{5} \div 3\frac{7}{10} = $
(29)	How many integers between 3 and 63 are divisible by 8?
*( <b>30</b> )	151222 ÷ 136 =
(31)	1776 × 24 + 576 =
(32)	If $x + y = 6$ and $x - y = 4$ , then $x^2 + y^2 = $
(33)	If $f(x) = 4x^2 - 12x + 9$ , then $f(15) = $
(34)	41.5 — 7.75 =
(35)	$41\frac{1}{2} - 7\frac{3}{4} - 9\frac{7}{8} = \underline{}$
(36)	$41\frac{1}{2} + 7.75 - 9\frac{7}{8} = \underline{\hspace{1cm}}$
(37)	Given: 0.125, $\frac{1}{4}$ , 0.375, $\frac{5}{8}$ , 1, m, 2.625, n, 6.875, Find m + n.
(38)	The smaller root of $(4x - 1)^2 = 9$ is
(39)	[ $\{a, l, g\} \cup \{g, e, o, m\} \cup \{t, r, i, g\}$ ] $\cap \{p, r, e, c, a, l\}$ contains how many distinct elements?

\*(40)  $\sqrt[3]{4202033} =$ 

#### **Regional Answers**

- (21) 42
- (22) 4,536
- (23) 2.45
- (24) 92
- (25) 8
- (26)  $136\frac{1}{2}$
- (27) 9.6,  $\frac{48}{5}$ ,  $9\frac{3}{5}$
- $(28) \frac{28}{37}$
- **(29)** 7
- \*(30) 1,057 1,167
- (31) 43,200
- (32) 26
- (33) 729
- (34) 33.75,  $\frac{135}{4}$ ,  $33\frac{3}{4}$
- $(35) \ \ 23.875, \frac{191}{8}, 23\frac{7}{8}$
- (36) 39.375,  $\frac{315}{8}$ ,  $39\frac{3}{8}$
- (37) 5.875,  $\frac{47}{8}$ ,  $5\frac{7}{8}$
- (38) .5,  $-\frac{1}{2}$
- (39) 4
- \*(40) 154 169

# The University Interscholastic League Number Sense Test • HS State • 2024

(21) 
$$39 \times 31 =$$

$$(22) \ 4\frac{2}{3} \div 2\frac{4}{9} = \underline{\hspace{1cm}}$$

(25) If 
$$f(x) = 16x^2 - 40x + 25$$
, then  $f(15) =$ 

(28) 
$$10\frac{5}{7} \times 10\frac{2}{7} =$$
 (mixed number)

\*(30) 
$$\sqrt{5504122} =$$

(32) If 
$$x + y = 15$$
 and  $x - y = 5$ , then  $x^2 + y^2 =$ 

(33) The reciprocal of 
$$-6\frac{2}{3}$$
 is \_\_\_\_\_ (decimal)

(34) 
$$[51 + 5 \times 20 - 24] \div 7$$
 has a remainder of \_\_\_\_\_

$$(35) \sqrt[3]{13824} = \underline{\hspace{1cm}}$$

$$(36) \ 3906 \times 6 - 36 =$$

(39) Find the digit 
$$B > 0$$
, such that  $39B9 = [9(13 - B)]^2$ .  $B =$ \_\_\_\_\_\_

\*(40) 
$$\sqrt[3]{515} \times \sqrt{515} \times 515 =$$

#### **State Answers**

- (21) 1,209
- $(22) \ \ \frac{21}{11}, 1\frac{10}{11}$
- (23) 8
- $(24) \frac{17}{33}$
- (25) 3,025
- (26) 1,512
- (27) 5,602,024
- $(28) \ 110\frac{10}{49}$
- (29) 4030
- \*(30) 2,229 2,463
- (31) 31.26
- (32) 125
- (33) .15
- (34) 1
- (35) 24
- (36) 23,400
- (37) 8
- (38) 40
- **(39)** 6
- \*(40) 88,996 98,363

