Motion Problems HS Calculator Applications Contest

23E-63. A punter kicks a football 45 yds with a hang time of 4.8 s. What was the football maximum elevation?------------------------63=___________ft

$$y = \frac{1}{2}(32.174)(2.4)^2 = 92.7$$

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Each year Dr. David Bourell writes at least nine UIL high school Calculator Application contests for competition. There are 21 stated problems and some of them involve motion of some sort. A discussion of the various motion or “rate” problems can be found in pages 29 – 42 of the UIL’s *Calculator Application Contest Manual* – revised 2023.

If you have not purchased a copy of the “UIL Calculator Applications Contest Manual – revised 2023” from the UIL’s online store by Dr. Bourell; **you need to do so!**

I will attempt to provide solutions for motion problems from tests for years 2020 – 2024.

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Appendix C - Acceleration and Trajectory Equations From Chapter 4Eii and 4Eiii

Acceleration Equations General.

a = constant acceleration, v = velocity, d = distance, t = time, v_o and d_o are associated values at which the acceleration initiates, and t_o is the time at which acceleration commences.

$$v = v_o + a(t - t_o) \quad \textbf{AND} \quad d = d_o + v_o\,(t - t_o) + \frac{1}{2}a(t - t_o)^2$$

Specific. When t_o, v_o and d_o are **all zero**, the standard equations simplify to the more common forms:

$$v = at \quad \textbf{AND} \quad d = \frac{1}{2}at^2 = \frac{1}{2}vt = \frac{1}{2}\frac{v^2}{a}$$

Appendix C - Acceleration and Trajectory Equations From Chapter 4Eii and 4Eiii

Trajectory Equations.

Initial and Final Elevations Equal. If v_o and θ are given, then the maximum horizontal "$d_{h(max)}$" and vertical "$d_{v(max)}$" distances are, respectively:

$$d_{h_{max}} = \frac{-V_o^2 \sin(2\theta)}{g} \quad \textbf{AND} \quad d_{v_{max}} = \frac{-V_o^2 \sin^2\theta}{2g}$$

$$d_{h_{max}} = \frac{-V_o^2 \sin(2\theta)}{g} \quad \text{and} \quad d_{v_{max}} = \frac{-V_o^2 \sin^2\theta}{2g} = \frac{d_v}{4\left[\left(\frac{d_h}{d_{h_{max}}}\right) - \left(\frac{d_h}{d_{h_{max}}}\right)^2\right]}$$

Where: $g = -32.174\ ft/sec^2$

Appendix C - Acceleration and Trajectory Equations From Chapter 4Eii and 4Eiii

Given d_{hmax} and d_{vmax}, the required initial velocity v_o and angle θ are given by:

$$v_o = \sqrt{\left(\frac{-g}{8d_{d_{max}}}\right)\left(d_{h_{max}}^2 + 16d_{v_{max}}^2\right)} = \sqrt{-d_{h_{max}} g \left(\frac{1+\tan^2\theta}{2\tan\theta}\right)} = \sqrt{\frac{d_h^2 g}{d_h \sin(2\theta) - 2d_v \cos^2\theta}}$$

$$\tan\theta = \frac{4d_{v_{max}}}{d_{h_{max}}} = \frac{d_v / d_h}{1 - \dfrac{d_h}{d_{max}}}$$

Where: $g = -32.174\ \text{ft/sec}^2$

Appendix C - Acceleration and Trajectory Equations From Chapter 4Eii and 4Eiii

The time-of-flight t_{of} is given by:

$$t_{of} = \frac{-2v_o \sin\theta}{g}$$

d_v given v_o, θ, and d_h:

$$d_v = d_h \tan\theta + \frac{g d_h^2}{2V_o^2 \cos^2\theta}$$

Where: $g = -32.174$ ft/sec^2

Appendix C - Acceleration and Trajectory Equations From Chapter 4Eii and 4Eiii

Initial and Final Elevations Unequal. The starting elevation is d_{vo}, the final elevation is d_{vf}. If t_o is set equal to zero, any horizontal distance d_h can be written as a function of time:

$$d_h = v_o t\cos\theta \quad \textbf{or} \rightarrow \quad t = \frac{d_h}{v_o\cos\theta} \qquad \textbf{and} \qquad t_{of} = \frac{d_{h_{max}}}{v_o\cos\theta}$$

