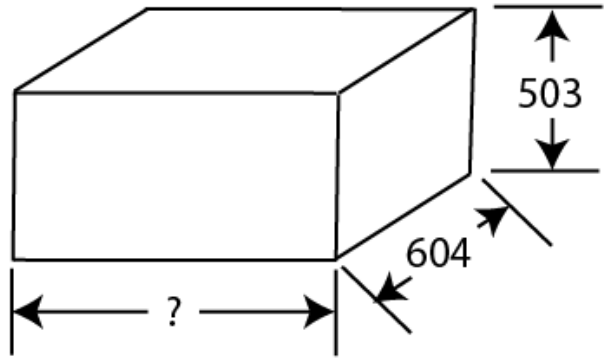
Selected Problems From 2024 HS Calculator Applications Contest

24A-29.

RECTANGULAR SOLID



Total Surface Area = 3.21×10^6

24A-29 =_____

$2[(604)(503) + 604x + 503x] = 3.21 \times 10^6$

$x = \mathbf{1180}$

Andy Zapata
adzapata74@gmail.com

Andy Zapata

Married
4 children
3 grandchildren
Retired Classroom Teacher 42 years
Co-founder Texas Math and Science Coaches Association (TMSCA)
Azle Junior High – (1974 – 1982)
Azle High School – (1982 – 2016)
Physics teacher (1982 – 2016)
AP Physics reader – (2004 – 2016)
AISD Grant Writer – (2017)
High School Aerospace Scholar counselor – (2006 – 2010)
Coached – JH slide rule (1974 – 1982)
HS slide rule, number sense, calculator applications, mathematics, science (1982 – 2016)
Coached numerous high school state champions and state championship teams.
Azle HS UIL academic coordinator
2001 – 2002 UIL sponsor excellence award winner
UIL A+ Number Sense, Calculator, Mathematics consultant (2007 – present)

adzapata74@gmail.com

Each year Dr. David Bourell writes at least nine UIL high school Calculator Application contests for competition. There are 21 stated problems and 14 geometry drawings. The stated and geometry problems range in difficulty from basic arithmetic to differential and integral calculus. I've selected some of the problems that have appeared from this past year's competition to show how they are worked. My solutions might not be unique, and in fact they are the work of other coaches, but they are accurate solutions – in that they yield answers that agree with the answers that Dr. Bourell gave.

I will confess that my knowledge of the math topic, calculus, is rudimentary; and I will also admit that when I saw some of the solutions that eluded me, I had several "aha" moments.

If you have not purchased a copy of the "UIL Calculator Applications Contest Manual – revised 2023" from the UIL's online store by Dr. Bourell; **you need to do so!**

In any case, I hope these particular solutions will be of help to you so that you can pass them on to the students you coach – since there really is no sense in keeping this information to yourself.

z.

Problem Types on High School Calculator Contest

Page 1 Stated Problems
Page 1 Geometry Problems
2D Problems

Page 2 Stated Problems
Page 2 Geometry Problems
Right Triangle Problems

Page 3 Stated Problems
Page 3 Geometry Problems
3D Problems

Page 4 Stated Problems
Page 4 Geometry Problems
Problem 39 (Triangles and Circles)
Problem 40 (Laws of Sines/Cosines)

Problem Types on High School Calculator Contest

Page 5 Stated Problems
- Problem 46 (Scaling)
- Problem 47 (Linear Regression)
- Problem 48 (Solver)

Page 5 Geometry Problems
- Difficult 3D Problems

Page 6 Stated Problems
- Problem 56 (Basic Calculus)
- Problem 57 (Calculus Applications)
- Problem 58 (Matrix)

Page 6 Geometry Problems
- Problem 59 (Calculus Geometry)
- Problem 60 (Difficult Plane Geometry)

Problem Types on High School Calculator Contest

Page 7 Stated Problems

Problem 61 (Difficult Stated Problem)

Problem 62 (Logarithmic Solution – Very Large/Small Number Problem)

Problem 63 (Difficult Stated Problem – Trajectory Problem)

Page 7 Geometry Problems

Difficult 2D Problems

Page 1 Problems

24A-7. What is the cube root of the result of 41.9 minus 17.8π? - 7=_____

$\sqrt[3]{41.9 - 17.8\pi}$ **- 2.41**

24B-8. A lizard grows from 4.32 in to 9.75 in over 2 months.
What is the positive change in length?---------- 8=_____in

9.75 – 4.32 **5.43**

Page 1 Problems

24B-7. What is the sum of 1.63 and the product of 2.1 and 0.796?

----------7=_____

1.63 + (2.1)(.796) **3.30**

24D-8. A plot of land is 150 ft by 248 ft. What is the area?------8=_____ft^2

(150)(248) **37,200**

Page 1 Problems

24F-7. What is the remainder of 5870 divided by 9.81?---------7=_____

5870 / 9.81 = 598.3690… remainder = .3690… (.3690…)(9.81)

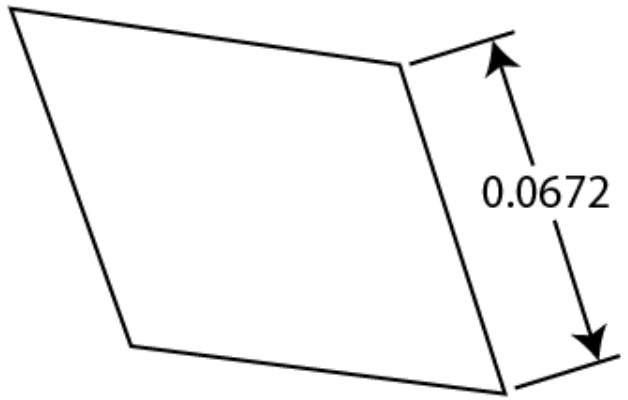
3.62

24H-8. A 30 oz jar of mayonnaise costs $4.58. What is the cost per oz?
----------8=_____$/oz

4.58 / 30 **.153**

Page 1 Problems

24D-10. RHOMBUS



Perimeter = ?

24D-10 =_____

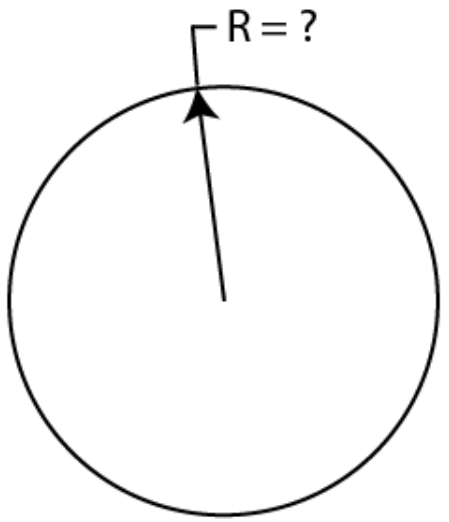
$P_{RHOMBUS} = 4(.0672)$

0.269

Page 1 Problems

24I-9.

CIRCLE



Circumference = 0.0995

24I-9 =_____

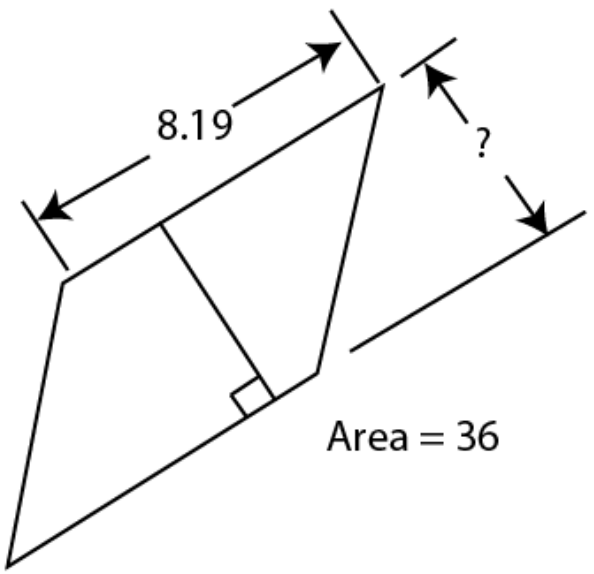
$2\pi R = .0995$

R = .0158

Page 1 Problems

24H-10.