# UIL Number Sense Contest

# Problems #41-60 from the Sequence Chart

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# Some interesting thoughts and ideas

#### **Right Triangles -- Pythagorean Triples -- Use in Trigonometry**

- 1.  $s^2 + m^2 = h^2$  (interesting labels ---  $a^2 + b^2 = c^2$ )
- 2. Area of right triangle  $A = (s \times m)/2$  (interesting labels ----  $A = (b \times h)/2$ )  $A = 1/2(a \times b \times \sin c)$
- 3. Altitude of right triangle  $h = (a \times b)/c$  (interesting labels ---  $a = (s \times m)/h$ )
- 4. Given m & n --- a triple can be created by  $m^2-n^2$ , 2mn, &  $m^2+n^2$  provided: m & n are relatively prime integers m > n m is even and n is odd or vice versa
- 5. Special note: the product of the integral sides of a right triangle is divisible by 60.
- 6. A 30-60-90 triangle has side ratios of x,  $\sqrt{3}$  x, & 2
- 7. A 45-45-90 triangle has side ratios of  $x, x, \sqrt{2}$
- 8. Pythagorean triples can be used to determine acute, obtuse, or right triangles.
- 9. Trig:  $\sin = \text{opp/hyp} = \text{y/r}$   $\cos = \text{adj/hyp} = \text{x/r}$  $\tan = \text{opp/adj} = \text{y/x}$

#### Sample problems

- 1. The legs of a right  $\triangle$  are 5 and 12. The length of the altitude to the hypotenuse is
- 2.The leg opposite the  $60^\circ$  angle in a right triangle is  $\sqrt{12}$  . The hypotenuse is
- 3. The hypotenuse of an isoscles right triangle is  $\sqrt{32}$  cm. The sum of the lengths of the two legs is
- 4. The legs of a right  $\triangle$  are 8 and 15. The length of the altitude to the hypotenuse is

#### **UIL High School Number Sense Test Problem Sequencing**

\_\_\_\_\_

#### Problems 41 - 60

- 1) Laws of Exponents
- 2) Right Triangle Problems
- 3) Coordinate Geometry Problems
- 4) Regular Polygon Problems
- 5) Inequalities
- 6) Applications of Theorems from Geometry
- 7) Direct and Inverse Variation
- 8) Sequences & Series (Finite & Infinite)
- 9) Complex Numbers
- 10) Logarithms & Logarithmic Equations
- 11) Factorials, Permutations & Combinations
- 12) Probability/Odds
- 13) Conics
- 14) Binomial Theorem (Expansion)
- 15) Base System Problems Using Operations
- 16) Roots of Equations
- 17) Polygonal Numbers
- \*\*\* A type of problem from this section could appear later in the test, but not in the previous sections. For example: A conics problem could appear in sections #41-60, amd/or #61-80, but should not appear in section #1-20 or #21-40.

# Any questions on any of these?

## The University Interscholastic League Number Sense Test • HS District • 2024

$$(41) (309)^2 =$$

$$(45) 54^2 + 66^2 = \underline{\hspace{1cm}}$$

(46) 
$$(2^5 + 7^5 - 2) \div 9$$
 has a remainder of \_\_\_\_\_

$$(47) (4x - 3y)^2 = ax^2 + bxy + cy^2 \text{ and } a + b - c = \underline{\hspace{1cm}}$$

(48) The product of the roots of 
$$3x^2 - 4x = 7$$
 is \_\_\_\_\_

$$(49) \ \ 327_8 + 322_8 - 24_8 = \underline{\hspace{1cm}} 8$$

$$(52) \ (11^3 - 13^3) \div (11 - 13) = \underline{\hspace{1cm}}$$

$$(53) \ \frac{1}{28} + \frac{1}{70} + \frac{1}{130} = \underline{\hspace{1cm}}$$

$$(54) \ 25 + 10 + 4 + 1.6 + \dots =$$

(55) If 
$$\sqrt{12 + \sqrt{8 + \sqrt{x - 8}}} = 4$$
, then  $x =$ \_\_\_\_\_

(57) Let 
$$3\frac{2}{m} \times n\frac{2}{23} = 20$$
, where m, n are natural numbers. Find n — m.