Where:
$g = -32.174$ ft/sec^2

Any vertical distance dv can likewise be written as

$$d_v = d_{vo} + v_o t\sin\theta + \frac{1}{2}gt^2.$$

Setting this equal to the final vertical elevation and substituting the time relationship for d_h,

$$d_{vf} = d_{vo} + d_{hmax}\tan\theta + \frac{g d_{h_{max}}^2}{2v_o^2\cos^2\theta}$$

No Acceleration

23A-18. A supersonic transport flies at 1.7 times the speed of sound. The speed of sound is 660 mph. How long does it take to fly from Los Angeles to Tokyo, if the distance is 5451 mi?-------------------------------18=________hr

5451 mi / [(1.7)(660 mph)]

4.86

23A-26. Sam runs a mile in 6 min 48 s. What is his velocity?
---26=_______mph

1 mi / [(6 min)(60 s/min) + 48 s] x (3600 s/ 1 hr)

8.82

No Acceleration

23C-17. Josh wants to drive 7 hr daily on a road trip. What is his daily mileage, if his average speed is 58 mph?------------------------17=_________mi

(7 hr/dy)(58 mph – dy)

406

23B-27. Fingernails grow at 1.64 in/yr. If Emily trims away 2 mm of fingernail when she trims her nails, how often should she trim her mails? --27=_______weeks

(1.64 in / yr) (2.54 cm / 1 in) (10 mm / 1 cm) = 41.656 mm

(41.656 mm / 2 mm/trim) = 20.828 trims

(365.256 dys/yr / 7 dys/wk) (1 yr / 20.828 trims)

2.51

No Acceleration

22A-36. Tammy leaves home and drives 85 mi in 1 hr 35 min 54 s. She increases her velocity by 8.1% and returns home. What was her driving time on the return?---36=_________hr(SD)

1 hr 35 m 54 s = 5754 sec {4SD}

85 {2SD} / [85 {2SD} x 1.081{4SD} / 5754{4SD}] = 5322.849 {4SD}

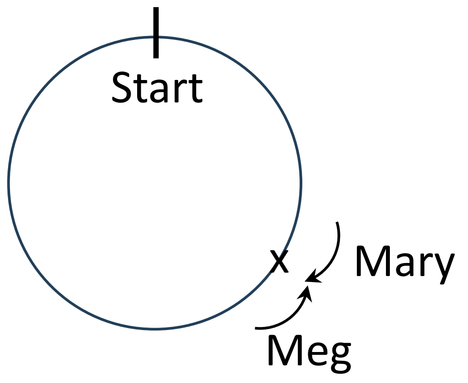
5322.849 {4SD} / 3600 s/hr

1.479

No Acceleration

22C-37. Meg runs a mile in 6 min 48 s. She starts running around a 440-yd track at the same time that Mary leaves her, running the opposite direction. They meet up after Meg ran 260 yd. What was Mary's speed?

---37=________mph



$(6 + 48/60 \text{ min}) / 1760 \text{ yds} = t_{Meg} / 260 \text{ yds}$

$t_{Meg} = 1.004545\ldots \text{ min}$ $\qquad$ $t_{Meg} = t_{Mary}$

$d_{Mary} = 440 \text{ yds} – 260 \text{ yds} = 180 \text{ yds}$

$v_{Mary} = [(180 \text{ yds})(3 \text{ ft}/1 \text{ yd})] \div [(1.004545\ldots \text{ min})(60 \text{ s/min})]$

$= 8.9592\ldots \text{ ft/s} \quad \text{x } (15/22)$

6.11

Acceleration

21C-27. A NASCAR racer accelerates from 0 to 60 mph in 3.4s. What is this acceleration?--27=_________ft/s^2

60 mph (22/15) = 88 ft/s

(88 ft/s – 0 ft/s)/3.4 s

25.9

21H-61. A ball is rolled on level ground at an initial velocity of 20 ft/s. It rolls to a stop 35 ft away. What was the deceleration, a negative number? ---61=_________ft/s^2

0 ft/s = $(20\text{ ft/s})^2 + 2a(35\text{ ft})$

-5.71

Acceleration

21I-38. An elevator has a traveling speed of 5 ft/s. It accelerates/decelerates at 4 ft/s^2. What is the percent error the time taken to travel 60 ft if one assumed the elevator accelerated/decelerated instantaneously?--38=____________%