PARALLELOGRAM



24H-10 =_____

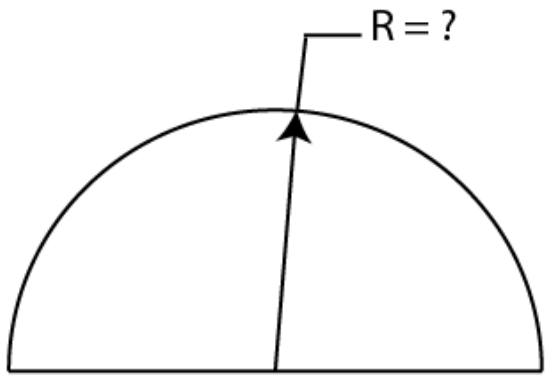
36 / 8.19

4.40

Page 1 Problems

24C-10.

SEMICIRCLE



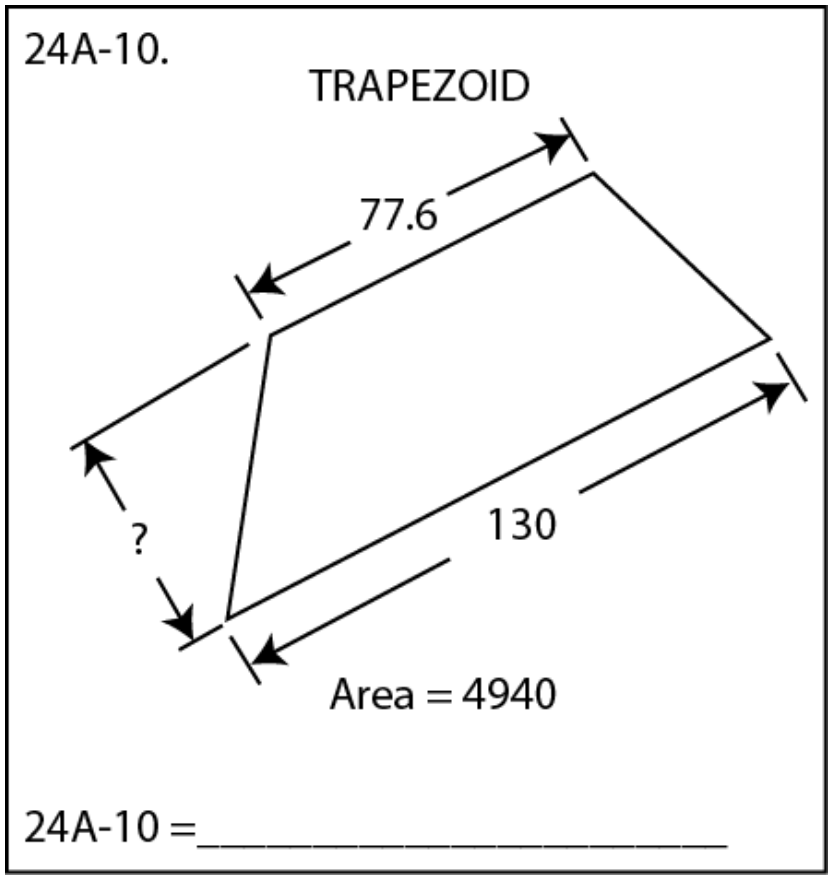
Perimeter = 9.15

24C-10 =_____

$2R + \pi R = 9.15$

R = 4.64

Page 1 Problems



$\frac{1}{2}\, h(77.6 + 130) = 4940$

47.6

Page 2 Problems

24D-17. Brad is planning a party for 117 guests. He and the guests will each use on average 1.35 drinking cups. If he buys fancy cups that come 6 to a package, how many packages will he need?----------17=_____integer

(117)(1.35) / 6 = 26.325

27

{Note: Brad + 117 guests = 118 people $\rightarrow$ 26.55 }

Page 2 Problems

24D-18. What is the percent error in using 24/17 for $\sqrt{2}$?-----18=_____%

% error = (100%)[(Approx / Exact) – 1]

$$\% \text{ error} = (100\%)\left[\frac{24/17}{\sqrt{2}} - 1\right]$$ **-.173**

OR Some calculators have a conversion button … ☺

$\sqrt{2}$, 24/7, **% change** = **-.173**

Page 2 Problems

24A-18. Half of the US 332 million population drink 12 oz of coffee daily. How many tanker trucks would this represent, if a tanker truck capacity is 7,500 gallons?----------18= _____

$$\frac{(.5)\left(332\times10^{6}\right)(12\,\text{oz})}{(128\,\text{oz}/\text{gal})(7500\,\text{gal})}$$

2080

Page 2 Problems

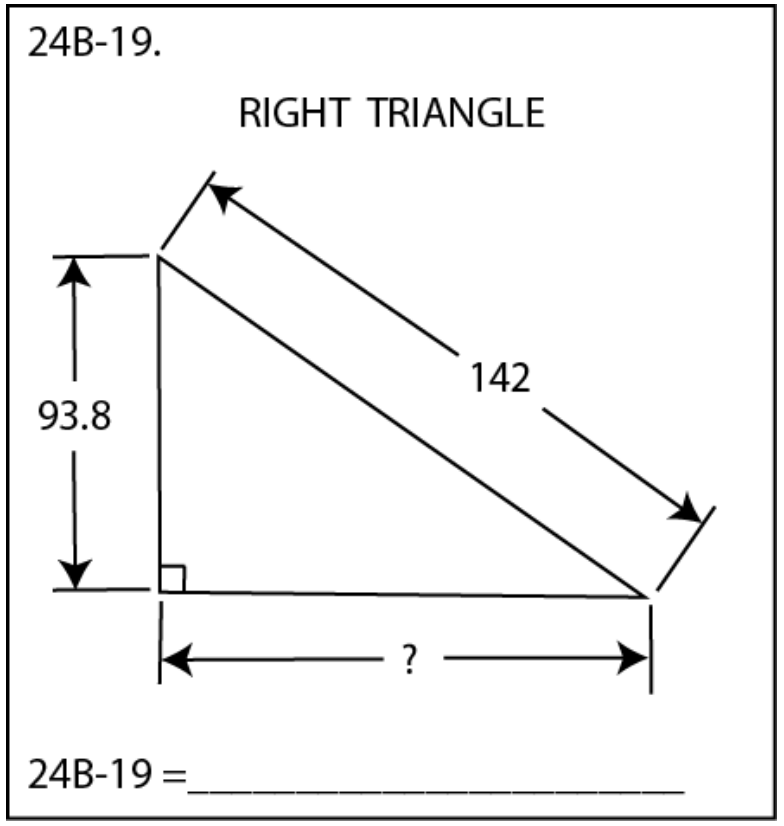
24H-17. The world land speed record was set by Andy Green driving a twin turbofan jet-powered car. The speed was 763.035 mph over one mile in October 1997. How long would it take to travel 1 mi at this speed?