(58) 
$$2024_6 \div 3_6$$
 has a remainder of \_\_\_\_\_

(59) Let 
$$4^5 \times 8^3 \div 128 = 2^k$$
. Find k.

\*(60) 
$$125^2 \div 25^3 \times 5^5 =$$
\_\_\_\_\_

#### **District Answers**

- (41) 95,481
- **(42)** 8
- (43) 45.08
- $(44) \ \frac{73}{3}, 24\frac{1}{3}$
- (45) 7,272
- **(46)** 7
- (**47**) **17**
- $(48) -\frac{7}{3}, -2\frac{1}{3}$
- (49) 625
- \*(50) 9,654 10,669
- (51) 144
- (52) 433
- $(53) \frac{3}{52}$
- $(54) \ \frac{125}{3}, 41\frac{2}{3}$
- (55) 72
- $(56) \frac{1}{11}$
- (57) 1
- **(58)** 1
- **(59) 12**
- \*(60) 2,969 3,281

# The University Interscholastic League Number Sense Test • HS Regional • 2024

(41)	75% of 37.5% of 64 is
(42)	$(3^5 + 5^5 - 7) \div 8$ has a remainder of
(43)	19200 = 144 + 1588 ×
(44)	Which is larger, $-\frac{11}{12}$ or $-\frac{10}{11}$ ?
(45)	83 <sup>2</sup> + 22 <sup>2</sup> =
(46)	The sum of the measures of the interior angles of a regular heptagon is degree
(47)	Let $8\frac{3}{m} \times n\frac{34}{35} = 26$ , where m, n are natural numbers. Find mn
(48)	$(3! \times 6!) \div (5! \times 4!) = $
(49)	$330_{11} - 42_{11} + A9_{11} = \underline{\hspace{1cm}}_{1}$
*(50)	33 <sup>3</sup> =
(51)	If $\frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \dots + \frac{1}{n} = \frac{11}{13}$ , then $n = \underline{\hspace{1cm}}$
(52)	$(708)^2 = $
(53)	$8\frac{1}{2}$ is what percent less than $12\frac{1}{2}$ ?
(54)	(3+7+10+17+27+44+71) + $(115+186+301) =$
(55)	33024 <sub>7</sub> ÷ 4 <sub>7</sub> has a remainder of
(56)	The perimeter of a square is decreased from 22 cm to 18 cm. Find the corresponding decrease in its area cm
(57)	$21 + 14 + 9\frac{1}{3} + 6\frac{2}{9} + \dots = $
(58)	150 fathoms = inche
(59)	The coefficient of the $x^3y^2$ term in the expansion of $(5x-2y)^5$ is

\*(60) A rectangular lot is  $\frac{3}{8}$  of a mile by  $\frac{7}{16}$  of a mile. The area of the lot is \_\_\_\_\_ square feet

## **Regional Answers**

- (41) 18
- **(42)** 1
- (43) 12
- $(44) \frac{10}{11}$
- (45) 7,373
- (46) 900
- **(47)** 8
- (48) 1.5,  $\frac{3}{2}$ ,  $1\frac{1}{2}$
- (49) 397
- \*(50) 34,141 37,733
- (51) 78
- (52) 501,264
- (53) 32
- (54) 781
- (55) 2
- (56) 10
- (57) 63
- (58) 10,800
- (59) 5,000
- \*(60) 4,345,110 4,802,490

# The University Interscholastic League Number Sense Test • HS State • 2024

(42) 
$$(7^5 + 3^5 - 2) \div 10$$
 has a remainder of \_\_\_\_\_

(44) The arithmetic mean of the set 
$$\{5, 15, 24, k\}$$
 is 16. Find k.