$t = (5\ \text{ft/s} – 0\ \text{ft/s}) / 4\ \text{ft/s}^2 = 1.25\ \text{s}$

$y = ½ (4\ \text{ft/s}^2)(1.25\ \text{s})^2 = 3.125\ \text{ft}$

$[60\ \text{ft} – 2(3.125\ \text{ft})] / 5\ \text{ft/s} + 2(1.25\ \text{s}) = 13.25\ \text{s}$

$[(60\ \text{ft}/5\ \text{ft/s} / 13.25\ \text{s}) – 1]100\%$

-9.43

20B-38. A car makes one loop around the Indianapolis 500 track, 2.5 mi. The car accelerates from rest to 85 mph and then travels at that speed to the finish line. What was the acceleration if the car's time for the loop was 1.9 min?--38=__________ft/s^2

$v_o = 0$ mph 85 mph(22/15) = 124.66… ft/sec = at_1

$x_1 = \frac{1}{2}at_1^2$ = ½ (124.66… ft/s) t_1^2 x_2 = (124.66…ft/s)t_2

$x_1 + x_2$ = 2.5(5280 ft/mi) = ½ (124.66…ft/s)t_1 + (124.66…ft/s)t_2

$t_1 + t_2$ = (1.9 min)(60 s/min) = 114 s $\rightarrow t_2 = 114 - t_1$ t_1 = 16.235… s

a = 124.66…/16.235… $\boxed{\mathbf{7.68}}$

20G-38. Two cars race over a 5-mi course. Both start from rest and race at 70 mph. One car accelerates at 20 ft/s^2, while the other accelerates at 8 ft/s^2. What is the positive difference in their course times?--38=______s

Car #1

70 mph (22/15) = 102.666... ft/s

$v = v_0 + at$ 102.666...ft/s = 0 + 20 ft/s$^2 t_1$ t_1 = 5.1333... s

x_1 = ½ (20)(5.1333...)^2 = 263.5111... ft

(5 mi)(5280 ft/mi) – x_1 = 26,136.4888... ft

26,136.4888...ft / 102.666...ft/s = 254.5761... s

$t_{Total\#1}$ = 5.1333... s + 254.5761... s = 259.70952...s

20G-38. Two cars race over a 5-mi course. Both start from rest and race at 70 mph. One car accelerates at 20 ft/s^2, while the other accelerates at 8 ft/s^2. What is the positive difference in their course times?--38=______s

Car #2

70 mph (22/15) = 102.666... ft/s

$v = v_0 + at$ 102.666...ft/s = 0 + 8 $ft/s^2 t_2$ t_2 = 12.8333... s

$x_2 = \frac{1}{2}(8)(12.8333...)^2 = 658.777...$ ft

(5 mi)(5280 ft/mi) – x_2 = 25,741.222 ... ft

25,741.222 ...ft / 102.666...ft/s = 250.72619 ... s

$t_{Total\#2}$ = 12.8333... s + 250.72619 ... s = 263.55952...s

$t_{difference}$ = 263.55952...s – 259.70952...s $\boxed{3.85}$

Projectile Motion

20G-63. Gilda tosses a ball that reaches a maximum vertical distance of 37 ft above the release point at a distance 58 ft away. What was the release angle relative to the horizontal?------------------63=_________degrees

$$\tan\theta = \frac{4d_{v_{max}}}{d_{h_{max}}}$$

$$\tan\theta = \frac{4(37)}{2(58)}$$

NOTE: the horizontal distance to the maximum vertical distance is only ½ the h_{Max}.

51.9

Projectile Motion

21C-63. Sammie wants to fire a cartridge exactly 100 yd. When the rifle was inclined at 24° relative to horizontal, the cartridge fell 8 ft short. What is the new inclination angle near 24° to hit the target?
---63=_____degrees

100 yards = 300 ft

$(300 \text{ ft} - 8 \text{ ft}) = v^2 \sin(2\text{x}24°) / 32.174 \text{ ft/s}^2$

$v = 112.4364.... \text{ ft/s}$

$300 \text{ ft} = v^2 \sin(2\theta) / 32.174 \text{ ft/s}^2 \quad \rightarrow 2\theta = 49.774 \ldots$

24.9

Projectile Motion

22F-63. Ellie stands on a tall ladder. She tosses a screwdriver from an elevation of 20 ft with a release velocity of 45 fps at a 35° angle relative to the horizontal. At what horizontal distance from the ladder does the screwdriver hit the ground?-------------------------------------63=__________ft