----------17=_____s(SD)

[1 mi/ 763.035 {6 SD} mi/hr] (3600 s/hr)

4.17800 {6 SD}

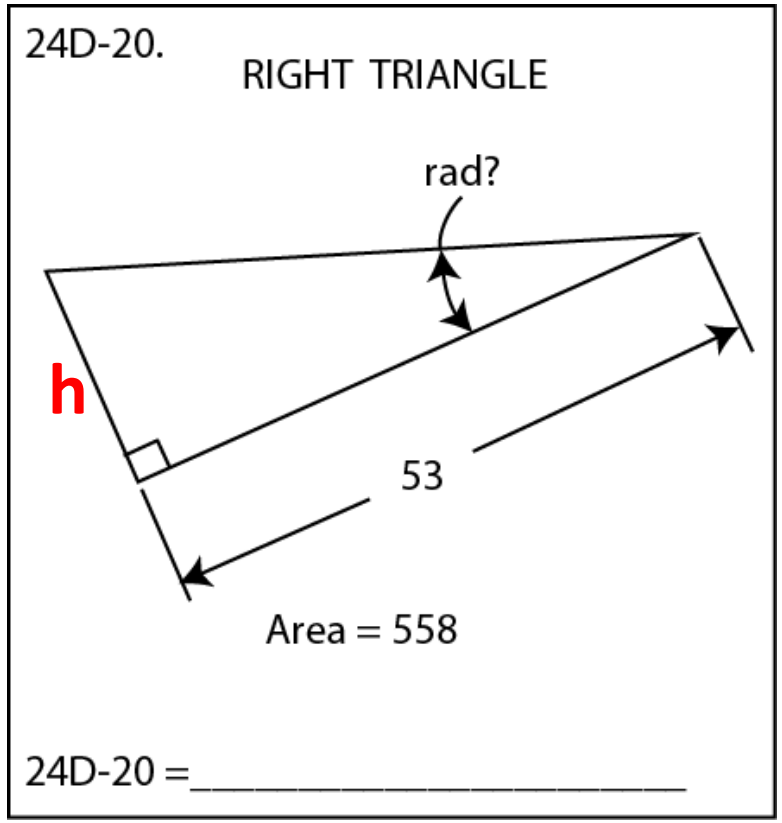
Page 2 Problems



$?^2 + (93.8)^2 = 142^2$

107

Page 2 Problems



½ (53)**h** = 558

h = 21.0566 …

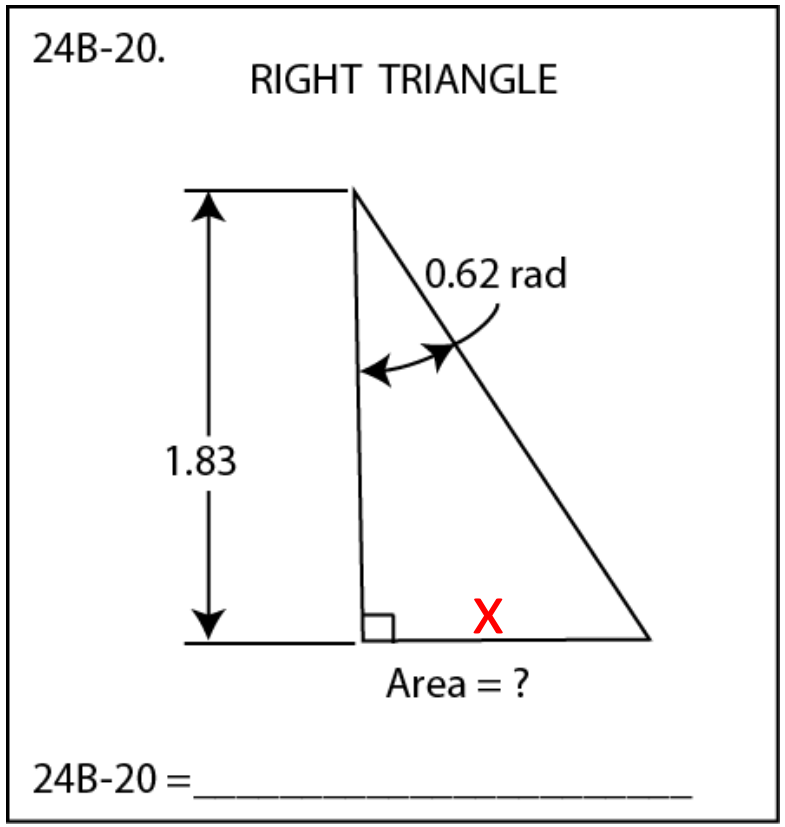
tan ? = 21.0566 … / 53

tan ? = 0.39729 ...

? = arctan(0.39729 ...)

0.378

Page 2 Problems



$\tan(.62) = x / 1.83$

$x = 1.3064 \ldots$

$A = \frac{1}{2}(1.3064 \ldots)(1.83)$

1.20

Page 3 Problems

24G-26. A car weighs 4100 lbs. It is composed of metal with an average density of 6.5 g/cm^3 and plastic with an average density of 1 g/cm^3. If a car is 1/3 plastic by mass, calculate the volume of material in a car.

----------26=_____ft^3

1 lb = 453.592 g $\rightarrow$ (4100 lbs)(453.592 g/lb) = 1859727.2 g **{Car Mass}**

(2.54 cm / 1 in)(12 in/1 ft) = 30.48 cm/ft $(30.48 \text{ cm/ft})^3 = 28316.846592 \text{ cm}^3 / 1 \text{ ft}^3$

$$\text{density} = \frac{\text{mass}}{\text{volume}} \rightarrow \text{volume} = \frac{\text{mass}}{\text{density}}$$

metal mass

$(2/3)(1859727.2 \text{ g})/(6.5 \text{ g / cm}^3)[(1 \text{ ft}^3 / 28316.846592 \text{ cm}^3)]$ **+**

plastic mass $(1/3)(1859727.2 \text{ g})/(1 \text{ g / cm}^3)[(1 \text{ ft}^3 / 28316.846592 \text{ cm}^3)]$

28.6

Page 3 Problems

24F-26. A Farmer Pat walked off a square, one-acre field. She estimated the side dimension to be 195 ft. What was the percent error in her measurement?----------26=_____%(SD)

$(5280\ ft)^2$ /640 acres = 43560 ft^2 / acre $\sqrt{43560\ ft^2}$ = 208.71…ft

% error = (100%)[(Approx / Exact) – 1] $\rightarrow$ 100%[(195 {3SD} / 208.71…) – 1]

% error = 100%[.93430… {3SD} – 1]

% error = 100%[-.06569…{2SD}]

-6.6 {2 SD}

Page 3 Problems

24E-28. Danny is 9 yr old, and Ruth is one third his age. After how many years will Ruth's age equal two thirds of Danny's?----------28=_____integer

D = 9, R = 3

(3+x) = (2/3)(9+x)

9

Page 3 Problems

24D-27. Due to word spreading quickly, the number of daily customers at a BBQ restaurant grows exponentially. The restaurant is open 7 days a week. On Day 7, there were 35 customers. On Day 12, there were 110. How many customers will eat there on Day 16?----------27=_____integer

Day 12 = 12 – 7 = 5 days

$110 = 35e^{5k} \rightarrow 110/35 = e^{5k}$

$\text{Ln}(110/35) = \text{Ln}(e^{5k}) \rightarrow 1.145 \ldots = 5k \rightarrow k = 0.229 \ldots$

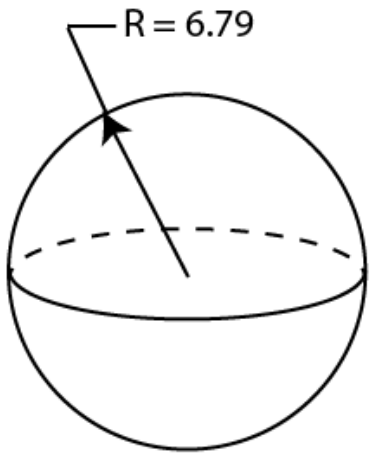
Day 16 = 16 – 7 = 9 days $\therefore A = 35e^{9(0.229\ldots)}$

275

Page 3 Problems

24I-29.

SPHERE



Volume = ?