(45) If 
$$3x + y = 5$$
 and  $x - 2y = 5$ , then  $x = ____$ 

(46) 
$$123_4 \times 2_4 = \underline{\hspace{1cm}}_2$$

$$(47) (2! \times 3! \times 5!) \div (4! \times 6!) = \underline{\hspace{1cm}}$$

(49) Let 
$$R_1$$
 and  $R_2$  be the roots of  $(2x-3)^2 = 5$ .  
Find  $R_1 + R_2 - R_1 \times R_2$ .

$$(51) 114 - 17\frac{1}{2} - 22.25 = \underline{\hspace{1cm}}$$

$$(52) 114 + 17.5 - 22\frac{1}{4} = \underline{\hspace{1cm}}$$

$$(53) 114.25 + 17\frac{1}{2} + 22 = \underline{\hspace{1cm}}$$

(54) 
$$2+7+9+16+25+41+66+m+$$
  
 $173+n+453=$ 

(55) If 
$$f(x) = 3x + \log_4(x)$$
, then  $f(8) =$ 

(56) 
$$4\frac{1}{5}$$
 is what percent more than  $3\frac{1}{2}$ ? \_\_\_\_\_\_\_\_%

$$(57) \ 513_6 - 1415_6 + 2024_6 = \underline{\hspace{2cm}}_6$$

$$(58) \ 48 + 32 + 21.333... + 14.222... + ... = \underline{\phantom{0}}$$

(59) 
$$37^{12} \div 23$$
 has a remainder of \_\_\_\_\_

\*(60) 
$$(10\pi^2 - 1)^2 =$$

#### **State Answers**

- (41)  $4\frac{4}{9}$
- (42) 8
- (43) 5
- (44) 20
- $(45) \ \frac{15}{7}, 2\frac{1}{7}$
- (46) 110110
- $(47) \frac{1}{12}$
- $(48) \ \frac{25}{3}, 8\frac{1}{3}$
- (49) 2
- \*(50) 24,242 26,793
- (51) 74.25,  $\frac{297}{4}$ ,  $74\frac{1}{4}$
- (52) 109.25,  $\frac{437}{4}$ ,  $109\frac{1}{4}$
- (53) 153.75,  $\frac{615}{4}$ , 153 $\frac{3}{4}$
- (54) 1,179
- $(55) \ \ 25.5, \frac{51}{2}, 25\frac{1}{2}$
- (56) 20
- (57) 1122
- (58) 144
- **(59) 9**
- \*(60) 9,068 10,021

# UIL Number Sense Contest

# Problems #61-80 from the Sequence Chart

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#### **Problems 61 - 70**

- 1) Volume & Surface Area
- 2) Greatest Integer
- 3) Application of Remainder Theorem
- 4) Trigonometry
- 5) Determinants
- 6) Matrices
- 7) Vectors
- 8) Composite Functions
- 9) Bases Involving Decimals or Fractions
- 10) Polar/Rectangle Coordinates

#### **Problems 71 - 80**

- 1) Function Domains and Ranges
- 2) Modular Arithmetic
- 3) Limits
- 4) Derivative
- 5) Slopes of Tangent Lines
- 6) Horizontal & Vertical Asymptotes
- 7) Determining Critical Values
- 8) Maximum & Minimum Problems
- 9) Definite Integration
- 10) Inverse Functions
- \*\*\* A type of problem from section #61-70 appear in this section and in section #71-80, but not in the previous sections. A type of problem from section #71-80 can appear only in its section and not in any previous sections.

For example: A vector problem could appear in section #61-80, but should not appear in section #1-20, #21-40, or #41-60. And, a limits problem can only appear in section #71-80.