Ground

$$0 \text{ ft} = 20\text{ft} + (45 \text{ ft/s} \sin 35°)t - \tfrac{1}{2}(32.174 \text{ ft/s}^2)t^2$$

$$t = 2.1758\ldots \text{ s}$$

$$d_h = (45 \text{ ft/s} \cos 35°)(2.1758\ldots \text{ s})$$

80.2

Projectile Motion

24H-63. Dirk throws a penny off the top of the Texas State Capitol Building with a velocity of 28 mph and a release angle of 64° relative to horizontal. It hits the ground in 5.63 s. What is the Capitol elevation?---63=________ft

28 mph (22/15) = 41.066… ft/s   time the penny is in the air = t = 5.63 s

$d_f = d_i + v_v(t) - \frac{1}{2} gt^2$ where d_f = ground = 0 **AND** d_i = height of capitol

v_v = vertical component of velocity

$v_v = (41.066\ldots)\sin 64° = 36.910\ldots$ ft/s

0 ft = d_i + (36.910… ft/s)(5.63 s) – ½ (32.174 ft/s²)(5.63 s)²

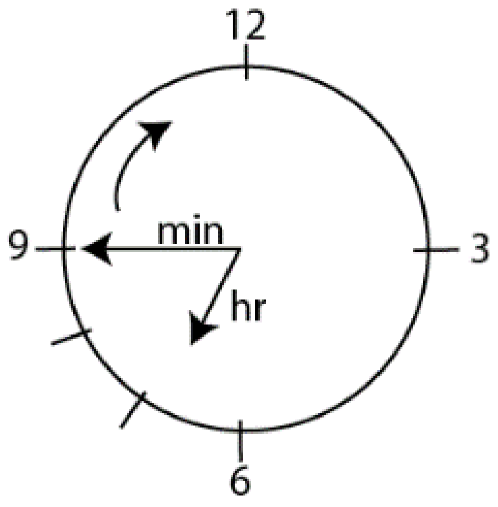
Clock Problems

24B-38. How many minutes after 6:45 do the minute and hour hands of a clock line up?--38=________min

*The fact that it takes an analog clock minute hand 1 hour to make 1 revolution and it takes the hour hand 12 hours to make the same 1 revolution with the clock hands moving in the same direction (clockwise) means their relative speed to each other is the factor **11/12**.*

time = distance / speed (relative)

Looking at the drawing to the right, we're going to measure the "distance" between the two clock hands in terms of minutes. From the 9th hour to the 6th hour mark is 45 minutes. From the 6th hour mark to the current location of the hour hand is ¾ of 5 minutes.



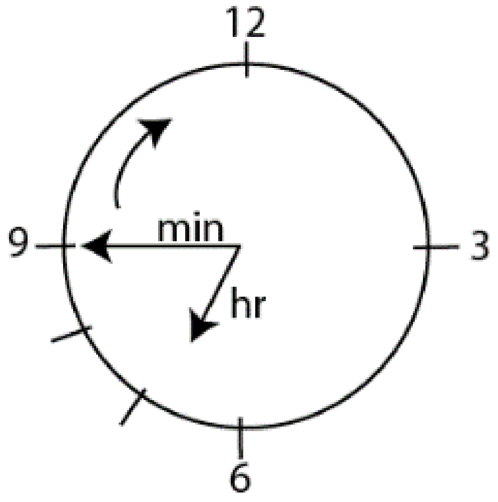
Clock Problems

24B-38. How many minutes after 6:45 do the minute and hour hands of a clock line up?--38=________min

So, the total time distance, $T = 45 + \frac{3}{4}(5) = 48.75$ min

$\therefore (11/12)(T) = 48.75$ min

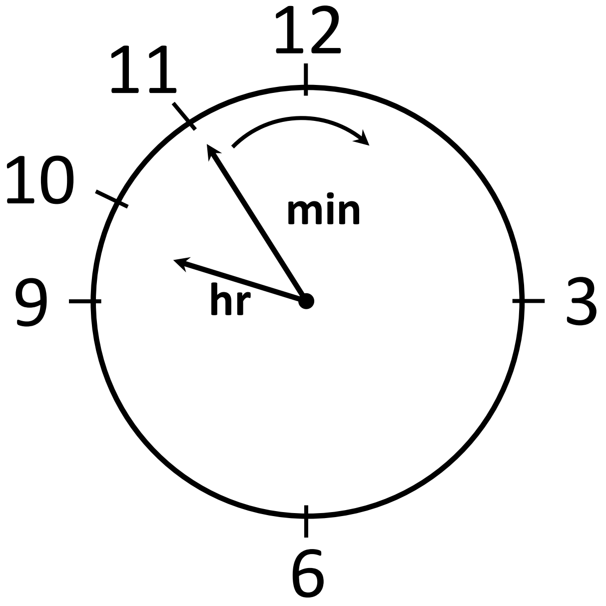
$\boxed{\mathbf{53.2}}$



Clock Problems

20C-61. How long after 9:55 do the hour and minute hands coincide?
--61=_____min

From the 11th hour to the 9th hour mark is 50 minutes. From the 9th hour mark to the current location of the hour hand is 55/60 (11/12) of 5 minutes.