24I-29 =_____

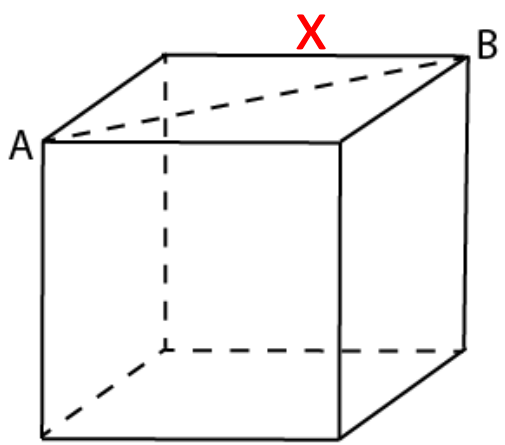
$V = (4/3)\pi(6.79)^3$

1310

Page 3 Problems

24I-30.

CUBE



Total Surface Area = 103

AB = ?

24I-30 =_____

Let x = edge

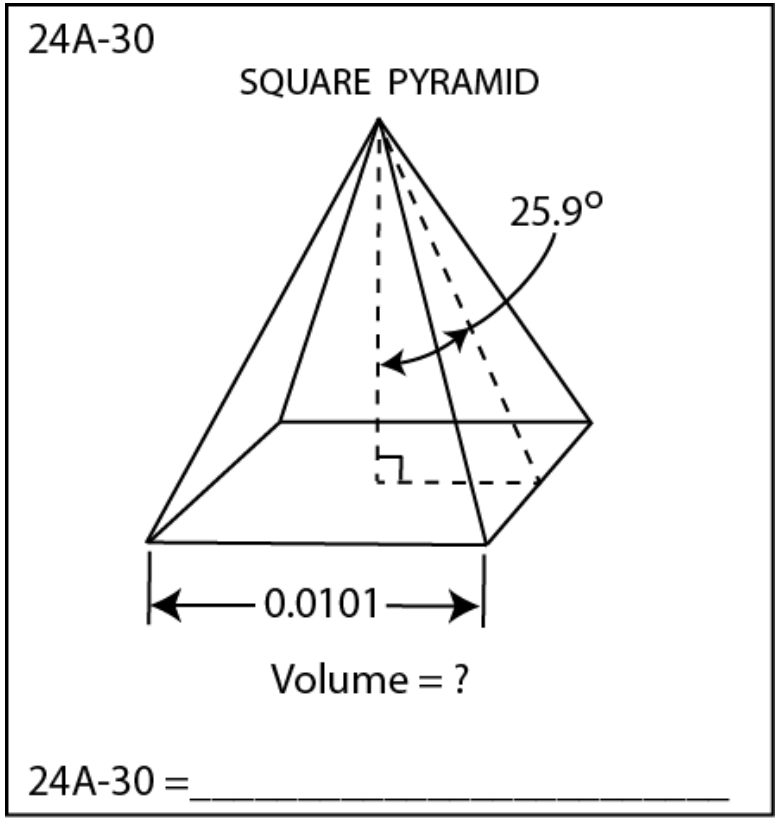
$103 = 6x^2$

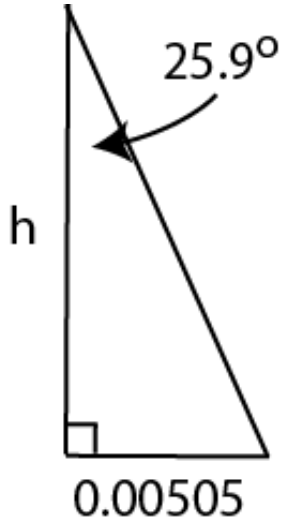
$x = 4.143\ldots$

$AB = (4.143\ldots)\sqrt{2}$

5.86

Page 3 Problems





$\tan 25.9° = .00505 / h$

$h = .01040\ldots$

Volume = 1/3(base area)(height)

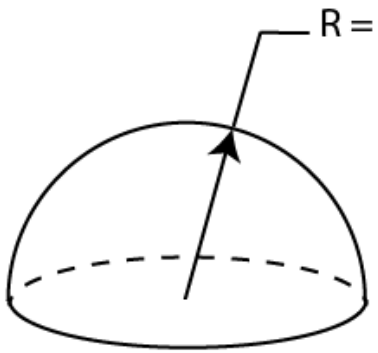
Volume $= 1/3\ (.0101)^2(.01040\ \ldots)$

3.54×10^{-7}

Page 3 Problems

24E-29.

HEMISPHERE



Total Surface Area = 884

24E-29 =_____

$$\pi R^2 + (\tfrac{1}{2})4\pi R^2 = 884$$

A_{CIRCLE} $SA_{HEMISPHERE}$

$$3\pi R^2 = 884$$

9.68

Page 4 Problems

24A-36. As a New Year's Resolution, on January 1, 2023, Charlie went on a diet. Her starting weight was 163 lbs. She averaged 3 lb loss each week. What is the percent decrease in her weight on March 6?----36=_____%

31 + 28 + 6 = 65 days

(65 dys/7 dys/wk) (3 lb/wk) = 27 6/7 lbs

Weight decrease = 163 – 27 6/7

Weight decrease = 135.142 … lbs

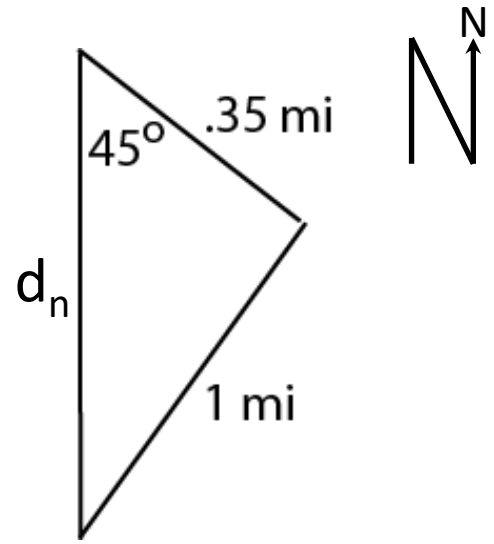
% decrease = (100%)[1 – (small / large)]

% decrease = (100%)[1 – (135.142 . . ./ 163)]

17.1

Page 4 Problems

24C-38. Mike leaves Nina, walking southeast at 3 mph. After time t_0, Nina starts biking south at 15 mph. What is t_0 if they are 1 mi apart 7 min after Mike started hiking?----------38=_____min



$d_M = (3 \text{ mph})(7/60) = .35 \text{ mi}$

$1^2 = d_N^2 + (.35)^2 - 2(d_N)(.35)\cos 45°$ ← **Law of Cosines**

$d_N = 1.2163\ldots \text{ mi}$

$t_N + t_0 = 7 \text{ min}$

$(15 \text{ mph})(t_N) = 1.2163 \ldots \text{ mi} \rightarrow t_N = .08109\ldots \text{ hrs}$

$.08109\ldots \text{ hrs} \times 60 \text{ min/hr} = 4.865 \ldots \text{ min}$

$t_0 = 7 - 4.865$

2.13

Page 4 Problems

24E-37. A nominal 1/4 mi track was erroneously constructed to be too long. The error in length was 1 ft 3.75 in. If a runner runs 4 laps in 6 min 33.3 s, what is their actual running velocity?----------37=_____mph(SD)

1 ft 3.75 in = 15.75 in {4SD} 6 min 33.3 s = 393.3 s {4SD}

(15.75 in / lap)(1 ft/ 12 in)(4 laps) = 5.25 ft / lap **{4SD}**

$$\left(\frac{5280\,\text{ft} + 5.25\,\text{ft}\{4\text{SD}\}}{393.3\,\text{s}\{4\text{SD}\}}\right)\left(\frac{15}{22}\right)$$

9.162 {4 SD}

Page 4 Problems

24H-37. Centrifugal force F equals $m\omega^2R$, where ω is the angular velocity of a mass m moving along an arc of radius R. If a 35000-lb_m car traveling at 50 mph skids when the centrifugal force equals 600 lb_f , what is the turning radius to initiate the skid? 1 lb_f = 32.174 lb_m ft/s^2.---37=_____ft