### Any questions on any of these?

### The University Interscholastic League Number Sense Test • HS District • 2024

(61) 
$$\sin(\frac{17\pi}{6}) =$$
\_\_\_\_\_

(62) If 
$$(\sqrt[3]{a^{22}})(\sqrt[3]{a^{27}}) = (\sqrt[n]{a^k})$$
, where n and k are relatively prime, then  $k =$ 

(64) 
$$f(x) = \frac{x+1}{2-3x}$$
 — 4 and  $f^{-1}(5) =$  \_\_\_\_\_

(65) If 
$$x = 7$$
 and  $y = -6$ , then
$$(x + y)(x^2 - xy + y^2) = \underline{\hspace{1cm}}$$

(67) If 
$$h(x) = x^2 - 3$$
 and  $g(x) = x - 1$ ,  
then  $h(g(0)) =$ \_\_\_\_\_

(68) Change 
$$\frac{12}{25}$$
 to a base 5 decimal. \_\_\_\_\_\_5

(71) Find k, 
$$0 \le x \le 22$$
, if  $11^4 + 2^6 \cong k \pmod{22}$ .

(72) Let 
$$h(x) = 3x^2 + 2x + 1$$
. Find  $h'(-4)$ .

(73) 
$$2\frac{2}{3}$$
 fathoms = \_\_\_\_\_\_ inches

(74) Let (a, b) be an inflection point for 
$$h(x) = 2x^3 - 3x + 5$$
. Find  $a + b$ .

(75) 
$$x^2 + y^2 = 4y$$
 has area of  $k\pi$  sq. units and  $k =$ \_\_\_\_

(76) 
$$\int_{-1}^{2} (x-3) dx = \underline{\hspace{1cm}}$$

(77) The domain of 
$$f(x) = \frac{\sqrt{3x-5}}{\sqrt{7-2x}}$$
 is  $w \le x < y$  and  $x \in \text{Reals. Find } w + y$ .

#### **District Answers**

- (61) .5,  $\frac{1}{2}$
- (62) 49
- (63) 45
- $(64) \frac{17}{28}$
- (65) 127
- (66) 17
- (67) 2
- (68) .22
- (69) 10
- \*(70) 7,357 8,131
- **(71)** 9
- (72) 22
- (73) 192
- (74) 5
- (75) 4
- $(76) -7.5, -\frac{15}{2}, \\ -7\frac{1}{2}$
- $(77) \ \frac{31}{6}, 5\frac{1}{6}$
- (78) 44
- (79) 105,294
- \*(80) 174,800 193,199

# The University Interscholastic League Number Sense Test • HS Regional • 2024

- (61) Write in figures: three hundred and four-fifths million three thousand thirty.
- (62) Given: y varies inversely with x and y = 12 when x = 7. Find y when x = 11.
- $(63) \begin{bmatrix} 1 & 3 \\ 6 & 10 \end{bmatrix} \times \begin{bmatrix} 0 & -2 \\ k & 5 \end{bmatrix} = \begin{bmatrix} 12 & 13 \\ 40 & 38 \end{bmatrix} . k = \underline{ }$
- (64)  $47^{15} \div 29$  has a remainder of \_\_\_\_\_\_
- (65) The first 4 digits after the decimal point in the decimal representation of  $\frac{13}{45}$  are \_\_\_\_\_
- (67) Let  $(6 + 4i) \div 2i = a + bi$ . Find b. \_\_\_\_\_
- (68) Arcsin $\left(\cos\left(\frac{\pi}{6}\right)\right)$  = \_\_\_\_\_\_ degrees
- (69) 0.77 base 8 = \_\_\_\_\_ base 10 (fraction)
- \*(70)  $5^3 \div 4! \times 3^5 \div 2! =$ \_\_\_\_\_\_
- (71)  $f(x) = \frac{5x-7}{3} + 2$  and  $f^{-1}(11) = \underline{\hspace{1cm}}$
- (72) Find  $f(g(-\frac{2}{3}))$  when f(x) = 3x + 5 and g(x) = 5x 3.
- (73) Let  $f(x) = \cos(2x)$ . Find  $f''(\frac{2\pi}{3})$ .
- (74) Given:  $f(x) = -x^2 + 4x + 1$  has a maximum point at (a, b). Find a + b.
- (75) Find the slope of the line tangent to  $f(x) = x^3 + 2x$  at the origin.
- (76)  $\int_{1}^{2} \int_{2}^{3} xy \, dy dx =$ \_\_\_\_\_
- (77) Let (x, y) be the focus of  $x = y^2 1$ . x =\_\_\_\_\_
- $(78) \ \ 330 \div 0.6875 = \underline{\hspace{1cm}}$
- (79) Given: 1, 1, 3, 5, 6, 12, 10, 22, T, P, ... . T P =
- \*(80)  $(\ln 100000)^3 =$