So, the total time distance = 50 min +(55/60)(5 min)

= 54.58333…min

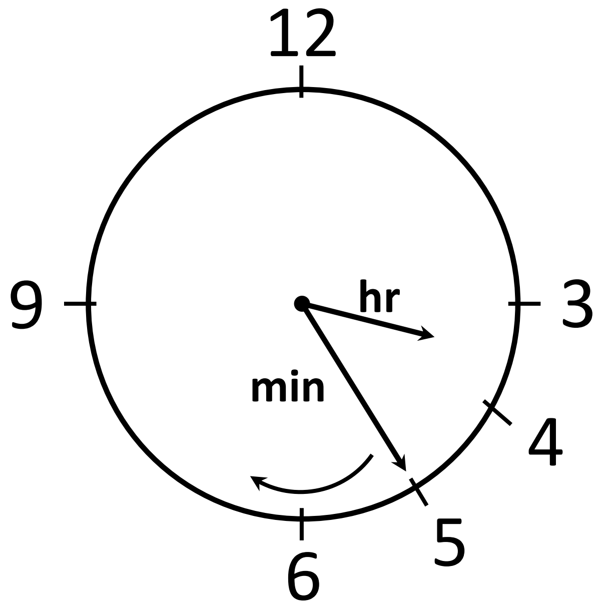
$\therefore$ (11/12)(T) = 54.58333 … min

59.5

Clock Problems

21G-38. How many minutes after 3:25 do the hour and minute hands of a clock first line up?-----------------------------------38=________min

From the 5^{th} hour to the 3^{rd} hour mark is 50 minutes. From the 3^{rd} hour mark to the current location of the hour hand is 25/60 (5/12) of 5 minutes.



So, the total time distance = 50 min +(25/60)(5 min)

= 52.08333…min

$\therefore$ (11/12)(T) = 52.08333 … min

$\boxed{56.8}$

20B-26. The Singapore Flyer is the second-largest Ferris Wheel in the world, 165 meters in diameter. If it takes 30 min to go around once, what is the car velocity?---26=_____m/s

Diameter = 165 m $\rightarrow$ radius = 82.5 m   t = 30 min

ω = 1 rev/30 min = 2π rads/[30 min $\times$ 60 s/min]

ω = π rads/900 s

v = $\omega \times$ r

$\therefore$ v = (π rads/900 s) $\times$ (82.5 m)

0.288

20E-26. A tire has a 28-in diameter. What is the tire rotational speed for a car driving 55 mph?---26=________RPM

Diameter = 28 in $\rightarrow$ radius = 14 in   v = 55 mph $\times$ 22/15 = 80.666 … ft/s

$v = \omega \times r \quad \rightarrow \quad \omega = v/r$

ω = 80.666 … ft/s/(14 in ÷ 12/ft)

ω = 80.666 … ft/s/(14 in ÷ 12/ft)

$$\omega = 69.1428\ldots \text{ rads/s} \times \left(\frac{60\,\text{s}/\text{min}}{2\pi\,\text{rads}/\text{Rev}} \right)$$

$\boxed{660}$

23D-28. A house table fan spins at 1300 RPM. If the blade tips are 15 in from the center of rotation, what total distance is traveled by a blade tip each hour?---28=__________mi

ω = 1300 RPM   r = 15 in   t = 1 hr   v = $\omega \times$ r

$$\omega = 1300\ \text{RPM} \times \left(\frac{60\ \text{s / min}}{2\pi\ \text{rads / Rev}} \right) = 136.135\ \ldots\ \text{rads/s}$$

v = (136.135 … rads/s) $\times$ [(15 in)(1 ft/12 in)] = 170.169 … ft/s

v = 170.169 … ft/s $\times$ (15/22) = 116.024 … mph

d = 116.024 … mph $\times$ 1 hr

116