$F_c = m\omega^2 r$; $v = \omega r \rightarrow \omega = v/r$; (50 mph)(22/15) = 73.333… ft/s

$F_c = m(v/r)^2(r) = mv^2/r \rightarrow r = mv^2 / F_c$

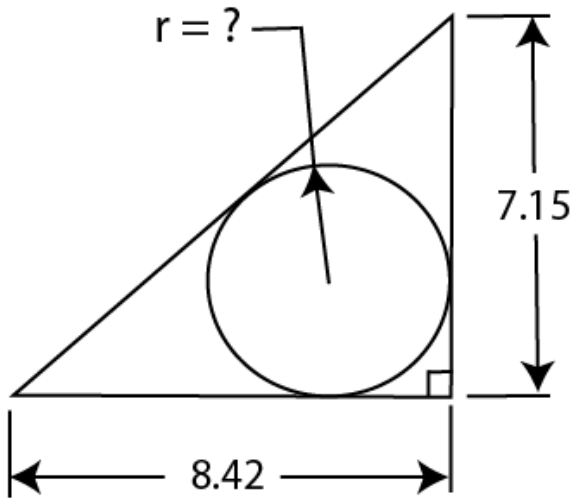
$$r = \frac{(3500\,\text{lb})(73.333\ldots\,\text{ft/s})^2}{(600\,\text{lb}_f)(32.174\,\text{ft/s}^2)}$$

975

Page 4 Problems

24A-39.

RIGHT TRIANGLE AND INSCRIBED CIRCLE



24A-39 =_____

$r = (a + b - c)/2$

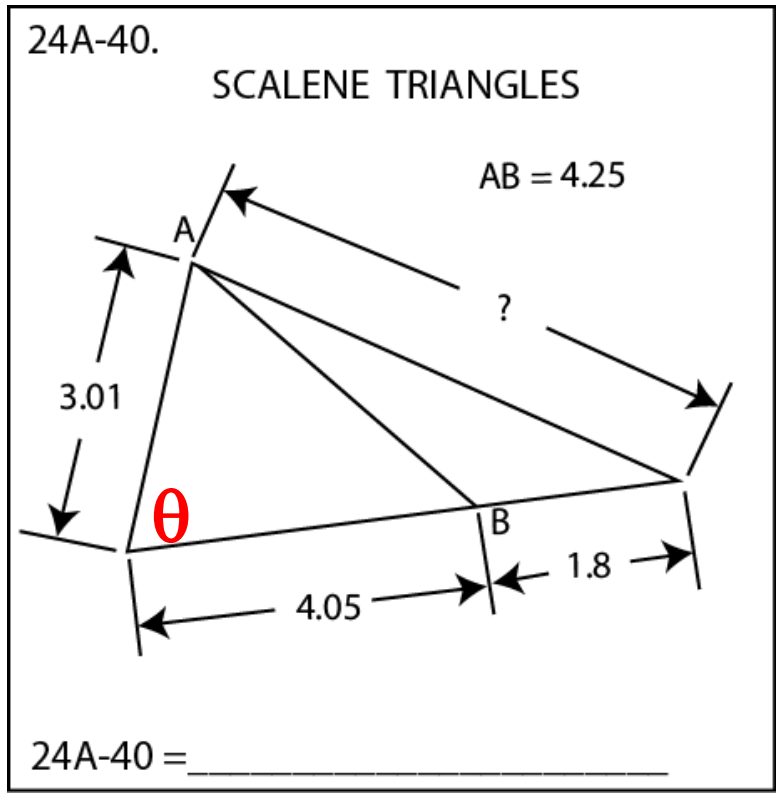
$c = \sqrt{7.15^2 + 8.42^2}$

$c = 11.046\ldots$

$r = (8.42 + 7.15 - 11.046\ldots)/2$

2.26

Page 4 Problems



Let θ = angle between 3.01 and 4.05

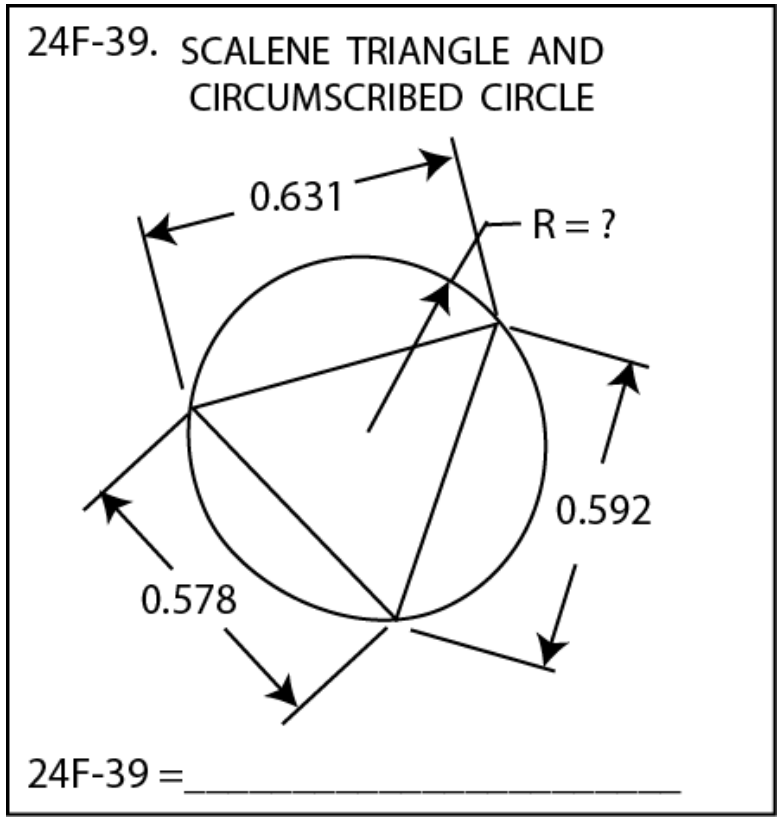
$$4.25^2 = 3.01^2 + 4.05^2 - 2(3.01)(4.05)\cos\theta$$

$$\theta = 72.3309\ldots^\circ$$

$$? = \sqrt{3.02^2 + (4.05+1.8)^2 - 2(3.01)(4.05+1.8)\cos 72.3309}$$

5.71

Page 4 Problems



$$R = abc / 4(\text{area})$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

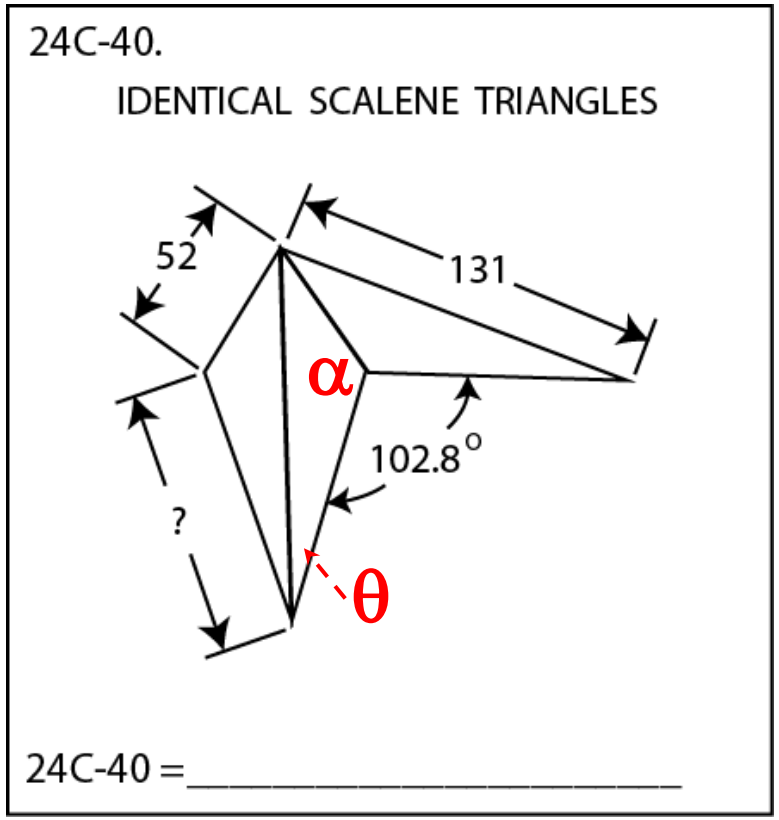
Where: $s = (0.631 + 0.578 + 0.592)/2$

$$s = .9005$$

$$R = \frac{(.631)(.578)(.592)}{4\sqrt{.9005(.9005-.631)(.9005-.578)(.9005-.592)}}$$