## **Regional Answers**

- (61) 300,803,030
- $(62) \ \frac{84}{11}, 7\frac{7}{11}$
- (63) 4
- (64) 11
- (65) 2888
- (66) 37.5,  $\frac{75}{2}$ ,  $37\frac{1}{2}$
- (67) 3
- (68) 60
- (69)  $\frac{63}{64}$
- \*(70) 602 664
- (71) 6.8,  $\frac{34}{5}$ ,  $6\frac{4}{5}$
- (72) 14
- (73) 2
- (74) 7
- (75) 2
- (76) 3.75,  $\frac{15}{4}$ ,  $3\frac{3}{4}$
- $(77) -.75, -\frac{3}{4}$
- (78) 480
- (79) 20
- \*(80) 1,450 1,602

## The University Interscholastic League Number Sense Test • HS State • 2024

(61) 
$$\begin{bmatrix} 2 & 5 \\ 3 & -7 \end{bmatrix} \times \begin{bmatrix} 1 & -6 \\ 3 & 10 \end{bmatrix} = \begin{bmatrix} a & c \\ b & d \end{bmatrix}. b + c = \underline{\qquad}$$

(62) 
$$\tan\left(\frac{\pi}{3}\right) \times \tan\left(\frac{5\pi}{3}\right) = \underline{\hspace{1cm}}$$

(64) If 
$$300^{\circ} = k\pi$$
 radians, then  $k =$ 

(67) 
$$\log_2(32) - \log_5(25) = \log_3(x)$$
 and  $x =$ 

(68) If 
$$x = 5$$
 and  $y = 15$ , then
$$(x - y)(x^2 + xy + y^2) = \underline{\hspace{1cm}}$$

(69) 
$$\sqrt{1000_4} =$$
\_\_\_\_\_4

\*(70) 
$$(4+5+6+...+11+12+13)^2 =$$

(71) If 
$$f(x) = \frac{5x}{6} + \frac{3}{4}$$
 and  $f^{-1}(x) = ax + b$ ,  
then  $a + b = \underline{\hspace{1cm}}$ 

(72) Let 
$$f(x) = \sin(2x)$$
. Find  $f''(\frac{\pi}{12})$ .

(74) 
$$h(x) = (x + 3)^{\frac{1}{2}}$$
 has a relative minimum at  $x =$ \_\_\_\_

(75) 
$$\int_{1}^{2} \int_{3}^{4} xy \, dy dx =$$
\_\_\_\_\_\_

(76) The axis of symmetry of the graph of 
$$f(x) = 5x^2 + 15x - 24$$
 is  $x =$ \_\_\_\_\_

(77) Given: 
$$5, 1, 4, -3, 7, -10, 17, k, 44, \dots k =$$

$$(78) (402)^3 = \underline{\hspace{1cm}}$$

(79) 
$$28146 \times 111 =$$

#### **State Answers**

- (61) 20
- (62) 3
- (63) 180
- (64)  $\frac{5}{3}$ ,  $1\frac{2}{3}$
- (65) .92
- (66) 45
- (67) 27
- (68) 3,250
- (69) 20
- \*(70) 6,864 7,586
- (71)  $.3, \frac{3}{10}$
- (72) 2
- (73) 340
- (74) 3
- (75) 5.25,  $\frac{21}{4}$ ,  $5\frac{1}{4}$
- $(76) -1.5, -\frac{3}{2}, -1\frac{1}{2}$
- (77) 27
- (78) 64,964,808
- (79) 3,124,206
- \*(80) 45,889 50,718