0.347

Page 4 Problems

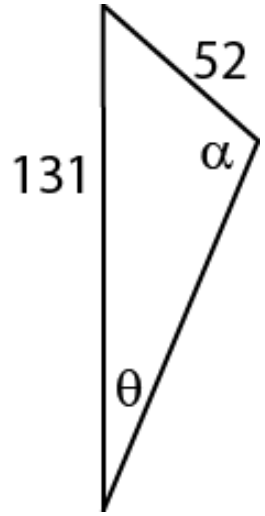


$\alpha = (360° - 102.8°) / 2$

$= 128.6°$

$\sin 128.6° / 131 = \sin \theta / 52$

$\theta = 18.0726°$



Last angle

$= 180° - 128.6° - 18.0726 = 33.3273°$

$\sin 33.3273° / ? = \sin 128.6° / 131$

92.1

Page 5 Problems

24B-46. Pizzas have the same thickness regardless of size. If a large 14-in pizza feeds 3 people, what sized pizza is needed to feed 100 people?

----------46=_____in

$14^2 / 3 = N^2 / 100$

80.8

For Scaling problems: see paged 58 – 62 in UIL *Calculator Applications Contest Manual (Revised 2023)*

Page 5 Problems

24H-47. A gift shop ran an unadvertised sale for one week. Their daily income from Monday through Thursday was $255, $410, $425, and $595, respectively. Estimate the Friday income.----------47=$_____

For Linear Regression problems: see paged 62 – 64 in UIL
Calculator Applications Contest Manual – Revised 2023

Monday thru Thursday = days 1, 2, 3 & 4

list 1:	1	2	3	4	5
list 2:	255	410	425	595	?

OR → (1, 255), (2, 410), (3, 425), (4, 595), (5, ?)

Use linear regression method **680.00**

Page 5 Problems

24D-46. If an 8-in tall book holds 12,800 12-point-font words, how many 14-point-font words does a similarly shaped 10-in tall book hold?

----------46=____words

Since we are comparing a volume of words, "holds" to an area of a words "12 – point-font", our scaling involves a ratio of volume to area calculation.

$8^3 /[(12^2)(12800)] = 10^3 /[(14^2)(w)]$

18400

Page 5 Problems

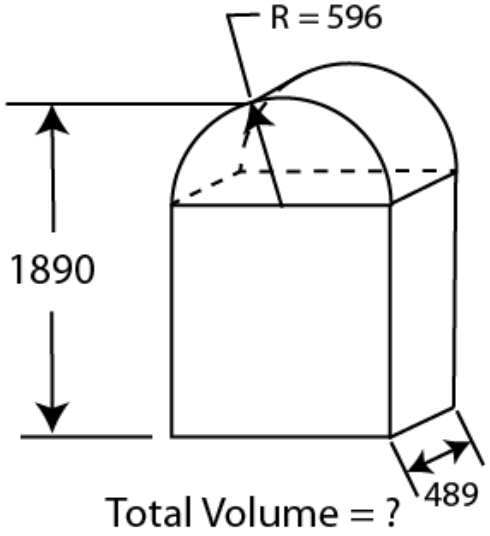
24G-47. A golfer wants to drive a ball at 50-yd increments starting at 50 yd. Her actual distances were 45 yd, 89 yd, 140 yd, and 185 yd. What distance should she aim for to drive the ball 250 yd?--------47=_____yd

(50, 45), (100, 89), (150, 140), (200, 185), (250, ?)

Use linear regression method

Page 5 Problems

24C-49.
HALF CYLINDER AND RECTANGULAR SOLID



24C-49 =_____

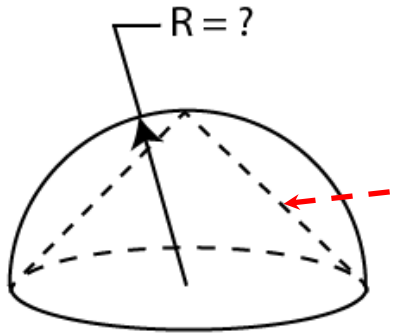
$$V = (489)(2x596)(1890-596) + \tfrac{1}{2}\pi(596)^2(489)$$

$L_{RECTSOLID}$ $W_{RECTSOLID}$ $H_{RECTSOLID}$ $V_{½CYLINDER}$

1.03×10^9

Page 5 Problems

24F-50.
HEMISPHERE WITH CONICAL CAVITY



Total Surface Area = 0.475

24F-50 =_____

$0.475 = 2\pi R^2 + \pi RL$

$SA_{HEMISPHERE}$ LSA_{CONE}

L = slant height of cone

$L = R\sqrt{2}$

$0.475 = 2\pi R^2 + \pi R(R\sqrt{2})$

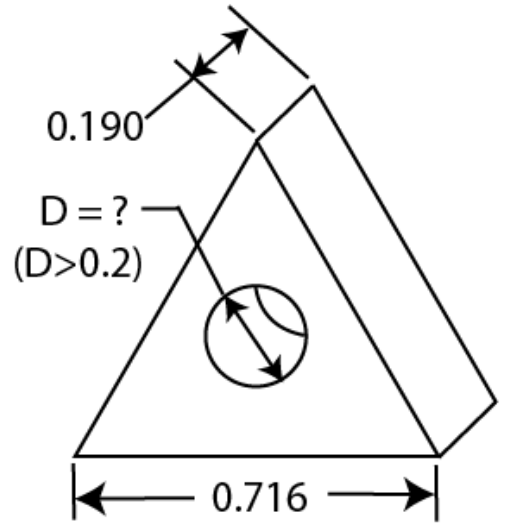
$0.475 = \pi R^2 (2 + \sqrt{2})$

0.210

Page 5 Problems

24A-50.

EQUILATERAL TRIANGLE PRISM WITH CYLINDRICAL CAVITY



Total Surface Area = 0.906

24A-50 =_____

$$0.906 = \frac{2(.716)^2\sqrt{3}}{4} + 3(.716)(.19) - 2\pi R^2 + 2\pi R(.19)$$

$A_{EQUITRIANGLE}$ LA_{RECT} $A_{CIRCLES}$ $LSA_{CYLINDER}$

R = .11612…

2R = **0.232**

Page 6 Problems

24H-56. What is the area between the curve $f(x) = -3x^2+9$ and the x axis?

----------56=_____

$f(x) = -3x^2 + 9$

To find values of x where function crosses the x-axis

Set: $-3x^2 + 9 = 0$ and solve. $\rightarrow x = \pm\sqrt{3}$

so area $= \int_{-\sqrt{3}}^{\sqrt{3}} -3x^2 + 9 \, dx$

20.8

Page 6 Problems

24E-57. Wanda eats a large bag of potato chips, but she eats at a rate that is proportional to the amount of remaining chips. If she wants to finish 90% of a full bag during a 1-hr TV program, what should her (positive) starting rate be?----------57=_____bags/hr

$A = 100e^{kt}$

$10 = 100e^{k} \rightarrow k = \ln(.1) / 100 \qquad k = -.0230258\ldots$

Now for $A = 100e^{kt}$, the rate of eating potato chips is $\frac{dA}{dt} = 100ke^{kt}$

So, at $t = 0$, $100(-.02302\ldots)e^{(-.02302)(0)} = -2.30$

$\therefore$ The positive rate is **2.30**

Page 6 Problems

24I-58. What is S_{23} if **S** = **TU**, **T** = $\begin{bmatrix} 1 & -5 & 13 \\ -5 & 17 & 4 \\ 13 & 4 & 11 \end{bmatrix}$ and **U** = $\begin{bmatrix} -6 & 15 & 18 \\ 15 & 2 & -7 \\ 18 & -7 & 3 \end{bmatrix}$ = ?

----------58=_____

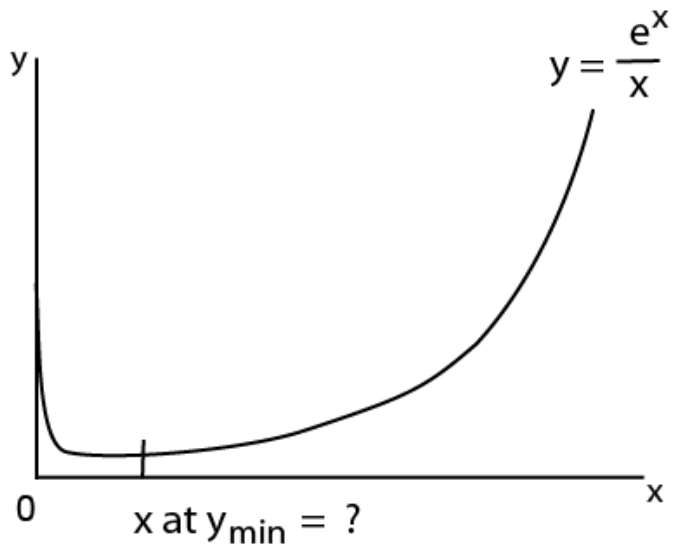
$$\begin{bmatrix} 1 & -5 & 13 \\ -5 & 17 & 4 \\ 13 & 4 & 11 \end{bmatrix} \mathbf{X} \begin{bmatrix} -6 & 15 & 18 \\ 15 & 2 & -7 \\ 18 & -7 & 3 \end{bmatrix} = \begin{bmatrix} 153 & -86 & 92 \\ 357 & -69 & -197 \\ 180 & 126 & 239 \end{bmatrix}$$

OR

S_{23} = (-5)(18) + (17)(-7) + (4)(3) **-197**

Page 6 Problems

24G-59.



24G-59 =_____

Trying to find "x" that gives **y** minimum.

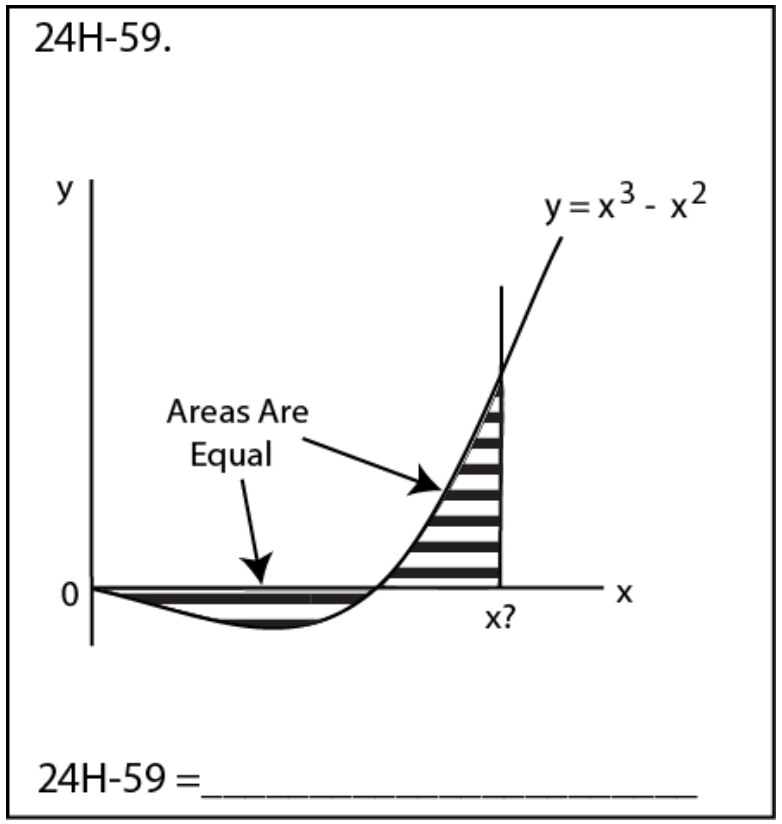
$$y = \frac{e^x}{x}$$

$$\frac{dy}{dx} = \frac{e^x(x-1)}{x^2}$$

$$\frac{e^x(x-1)}{x^2} = 0$$

1.00

Page 6 Problems



Function crosses x-axis at $x = 0$ and $x = 1$

$$\text{area} = -\int_0^1 x^3 - x^2 \, dx = \frac{1}{12}$$

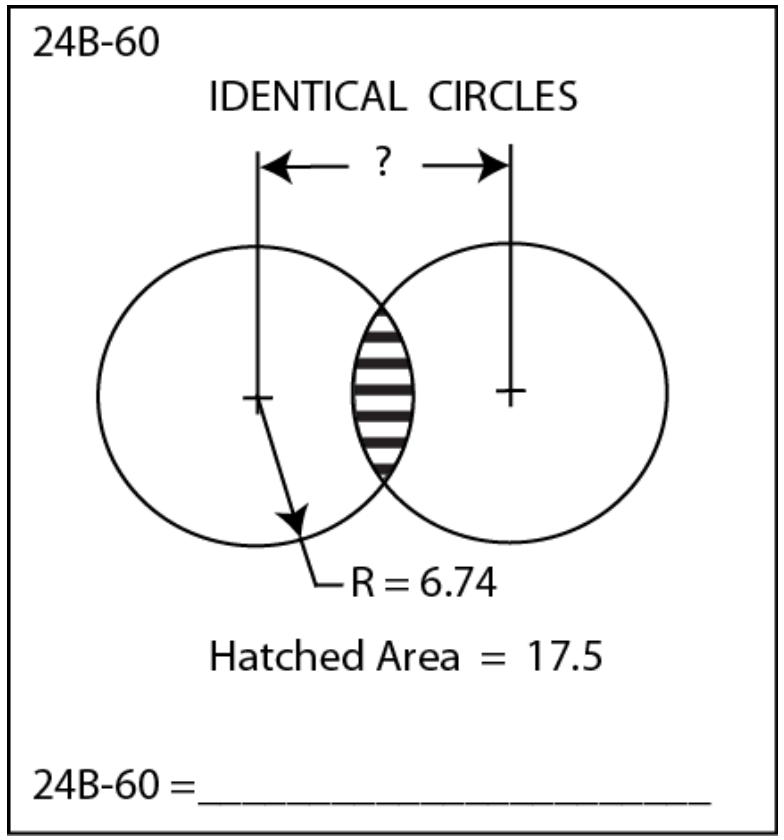
since areas are equal:

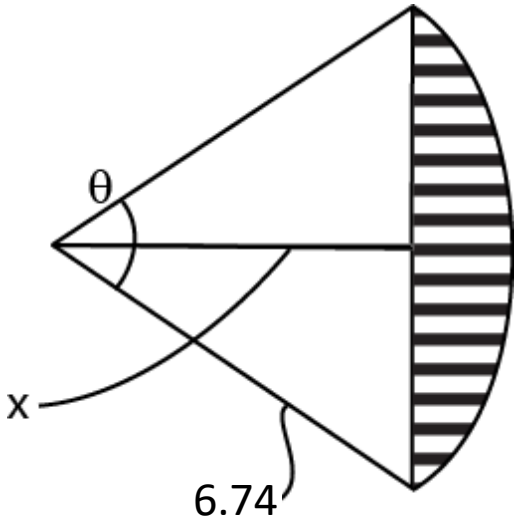
$$\int_0^c x^3 - x^2 \, dx = \frac{c^4}{4} - \frac{c^3}{3} + \frac{1}{12} = \frac{1}{12}$$

$$\frac{c^4}{4} - \frac{c^3}{3} = 0$$

1.33

Page 6 Problems





$\frac{1}{2}(6.74^2)(\theta - \sin\theta) = 17.5/2$
(calc. in rad mode)

$\theta = 1.3639\ldots$rads

$\theta/2 = .68195\ldots$ rads

$\cos(.68195\ldots) = x / 6.74$

$x = 5.232\ldots$

$2x =$ **0.361**

Page 7 Problems

24C-61. Quincy rows a canoe in still water at 2 mph. On a flowing river, he takes twice as long to row upstream as he does to row downstream. What is the river's flow rate?----------61=_____mph

Distance = Rate x Time

(2 mph + Current)(1) = (2mph – Current)(2)

2 + C = 4 – 2C

0.667

Page 7 Problems

24I-62. The odds of being hit by a meteorite in a lifetime is $1/(8.4 \times 10^8)$. What is this fraction raised to the -64,826th power?----------62=_____

$1 / (8.4 \times 10^8) = (8.4 \times 10^8)^{-1}$

$[(8.4 \times 10^8)^{-1}]^{-64826} = (8.4 \times 10^8)^{64826}$

$64826 \log (8.4 \times 10^8) = 578525.328998$

subtract $578525 \rightarrow .328998$

$10^{.328998} = 2.13$

2.13×10^{578525}

Page 7 Problems

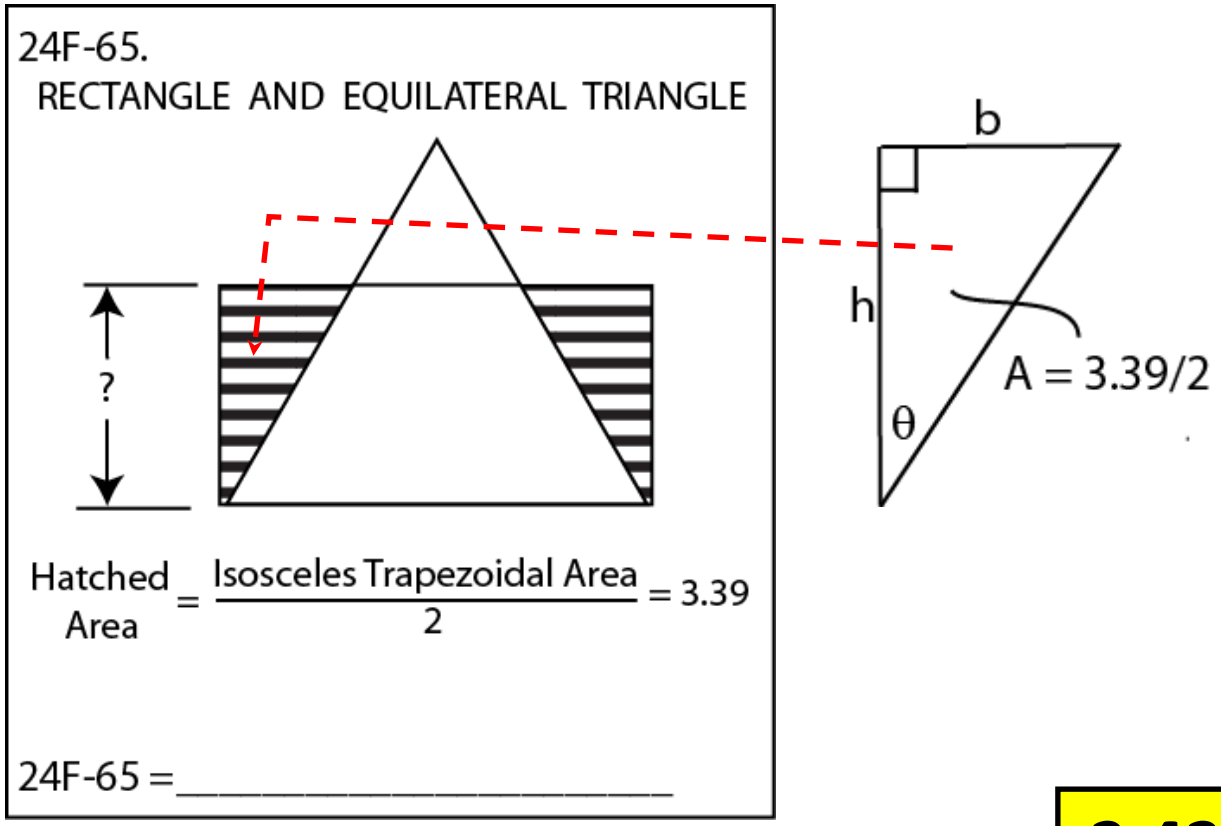
24C-63. Quarterback George wants to throw a pass 35 yd to a receiver. He wants the ball velocity v_0 to be as slow as possible. What is this velocity?----------63=_____mph

$$(35 \text{ yds})(3 \text{ ft/yd}) = v^2 \sin(2 \times 45)/32.174 \text{ ft/s}^2$$

$$v = 58.122\ldots\text{ft/sec} \times (15/22)$$

39.6

Page 7 Problems



$\theta = 90° - 60° = 30°$

$\tan 30° = b / h$

$b = h \tan 30°$

$(1/2)(b)(h) = 3.39/2$

$(1/2)(h \tan 30°)(h) = 3.39 / 2$

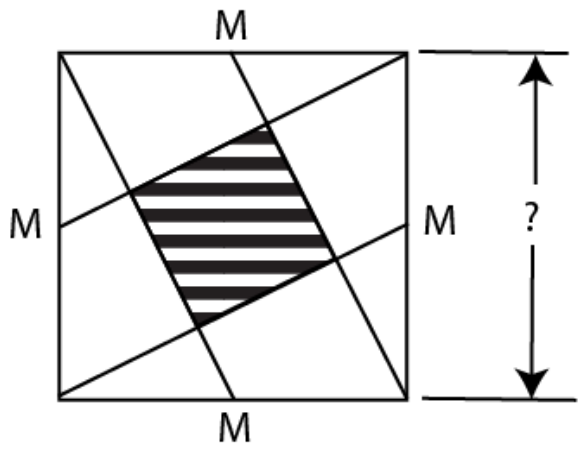
2.42

Page 7 Problems

24E-65.

SQUARE

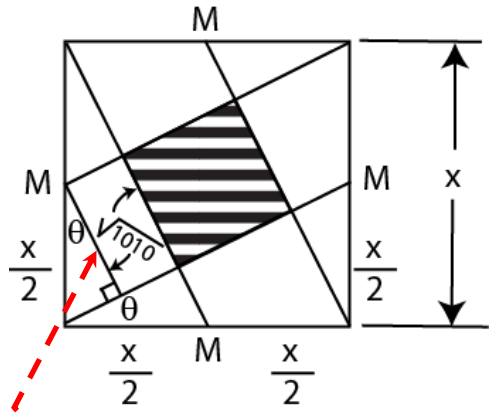
M = midpoints



Hatched Area = 1010

24E-65 =_____

$\angle\theta = \arctan\left(\frac{x/2}{x}\right) = 26.565°$



Side of shaded square $= \sqrt{1010}$

Draw a line parallel to one of the sides of the shaded square starting at one of the midpoints. Acute angle of created triangle is also θ and parallel side is also $\sqrt{1010}$.

$\therefore \cos\theta = \sqrt{1010}/(x/2) \rightarrow x = 2\sqrt{1010}/\cos 26.565°$

71.1